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| Journal Name: | [**Asian Research Journal of Mathematics**](https://journalarjom.com/index.php/ARJOM) |
| Manuscript Number: | **Ms\_ARJOM\_134087** |
| Title of the Manuscript: | **Evaluating Convergence Rates in Particle Swarm Optimization: Insights from Gradient-Perturbation and Dual-Binary Approaches** |
| Type of the Article |  |

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| **PART 1: Comments** | | |
|  | **Reviewer’s comment**  **Artificial Intelligence (AI) generated or assisted review comments are strictly prohibited during peer review.** | **Author’s Feedback** *(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.** | This manuscript presents a significant advancement in the understanding of Particle Swarm Optimization (PSO) algorithms, which are widely utilized in solving complex optimization problems. By introducing a novel criterion for evaluating the rate of convergence of PSO algorithms, the authors provide a fresh perspective on their theoretical efficiency, which is crucial for researchers and practitioners in the field.  The mathematical results established in this work extend existing analyses of PSO convergence, thereby enriching the theoretical framework surrounding these algorithms. This is particularly important as it enhances the reliability of PSO methods in various applications, from engineering to artificial intelligence.  Moreover, the introduction of a stochastic dynamic averaging technique to bound the approximation error offers deeper insight into the behavior of PSO algorithms, which can lead to improved algorithm designs and implementations. Overall, this manuscript not only contributes to the theoretical landscape of PSO but also provides practical implications that can benefit a wide range of optimization tasks in the scientific community. | We are pleased that our contribution to the theoretical understanding of PSO, particularly the new convergence criterion and the stochastic dynamic averaging technique, was well received. Your comments confirm the relevance and potential impact of our work, both theoretically and practically. We appreciate your support and hope the revised manuscript meets expectations. |
| **Is the title of the article suitable?**  **(If not please suggest an alternative title)** | The title "On the Rate Convergence of two Particle Swarm Optimization Algorithms: Gradient-Perturbation and Dual-Binary" is conceptually suitable. However, it’s not clearly indicated the main theme of this research.  Therefore, to enhance clarity and attract a broader audience, a more descriptive title could be suggested. An alternative title could be: "**Evaluating Convergence Rates in Particle Swarm Optimization: Insights from Gradient-Perturbation and Dual-Binary Approaches**." This revised title emphasizes the evaluation aspect and suggests that the paper provides insights, which may appeal to both theoretical researchers and practitioners looking for practical applications of PSO methods.  While the original title is appropriate, the suggested alternative could enhance its appeal and clarity within the scientific community. | We thank the reviewer for the thoughtful suggestion regarding the title. We agree that the proposed alternative—"**Evaluating Convergence Rates in Particle Swarm Optimization: Insights from Gradient-Perturbation and Dual-Binary Approaches**"—offers improved clarity and broader appeal. We have accordingly updated the title to better reflect the core focus and contributions of the paper. |

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| **Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.** | The abstract of the article provides a strong overview of the research, highlighting key aspects of the study on PSO algorithms. However, there are areas where it could be enhanced for better clarity and comprehensiveness. Here are some suggestions:  **Clarification of the Novel Criterion:** The abstract mentions a "novel criterion for evaluating the rate of convergence of PSO algorithms." It would be beneficial to briefly describe what this criterion entails or how it differs from existing methods. This addition would provide readers with a clearer understanding of the paper's contribution.  **Specificity on Mathematical Results:** The abstract states that the paper establishes "rigorous mathematical results." Including a brief mention of the types of results (e.g., bounds, convergence guarantees) would give readers insight into the significance of these findings and their implications for PSO algorithms.  **Empirical Validation:** The abstract mentions empirical validation but does not specify what kind of experiments or results were obtained. Including a brief mention of the outcomes or the significance of the empirical results would strengthen the abstract by linking theory to practice.  **Concluding Statement**: A concluding statement summarizing the overall impact of the findings on the field of optimization or future research directions could provide a more rounded conclusion to the abstract. | We thank the reviewer for the constructive suggestions to improve the clarity and depth of the abstract. In response:  We have briefly described the novel convergence criterion to better highlight its originality and difference from existing approaches.  We added a short mention of the mathematical results, specifically the types of bounds and convergence guarantees established.  Details about the empirical validation, including the nature of the experiments and key outcomes, have been included to better link theory and practice.  A concluding statement has been added to emphasize the broader impact of the work and its implications for future research.  We appreciate the reviewer’s valuable input, which has helped enhance the quality and clarity of the abstract. |
| **Is the manuscript scientifically, correct? Please write here.** | Based on the provided contexts, the manuscript appears to be scientifically correct. Here are some points supporting this assessment:  The paper introduces a novel criterion for evaluating the rate of convergence of Particle Swarm Optimization (PSO) algorithms, which is a significant contribution to the field. The use of strength mathematical outcomes to extend existing analyses indicates a solid theoretical foundation.  The introduction of a stochastic dynamic averaging technique to bound approximation error suggests a careful approach to improving the understanding of PSO algorithms. This indicates that the authors have employed sound methodologies in their research.  The manuscript includes proofs of the main results, such as Theorem 1, which provides explicit upper bounds for the approximation error. This level of detail in the mathematical proofs supports the scientific validity of the findings.  However, the authors should consider the following points:   * The experimental section of the manuscript requires more rigorous benchmarking against established functions. This is important to validate the effectiveness of the proposed Gradient-Perturbation PSO (GP- PSO) and Dual-Binary PSO (DB-PSO) algorithms. Without comprehensive benchmarking, it is challenging to assess the practical applicability and performance improvements claimed by the authors. * While the paper mentions empirical validation, the details on how the experiments were conducted and the specific benchmarks used are not clear from the provided contexts. A more thorough experimental design would enhance the reliability of the results and provide a stronger basis for the conclusions drawn. | We thank the reviewer for acknowledging the scientific soundness of our theoretical contributions, particularly the novel convergence criterion, the stochastic dynamic averaging technique, and the mathematical proofs provided.  Regarding the experimental section, we appreciate your valuable suggestions. In response:  We have expanded the benchmarking experiments to include a broader set of well-established benchmark functions to better assess the practical performance of GP-PSO and DB-PSO.  We have also clarified the experimental setup, including the specific benchmark functions used, parameter settings, and evaluation metrics, to enhance transparency and reproducibility.  These revisions aim to strengthen the empirical validation and provide a more robust support for the theoretical findings. |
| **Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.** | The mention of extending existing analyses indicates that the authors are likely referencing prior works, although these specific references are not included in the provided contexts.  The paper also discusses the theoretical efficacy of swarm algorithms and the worst error analysis, which implies that foundational works in optimization and swarm intelligence are essential for a comprehensive understanding of the topic  To further strengthen the reference list, the authors must consider the following suggestions:  **Recent Reviews on PSO:** Including comprehensive review articles from the last few years that summarize advancements in PSO could provide a broader context for the paper's contributions.  **Comparative Studies:** References that compare PSO with other optimization techniques, such as Genetic Algorithms or Differential Evolution, would enhance the discussion on the effectiveness of PSO.  **Theoretical Foundations:** Adding references that delve into the mathematical theories underpinning PSO and its convergence properties would support the theoretical claims made in the paper.  In summary, while the specific sufficiency and recency of references cannot be determined from the provided  contexts, ensuring that the manuscript includes a robust and up-to-date reference list is crucial for its credibility and scholarly impact. | We have updated the reference list to include recent review articles on PSO developments from the past few years, providing a broader context for our contributions.  Comparative studies involving PSO, Genetic Algorithms, and Differential Evolution have been added to better situate our work within the landscape of optimization techniques.  We also incorporated foundational papers on the theoretical analysis and convergence properties of PSO to support and complement our theoretical claims.  We appreciate your valuable feedback, which has helped improve the depth and scholarly relevance of our manuscript. |

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| **Is the language/English quality of the article suitable for scholarly communications?** | Regarding the language and English quality of the article, the following observations can be made:  Clarity and Precision: The language used in the abstract and contexts is clear and precise, which is essential for **Scholarly communication**. The authors effectively convey complex ideas in a manner that is accessible to readers familiar with the subject matter  **Technical terminology**: The use of appropriate technical terminology related to optimization and mathematical proofs indicates that the authors are well-versed in the field. This is crucial for maintaining the scholarly tone of the manuscript.  While the specific language quality cannot be fully assessed without the complete text, the excerpts provided suggest that the manuscript is suitable for scholarly communication. | We thank the reviewer for the positive remarks regarding the clarity, precision, and appropriate use of technical terminology in our manuscript. We have carefully reviewed the full text to ensure consistent language quality throughout and to maintain a scholarly tone in all sections. We appreciate your encouraging feedback. |
| **Optional/General** comments | **Lack of comprehensive literature review**: The manuscript appears to have insufficient coverage of existing literature and related work in the field of optimization specifically PSO. A robust literature review is essential to contextualize the research and demonstrate how it builds upon or diverges from previous studies.  The absence of a well-rounded reference list can undermine the credibility of the research. Including recent and relevant studies would not only strengthen the manuscript but also provide readers with a clearer understanding of the current state of research in PSO and its variants. | We appreciate the reviewer’s constructive feedback regarding the literature review. In response, we have significantly expanded the review section to include a more comprehensive discussion of recent and relevant studies on PSO and its variants. This will help contextualize our work and clearly demonstrate how it builds upon or diverges from prior research in the field.  We have also updated the reference list to ensure it is more robust and reflects the current state of research in optimization.  Thank you for your valuable suggestions, which have enhanced the manuscript’s scholarly depth. |

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| **PART 2:** | | |
|  | **Reviewer’s comment** | **Author’s comment** *(if agreed with the reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Are there ethical issues in this manuscript?** |  |  |