Numerical Solution of Fractional order Epidemic Mathematical Model on Dengue Using FDTM and FADM

Abstract

In this article, we solve non-linear fractional order mathematical models on dengue using two alternative methods: FDTM (Fractional Differential Transform Method) and FADM (Fractional Adomian Decomposition Method). Additionally, for various fractional orders, the fractional model solution that is produced by FDTM is linked to the fractional model solution that is produced by FADM. Additionally, numerical and graphical analysis of the result is performed using Python software.

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Keywords: Fractional Derivative, Fractional Differential Equation, Fractional Differential Transform Method, Fractional Adomian Decomposition Method, Adomian Polynomials.

1 Introduction

Dengue fever is a potentially fatal disease caused by bacteria and viruses. This epidemic disease is linked to climate change, and more awareness of dengue fever is required. This hazardous disease has become a major health issues in several countries throughout the world in recent years. As a result, a dengue fever model is required. Various writers have examined and explored mathematical models of Dengue disease, which may be found in [1, 2, 20, 25]. In the last few decades, many researchers have been interested in the area of fractional calculus. This is due to the fact that Fractional calculus can more correctly explain the detained and transmission properties of diverse materials and processes than integer order models. Many researchers solved the linear and non-linear mathematical-biological fractional model on various disease by different methods like HPM, VIM and LADM [23,24,27] and so many variety of transform methods [14–18] to solved FDEs . Now, in this work, we'll look at a Dengue mathematical model of fractional order that goes like this:

$$\begin{cases} \mathbf{D}_\xi^\alpha y_1(\xi) = \mu - [\mu + \vartheta y_3(\xi)]y_1(\xi) \\ \mathbf{D}_\xi^\alpha y_2(\xi) = \vartheta y_1(\xi)y_3(\xi) - \eta y_2(\xi) \\ \mathbf{D}_\xi^\alpha y_3(\xi) = \sigma y_2(\xi) - [\sigma y_2(\xi) + \rho]y_3(\xi) \end{cases} \tag{1.1}$$

with given initial conditions, $y_1(0) = 0.9999400528$, $y_2(0) = 0.0000599472$ and $y_3(0) = 0.1$, where $0 < \alpha \leqslant 1$. Further the involve functions in the model obey $N(\xi) = y_1(\xi) + y_2(\xi) + y_3(\xi)$, where the total population is $N = 5071126$ in [20].

In this work, we will solve above stated model and find an approximate solution by using FDTM and FADM. In addition, Python software is used to analyse the dengue model's solution numerically and graphically.

2 Basic Ideas of the FDTM and FADM

In this part, we review several key conclusions from the FDTM and FADM, both of which are utilised to generate approximate analytical solutions for the system in this work (1.1).

2.1 Basic ideas of the FDTM

Let the fractional power series of an analytical and continuous function $\varphi(\zeta)$ in Riemann-Liouville sense is as follows [16]:

$$\varphi(\zeta) = \sum_{k=0}^{\infty} \Phi(k) \left(\zeta - \zeta_0\right)^{k/\alpha}, \tag{2.1.1}$$

where α and $\Phi(k)$ are the order of fraction and FDT of $\varphi(\zeta)$ respectively. Let fractional IVP, in terms of the Caputo sense are as follows.

$$\Phi(k) = \begin{cases} \text{If } k/\alpha \in Z^+, \frac{1}{(k/\alpha)!}\left[\frac{\mathrm{d}^{k/\alpha}\varphi(\zeta)}{\mathrm{d}\zeta^{k/\alpha}}\right]_{\zeta=\zeta_0} & \text{for } k = 0, 1, 2, \ldots, (q\alpha - 1) \\ \text{If } k/\alpha \notin Z^+ & 0, \end{cases} \tag{2.1.2}$$

where, q denotes the order of the fractional differential equation under consideration. Now we recall some important theorems of FDTM which can be used to find an analytical solution of the model of dengue.

Theorem 2.1 *[4, 16] If $\varphi(\zeta) = \psi(\zeta) \pm w(\zeta)$, then $\Phi(k) = \Psi(k) \pm \omega(k)$.*

Theorem 2.2 *[4, 16] If $\varphi(\zeta) = \psi(\zeta)w(\zeta)$, then $\Phi(k) = \sum_{l=0}^{k} \Psi(l)\omega(k-l)$.*

Theorem 2.3 *[4, 16] If $\varphi(\zeta) = \psi_1(\zeta)\psi_2(\zeta)\ldots\psi_{n-1}(\zeta)\psi_n(\zeta)$, then*

$$\Phi(k) = \sum_{k_{n-1}=0}^{k} \sum_{k_{n-2}=0}^{k_{n-1}} \cdots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} \Psi_1(k_1)\,\Psi_2(k_2-k_1)\ldots\Psi_{n-1}(k_{n-1}-k_{n-2})\,\Psi_n(k-k_{n-1})$$

Theorem 2.4 *[4, 16] I f $\varphi(\zeta) = (\zeta - \zeta_0)^r$, then $\Phi(k) = \delta(k - \alpha r)$ where,*

$$\delta(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Theorem 2.5 *[4, 16] If $\varphi(\zeta) = D^q_{\zeta_0}[\psi(\zeta)]$, then $\Phi(k) = \frac{\Gamma(q+1+k/\alpha)}{\Gamma(1+k/\alpha)}\Psi(k+\alpha q)$.*

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2.2 Basic ideas of the FADM

Now have a look at the fractional differential equations system [19]

$$D^{\alpha_i} y_i(\zeta) = N_i(\zeta, y_1, \ldots, y_n), \quad y_i^{(k)}(0) = c_k^i, \quad 0 \leqslant k \leqslant [\alpha_i] \tag{2.2.1}$$

where $1 \leqslant i \leqslant n$, and $\alpha_i \in \mathbb{R}^+$. After applying Caputo integral operator I^{α_i} found in [9] to the equation 2.2.1, we obtain,

$$y_i = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{\zeta^k}{k!} + I^{\alpha_i} N_i(\zeta, y_1, \ldots, y_n), \quad 1 \leqslant i \leqslant n. \tag{2.2.2}$$

We adopt ADM to solve the system 2.2.1. By representing the solution as an infinite series given by in the second phase of the decomposition approach, we can:

$$y_i = \sum_{m=0}^{\infty} y_{im}, \tag{2.2.3}$$

and

$$N_i(\zeta, y_1, \ldots, y_n) = \sum_{m=0}^{\infty} A_{im} \tag{2.2.4}$$

where A_{im} which are depend upon $y_{10}, \ldots, y_{1m}, y_{20}, \ldots, y_{2m}, \ldots, y_{n0}, \ldots, y_{nm}$. By using equations. 2.2.3 and 2.2.4, the equation 2.2.2, can be written as,

$$\sum_{m=0}^{\infty} y_{im} = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{\zeta^k}{k!} + I^{\alpha_i} \sum_{m=0}^{\infty} A_{im}(y_{10}, \ldots, y_{1m}, \ldots, y_{n0}, \ldots, y_{nm}), \quad 1 \leqslant i \leqslant n. \tag{2.2.5}$$

This can be expressed as

$$y_{i0}(\zeta) = \sum_{k=0}^{[\alpha_i]} c_k^i \frac{\zeta^k}{k!},$$

$$y_{i,m+1}(\zeta) = I^{\alpha_i} \left[\frac{1}{m!} \frac{\mathrm{d}^m}{\mathrm{d}\lambda^m} N_i \left(\zeta, \sum_{m=0}^{\infty} y_{1m} \lambda^m, \ldots, \sum_{m=0}^{\infty} y_{nm} \lambda^m \right) \right]_{\lambda=0}, \quad m = 0, 1, \ldots \tag{2.2.6}$$

The shortened series can be used to approximate the result as y_i.

$$\varphi_{ik} = \sum_{m=0}^{k-1} y_{im}, \quad \lim_{k \to \infty} \varphi_{ik} = y_i(\zeta).$$

We employ the define H. Jafari [19] , for convergence of this approach.

3 Solution of the Fractional Order Dengue Model

In this part, we'll use the FDTM and FADM algorithms to discover a solution of 2.2.1. Consider the system of non-linear fractional order dengue model.

$$\begin{cases} \mathbf{D}_\xi^\alpha y_1(\xi) = \mu - [\mu + \vartheta y_3(\xi)] y_1(t) \\ \mathbf{D}_\xi^\alpha y_2(\xi) = \vartheta y_1(\xi) y_3(\xi) - \eta y_2(\xi) \\ \mathbf{D}_\xi^\alpha y_3(\xi) = \sigma y_2(\xi) - [\sigma y_2(\xi) + \rho] y_3(\xi) \end{cases} \tag{3.1}$$

3.1 Solution of system 3.1 by FDTM

Now apply the FDTM and by using Theorems 2.1, 2.2, 2.4, and 2.5, then the equation 3.1 can be expressed as follows:

$$\begin{aligned}
Y_1(k+\alpha\theta) &= \frac{\Gamma(1+k/\theta)}{\Gamma(\alpha+1+k/\theta)}\left[\mu\delta(k)-\mu Y_1(k)+\sum_{l=0}^{k}\vartheta Y_3(l)Y_1(k-l)\right]\\
Y_2(k+\alpha\theta) &= \frac{\Gamma(1+k/\theta)}{\Gamma(\alpha+1+k/\theta)}\left[\sum_{l=0}^{k}\vartheta Y_1(l)Y_3(k-l)-\eta Y_2(k)\right]\\
Y_3(k+\alpha\theta) &= \frac{\Gamma(1+k/\theta)}{\Gamma(\alpha+1+k/\theta)}\left[\sigma Y_2(k)-\sum_{l=0}^{k}\sigma Y_2(l)Y_3(k-l)+\rho Y_3(k)\right],
\end{aligned} \tag{3.1.1}$$

where θ is the fraction of order α and $Y_1(k), Y_2(k)$ and $Y_3(k)$ are FDT of $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$ respectively. The approximate solution of given system 3.1 by using FDTM can be written as

$$y_i(\xi)=\sum_{k=0}^{\infty}Y_i(k)\xi^{k/\theta}. \quad i=1,2 \textit{ and } 3 \tag{3.1.2}$$

3.2 Solution of system 3.1, by FADM

By using the result of FADM, the system 3.1 is transferred as follows,

$$\begin{aligned}
N_1(\bar{y}) &= \mu-\mu y_{10}-\vartheta y_{30}y_{10}=\sum_{j=0}^{\infty}A_{1j},\\
N_2(\bar{y}) &= \vartheta y_{10}y_{30}-\eta y_{20}=\sum_{j=0}^{\infty}A_{2j},\\
N_3(\bar{y}) &= \sigma y_{20}-\sigma y_{20}y_{30}-\rho y_{30}=\sum_{j=0}^{\infty}A_{3j}.
\end{aligned}$$

Now we obtained the corresponding Adomian polynomials $A_{ij}, i=1,2,3$ and $j=0,1,\dots$

$$\begin{aligned}
A_{10} &= \mu-\mu y_{10}-\vartheta y_{30}y_{10},\\
A_{11} &= -\mu y_{11}-\vartheta[y_{31}y_{10}+y_{30}y_{11}],\\
A_{12} &= -\mu y_{12}-\vartheta[y_{32}y_{10}+y_{31}y_{11}+y_{30}y_{12}],\\
A_{13} &= -\mu y_{13}-\vartheta[y_{33}y_{10}+y_{32}y_{11}+y_{31}y_{12}+y_{30}y_{13}],\\
&\vdots
\end{aligned}$$

$$\begin{aligned}
A_{20} &= \vartheta y_{10}y_{30}-\eta y_{20},\\
A_{21} &= \vartheta[y_{11}y_{30}+y_{10}y_{31}]-\eta y_{21},\\
A_{22} &= \vartheta[y_{12}y_{30}+y_{11}y_{31}+y_{10}y_{32}]-\eta y_{22},\\
A_{23} &= \vartheta[y_{13}y_{30}+y_{12}y_{31}+y_{12}y_{31}+y_{10}y_{33}]-\eta y_{23},\\
&\vdots
\end{aligned}$$

$$\begin{aligned}
A_{30} &= \sigma y_{20} - \sigma y_{20} y_{30} - \rho y_{30}, \\
A_{31} &= \sigma y_{21} - \sigma[y_{21} y_{30} + y_{20} y_{31}] - \rho y_{31}, \\
A_{32} &= \sigma y_{22} - \sigma[y_{22} y_{30} + y_{21} y_{31} + y_{20} y_{32}] - \rho y_{32}, \\
A_{33} &= \sigma y_{23} - \sigma[y_{23} y_{30} + y_{22} y_{31} + y_{21} y_{32} + y_{20} y_{33}] - \rho y_{33}, \\
&\vdots
\end{aligned}$$

The few terms of Adomian decomposition series 2.2.6 are as follows

$$\begin{cases} y_{10} = 0, \\ y_{1,m+1} = I^{\alpha} A_{1m}, \end{cases} \quad \begin{cases} y_{20} = 1, \\ y_{2,m+1} = I^{\beta} A_{2m}, \end{cases} \quad \begin{cases} y_{30} = 1, \\ y_{3,m+1} = I^{\gamma} A_{3m}, \end{cases} \quad m = 0, 1, \ldots.$$

The approximate solution of given dengue model in series form is as follows.

$$\varphi_{ik} = \sum_{m=0}^{k-1} y_{im}, \quad \lim_{k \to \infty} \varphi_{ik} = y_i(\zeta) \quad i = 1, 2, 3.$$

4 Numerical Solution of Dengue Model

In terms of an infinite power series, the FDTM and FADM give an analytical approximation answer. However, evaluating this solution and obtaining numerical numbers from the infinite power series is necessary in practise. To complete this work, the series is truncated as a result, and the practical approach is used.
For sake of convenience we use following parameter in given model

PARAMETER	NOMENCLATURE
y_1	Represents those who are susceptible to infection.
y_2	Represents people who have been infected with the Dengue virus.
y_3	Represents people who have recovered from the Dengue virus.
η	It denotes the rate of infection.
ϑ	It denotes the average number of bites received by an infected mosquito.
μ	It represents the susceptible host's death rate.
σ	It denotes the rate at which an infection can be recovered.
ρ	It is the number of people who have died as a result of being infected by a mosquito.

We utilise the following numerical values for parameters from [20] to derive the approximate series solution of the above model.

4.1 Numerical solution by FDTM

Let $y_1(0) = 0.9999400528$, $y_2(0) = 0.0000599472$ and $y_3(0) = 0.1$. Now by using equation 2.1.2, the initial conditions can be written as follows
$Y_1(k) = 0, Y_2(k) = 0, Y_3(k) = 0$ for $k = 1, 2, \ldots, \alpha\theta - 1$
$Y_1(0) = 0.9999400528$, $Y_2(0) = 0.0000599472$ and $Y_3(0) = 0.1$
$Y_1(k), Y_2(k)$ and $Y_3(k)$ are calculated using equation 2.1.1 upto 4 iterations.

Case I: If we taking the values of $\alpha = 1$ and $\theta = 1$ which gives the system of ordinary differential equations and values of other parameters are $\eta = 0.333, \rho = 0.02941, \mu = 0.0045, \vartheta = 0.0006, \sigma = 0.375$. We have Using the equation 2.1.1 the fourth approximations are calculated for $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$, respectively.
In view of these values, we have

$$\begin{aligned} y_1(\xi) &= 0.9999400528 - 5.97266407680003e-05\xi + 1.0123545593952428e-03\xi^2 \\ &\quad - 1.1520763240136989e-08\xi^3 + \cdots \\ y_2(t) &= 0.0000599472 + 4.0033985567999995e-05\xi - 7.543628214739242e-06\xi^2 \\ &\quad + 8.473449632371e-07\xi^3 + \cdots \\ y_3(\xi) &= 0.1 - 0.00292076782\xi + 4.973845558007358e-05\xi^2 - 1.6086908718666682e-06\xi^3 \\ &\quad + \cdots \end{aligned}$$

Case II: If $\alpha = 0.5$ and $\theta = 2$, and values of other parameters are $\eta = 0.333, \rho = 0.02941, \mu = 0.0045, \vartheta = 0.0006, \sigma = 0.375$. We have Using the equation 2.1.1 the fourth approximations are calculated for $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$, respectively as follows;

$$\begin{aligned} y_1(\xi) &= 0.9999400528 - 6.739429716320906e-05\xi^{1/2} + 2.0247091187904856e-03\xi \\ &\quad - 5.1941816648317056e-08\xi^{3/2} + \cdots \\ y_2(\xi) &= 0.0000599472 + 4.51735152907336e-05\xi^{1/2} - 1.5087256429478483e-05\xi \\ &\quad + 3.824448275171701e-06\xi^{3/2} + \cdots \\ y_3(\xi) &= 0.1 - 0.003295733560010976\xi^{1/2} + 9.947691116014716e-05\xi - 5.990968624421687e-06\xi^{3/2} \\ &\quad + \cdots \end{aligned}$$

Case III: If $\alpha = 0.4$ and $\theta = 5$ and values of other parameters are $\eta = 0.333, \rho = 0.02941, \mu = 0.0045, \vartheta = 0.0006, \sigma = 0.375$. We have Using the equation 2.1.1 the fourth approximations are calculated for $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$, respectively as follows

$$\begin{aligned} y_1(\xi) &= 0.9999400528 - 6.7315537487015e-05\xi^{2/5} + 2.3449325427454744e-03\xi^{3/5} \\ &\quad - 7.616053066560027e-08\xi^{4/5} + \cdots \\ y_2(\xi) &= 0.0000599472 + 4.512072371063551e-05\xi^{2/5} - 1.7473422850668217e-05\xi^{3/5} \\ &\quad + 4.429298427619285e-06\xi^{4/5} + \cdots \\ y_3(\xi) &= 0.1 - 0.003291882033710764\xi^{2/5} + 0.00011520995488506399\xi^{3/5} - 8.795086986354861e-06\xi^{4/5} \\ &\quad + \cdots \end{aligned}$$

4.2 Numerical solution by FADM

Case I: If we taking the values of $\alpha = 1$ and $\theta = 1$ which gives the ordinary system and values of other parameters are $\eta = 0.333, \rho = 0.02941, \mu = 0.0045, \vartheta = 0.0006, \sigma = 0.375$. By using the equation 2.2.3 and above parameters the fourth approximations for $y_1(\xi), y_2(v)$ and $y_3(\xi)$ are as follows.
In view of these values, we have

$$y_1(\xi) = 0.9999400528 - 5.97266407680003e - 05\xi + 0.02687874429\xi^2 + \cdots$$

$$y_2(\xi) = 0.0000599472 + 4.0033985567999995e - 05\xi - 1.755939235e - 06\frac{\xi^2}{2} + \cdots$$

$$y_3(\xi) = 0.1 - 0.00292076782\xi + 0.00009947691117\frac{\xi^2}{2} + \cdots$$

Case II: If $\alpha = 0.5$ and $\theta = 2$ and values of other parameters are $\eta = 0.333, \rho = 0.02941, \mu = 0.0045, \vartheta = 0.0006, \sigma = 0.375$. We have Using the equation 2.1.1 the fourth approximations are calculated for $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$, respectively are as follows

$$y_1(\xi) = 0.9999400528 - 5.97266407680003e - 05\frac{\xi^{1/2}}{\Gamma(1.5)} + 0.02687874429\xi + \cdots$$

$$y_2(\xi) = 0.0000599472 + 4.0033985567999995e - 05\frac{\xi^{1/2}}{\Gamma(1.5)} - 1.755939235e - 06\xi + \cdots$$

$$y_3(\xi) = 0.1 - 0.00292076782\frac{\xi^{1/2}}{\Gamma(1.5)} + 0.00009947691117\xi + \cdots$$

Case III: If $\alpha = 0.4$ and $\theta = 5$ and using equation 2.2.3, we obtain one more solution $y_1(\xi), y_2(\xi)$ and $y_3(\xi)$ are as follows;

$$y_1(\xi) = 0.9999400528 - 5.97266407680003e - 05\frac{\xi^{2/5}}{\Gamma(1.4)} + 0.02687874429\frac{\xi^{4/5}}{\Gamma(1.8)} + \cdots$$

$$y_2(\xi) = 0.0000599472 + 4.0033985567999995e - 05\frac{\xi^{2/5}}{\Gamma(1.4)} - 1.755939235e - 06\frac{\xi^{4/5}}{\Gamma(1.8)} + \cdots$$

$$y_3(\xi) = 0.1 - 0.00292076782\frac{\xi^{2/5}}{\Gamma(1.4)} + 0.00009947691117\frac{\xi^{4/5}}{\Gamma(1.8)} + \cdots$$

5 Numerical Interpretation and Discussion

The numerical graphs of various categories relating to fractional order derivatives are presented in this section. In Fig 1, Fig 2, and Fig 3, we plot the different components of approximate solutions of a given non-linear fractional dengue model for different order α, by using FDTM and FADM.From Figs. $1a - 3a$ and $1b - 3b$, when the Dengue virus spreads in a healthy society, the number of people who are infected and the number of people who are susceptible to infection increases. It is also observed that, if there is no cure of disease in the infected population then the regained population then continues to decline. This fluctuation of infection can be observed in Figs. $1a - 3a$ and $1b - 3b$, at different fractional orders which are calculated by FDTM and FADM. From these figures it is observed that, if the order of the fractional derivative is lower order, then the fluctuation rate of different categories is rapid. On the other hand, if the order of the fractional derivative is higher then the fluctuation rate which is calculated by using both methods FDTM and FADM of different categories becomes slower. From these graphical interpretations, we can deduce how a disease spreads in a society that is assumed to be open to infection. When a virus strikes, the receptive rate decreases as infected people get more afflicted. Some people will recover if they receive adequate treatment or vaccination, and their population will grow. As a result, this graphical analysis shows us the most effective methods of illness transmission and recovery.

(a) In a model 3.1, the Dengue virus is transmitted to a susceptible population by FDTM

(b) In a model 3.1, the Dengue virus is transmitted to a susceptible population by FADM.

Figure 1: FDTM and FADM were used to compare geometrical interpretations of Dengue virus transmission in a susceptible population in model 3.1 at various α values.

(a) Dengue virus transmission of infected population in model 3.1 via FDTM.

(b) Dengue virus transmission of infected population in model 3.1 via FADM.

Figure 2: FDTM and FADM were used to compare geometrical interpretations of Dengue virus transmission in an infected human population in model 3.1 at different values of α.

(a) Dengue virus transmission in a recovered society in model 3.1 using FDTM.

(b) Dengue virus transmission in a recovered society in model 3.1 using FADM.

Figure 3: FDTM and FADM were used to compare geometrical interpretations of Dengue virus transmission in a recovered human population in model 3.1 at various α values.

6 Conclusion

This work uses FDTM and FADM to solve a non-linear fractional order mathematical model on dengue. Furthermore, the fractional model solution produced by FDTM is associated with the solution of the same model estimated by FADM for different fractional orders. Two alternative strategies FADM and FDTM have been used to solve and analyse a non-linear fractional order mathematical model of dengue fever. In terms of infinite series for various orders and by specifying fixed components with various time intervals, an approximate solution to the specified model is established. The Python programme is used to analyse the solution numerically and visually. The outcomes of these numerical simulations have been positive.

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