***Original Research Article***

**Second Order Slope Rotatable Designs of Second Type utilizing Balanced Incomplete Block Designs**

**ABSTRACT**

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| --- |
| Kim [12] proposed second order rotatable designs (SORD) of second type utilizing Central composite design (CCD), wherein the two digits  denotes the position of star points. Kim and Ko [13] introduced second order slope rotatable designs (SOSRD) of second type utilizing CCD, wherein the two digits  denotes the position of star points. In this study, we propose SOSRD of second type utilizing balanced incomplete block designs (BIBD) for 3≤v≤16 (v-stands for number of factors). In specific cases, the recommended procedure results in fewer design points than SOSRD of second type acquired utilizing CCD of Kim and Ko [13], Ravikumar and Victorbabu [16], Pairwise balanced designs (PBD) of Ravikumar and Victorbabu [18], Symmetrical unequal block arrangements (SUBA) with two unequal block sizes of Ravikumar and Victorbabu [20] and BIBD with unequal block sizes of Ravikumar and Victorbabu [21]. The variance of the estimated response of first order partial derivative of SOSRD of second type using BIBD for different factors 3≤v≤16 are also obtained. |

***Keywords:*** *Response surface designs, Second order slope rotatable designs, Second order slope rotatable designs of second type.*

1. **INTRODUCTION**

Response surfaces are set of statistical and mathematical models used to analyze the issues when several explanatory variables have an impact on a response variable. Box and Hunter [1] suggested the rotatability property for exploring response surface models and developed rotatable central composite designs (CCD). Das and Narasimham [7] studied second order rotatable designs (SORD) utilizing balanced incomplete block designs (BIBD). Das [6] developed SORD through BIBD with blocks of unequal sizes. Kim [12] introduced extended CCD with the axial points indicated by two numbers Victorbabu and Vasundharadevi [31] developed modified quadratic response surface models utilizing BIBD. Jyostna and Victorbabu [10] constructed modified rotatability measure for a quadratic polynomial utilizing BIBD. Jyostna et al. [11] studied modified rotatability measure for quadratic polynomial models utilizing CCD. Rohit et al. [22] studied response surface designs with four and six equi-spaced levels. Chiranjeevi et al. [5] developed SORD of second type utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] constructed SORD of second type utilizing BIBD, pairwise balanced designs (PBD), symmetrical unequal block arrangements (SUBA) with two unequal block sizes respectively. Ravikumar and Victorbabu [19] developed SORD of second type utilizing BIBD with unequal block sizes.

Slope rotatable central composite designs (SRCCD) was first developed by Hader and Park [9]. Victorbabu and Narasimham [26, 27, 28] studied second order slope rotatable designs (SOSRD) utilizing Incomplete block designs with unequal block sizes, BIBD and pair of incomplete block designs respectively. Victorbabu [23, 24] introduced modified SOSRD utilizing CCD and BIBD respectively. A review was proposed by Victorbabu [25] on SOSRD. Das et al. [8] developed modified robust SOSRD. Park and Kim [14] developed a measure of slope rotatability for second order response surface experimental designs. Victorbabu and Surekha [29, 30] constructed SOSRD measure utilizing CCD and BIBD. Specifically, Kim and Ko [13] introduced SOSRD of second type using CCD for the factors 2≤v≤5 by taking  (‘na’ denotes the number of replications of axial points), where in the two numbers  represent the positions of the star points. Further, Ravikumar and Victorbabu [16] extended the work of Kim and Ko [13] and developed SOSRD of second type utilizing CCD for 6≤v≤17 by taking Ravikumar and Victorbabu [17, 18, 20, 21] studied SRCCD of second type for 2≤v≤17 with , SOSRD of second type utilizing PBD, SUBA with two unequal block sizes and BIBD with unequal block sizes respectively.

In this study, we suggest SOSRD of second type utilizing BIBD. It is found that in some cases this suggested method leads to SOSRD of second type with fewer number of design points compared to SOSRD of second type utilizing CCD of Kim and Ko [13], Ravikumar and Victorbabu [16], PBD of Ravikumar and Victorbabu [18], SUBA with two unequal block sizes of Ravikumar and Victorbabu [20] and BIBD with unequal block sizes of Ravikumar and Victorbabu [21]. Specifically for v=7 factors the new design needs 85 design points whereas corresponding SOSRD of second type utilizing CCD need 93 design points and BIBD with unequal block sizes need 127 design points. Similarly for v=9 factors the new design needs 133 design points whereas corresponding SOSRD of second type utilizing CCD need 165 design points, PBD need 213 design points, SUBA with two unequal block sizes need 181 design points and BIBD with unequal block sizes need 199 design points, For v=13 factors the new design needs 261 design points whereas corresponding SOSRD of second type utilizing CCD need 309 design points, PBD need 565 design points. Further, for v=3, 4, 5 and 11 factors these new design needs 25, 41, 61 and 221 design points where as corresponding SOSRD of second type using BIBD with unequal block sizes need 31, 49, 71 and 353 design points respectively. For v=8 and 12 factors these new design needs 145 and 401 design points where as corresponding SOSRD of second type using PBD need 273 and 561 design points respectively.

1. **STIPULATIONS FOR SECOND ORDER SLOPE ROTATABLE DESIGNS**

A general quadratic polynomial model  for fitting

 (2.1)

In which indicates the sth factor level in the experiment’s wth run (w=1,2,…,N) and ’s are uncorrelated random errors having a mean ‘0’ and variation of . D is then referred to as SOSRD if  with regard to every explanatory variable  is a of the point from the origin (center) of the design.

The general circumstances for SOSRD are given below(cf. [1, 9, 27]).

All moments of odd order are vanish. In simple terms when minimum of one odd power X equals to zero. i.e;

1. 
2. (i) 

(ii) 

1.  (2.2)

where c, and are constants.

The variances and covariances of the estimated parameters are











 (2.3)

and the remaining covariances vanish.

An inspection of shows an essential condition for the existence of a non singular second order slope rotatable design is

1.  (Non singularity condition) (2.4)

For the second order model

 (2.5)

 (2.6)

The criteria for R.H.S of (2.6) to be a alone (for slope rotatability) is

 (cf. [9]) (2.7)

Simplifying (2.7) using (2.3), we get

1.  (cf. [27]) (2.8)

Therefore A, B, C of (2.2), (2.4) and (2.8) suggest a set of conditions for slope rotatability in any general quadratic model. (cf. [9, 27]).

On simplification of equation (2.6) we get  (2.9)

1. **NEW SOSRD OF SECOND TYPE UTILIZING BALANCED INCOMPLETE BLOCK DESIGNS**

Kim [12] developed second type of rotatable CCD, in which the two digits  are used to denote the star points for 2≤v≤8. Chiranjeevi et al. [5] introduced SORD of second type utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] studied SORD of second type utilizing BIBD, PBD and SUBA with two unequal block sizes respectively. Specifically, Kim and Ko [13] constructed SOSRD of second type using CCD 2≤v≤5 by taking. Ravikumar and Victorbabu [16] extended the results of Kim and Ko [13] and developed SOSRD of second type utilizing CCD for by taking. Ravikumar and Victorbabu [17, 18, 20, 21] studied SRCCD of second type for 2≤v≤17 with , SOSRD of second type utilizing PBD, SUBA with two unequal block sizes and BIBD with unequal block sizes respectively.

Definition of BIBD: An arrangement of v treatments in m blocks each block containing k (<v) treatments, if

(i) every treatments occurs at most once in each block,

(ii) every treatment occurs in exactly r blocks and

(iii) every pair of treatments occurs together in blocks.

The parameters of BIBD are denoted by (v, m, r, k, λ). The design plan of SOSRD of second type using BIBD in which the position of the axial points are indicated by two numbers  and  (≥≥0). Let (v, m, r, k, λ) denote a BIBD, 2t(k) indicate the fractional replicate of  in ±1 levels, wherein no interaction is confounded with fewer than five factors.  represents the central points.

Let  indicates the design points produced from transposed incidence matrix of BIBD. Let are the design points produced from BIBD by multiplication (cf. [15]). We employ the extra set of points like are two axial points sets. Here  indicate the 4v axial points produced from and point sets. Let  indicate the union of design points produced from various point sets and central points represented by. Following the methods of [13], [16] we suggest a method of construction on SOSRD of second type utilizing BIBD as shown in below theorem.

**THEOREM (3.1):**

The design points,  will result in a v-dimensional SOSRD of second type utilizing BIBD in  design points with the following biquadratic equation

 (3.1)

The design exists if the equation mentioned above (3.1) contains at least one positive real root.

c can be obtained from (3.2)

(Evolution of (3.1) is explained in (3.4) below)

**Proof:** Regarding the design points produced from SOSRD of second type utilizing BIBD, simple symmetry stipulations A, B and C of equation (2.2) are true. Since condition A of equation (2.2) is obviously true, condition B and C of (2.2) are also true as follows.

1. (i) 

(ii) 

1.  (3.3)

From B(ii) and C of (3.3), we have .

The result of simplifying equation (2.8) by substituting c, and is

 (3.4)

The biquadratic equation shown in (3.1) is obtained by simplifying (3.4).

**EXAMPLE:**

Construction on SOSRD of second type for 7-factors with the help of a BIBD (v=7, b=7, r=3, k=3, λ=1). The design points,

 will result in a SOSRD of second type utilizing BIBD in N=85 design points with .

Here B and C of equation (3.3) are

1. (i) 

(ii) 

1.  (3.5)

From B (ii) and C of equation (3.5), we have 

Substitute c,  and  in equation (2.8) and after simplifying, we obtain the biquadratic equation that follows.

(3.6)

Substitute  in the above equation (3.6) and on simplification, we get

 (3.7)

(This can be alternatively written directly from equation (3.1) of theorem 3.1)

Equation (3.7) has only one positive real root .

From equation (3.5), we get c=7.0986, =0.3982, and =0.0941.

The non-singularity condition D of (2.4) is also satisfied. i.e; 0.5935>0.5344.

In the context of 7-factors, this new approach contains 85 design points, while the corresponding SOSRD of second type obtained utilizing CCD of Ravikumar and Victorbabu [16] need 93 design points and BIBD with unequal block sizes of Ravikumar and Victorbabu [21] need 127 design points. (please see Appendix for other factors)

Discussion:

In this study, we suggest SOSRD of second type utilizing BIBD. It is found that in some cases this suggested method leads to SOSRD of second type with fewer number of design points compared to SOSRD of second type utilizing CCD of Kim and Ko [12], Ravikumar and Victorbabu [15], PBD of Ravikumar and Victorbabu [17], SUBA with two unequal block sizes of Ravikumar and Victorbabu [19] and BIBD with unequal block sizes of Ravikumar and Victorbabu [20]. Specifically for v=7 factors the new design needs 85 design points whereas corresponding SOSRD of second type utilizing CCD need 93 design points and BIBD with unequal block sizes need 127 design points. Similarly for v=9 factors the new design needs 133 design points whereas corresponding SOSRD of second type utilizing CCD need 165 design points, PBD need 213 design points, SUBA with two unequal block sizes need 181 design points and BIBD with unequal block sizes need 199 design points, For v=13 factors the new design needs 261 design points whereas corresponding SOSRD of second type utilizing CCD need 309 design points, PBD need 565 design points. Further, for v=3, 4, 5 and 11 factors these new design needs 25, 41, 61 and 221 design points where as corresponding SOSRD of second type using BIBD with unequal block sizes need 31, 49, 71 and 353 design points respectively. For v=8 and 12 factors these new design needs 145 and 401 design points where as corresponding SOSRD of second type using PBD need 273 and 561 design points respectively.

The work is in progress to study some new methods of SOSRD of second type to reduce the number of design points.

1. **CONCLUSION**

In this paper, SOSRD of second type utilizing BIBD is suggested. It is observed that sometimes the proposed method may generate designs with fewer design points than SOSRD of second type acquired utilizing CCD, PBD, SUBA with two unequal block sizes and BIBD with unequal block sizes.

The Appendix table 1 gives the appropriate slope rotatability values of the parameter with  for designs using a BIBD and star points and for different number of central points for 3≤v≤16.

Table 2 gives the variance of estimated response of the first order partial derivative of SOSRD of second type utilizing BIBD for different factors for 3≤v≤16.

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**APPENDIX**

**Table 1:** Values of a2 for SOSRD of second type using BIBD for 3≤v≤16 with 

[These are SOSRDs of second type with design points ]

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| (3, 3, 2, 2, 1) | | |  | (4, 6, 3, 2, 1) | | |  | (4, 4, 3, 3, 2) | | |  | (5, 10, 4, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 24 | 1.9337 |  | 0 | 40 | 1.8605 |  | 0 | 48 | 2.5563 |  | 0 | 60 | 1.7995 |
| 1 | 25 | 1.8837 |  | 1 | 41 | 1.8185 |  | 1 | 49 | 2.5338 |  | 1 | 61 | 1.7601 |
| 5 | 29 | 1.7346 |  | 5 | 45 | 1.6723 |  | 5 | 53 | 2.4609 |  | 5 | 65 | 1.6044 |
| 10 | 34 | 1.6281 |  | 10 | 50 | 1.5417 |  | 10 | 58 | 2.3989 |  | 10 | 70 | 1.4284 |
| 15 | 39 | 1.5685 |  | 15 | 55 | 1.4583 |  | 15 | 63 | 2.3574 |  | 15 | 75 | 1.2937 |
| 20 | 44 | 1.5319 |  | 20 | 60 | 1.4042 |  | 20 | 68 | 2.3285 |  | 20 | 80 | 1.1982 |

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| (5, 5, 4, 4, 3) | | |  | (5, 10, 6, 3, 3) | | |  | (6, 15, 5, 2, 1) | | |  | (6, 10, 5, 3, 2) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 100 | 3.2614 |  | 0 | 100 | 2.7125 |  | 0 | 84 | 1.7459 |  | 0 | 104 | 2.4233 |
| 1 | 101 | 3.2512 |  | 1 | 101 | 2.6997 |  | 1 | 85 | 1.7064 |  | 1 | 105 | 2.4061 |
| 5 | 105 | 3.2150 |  | 5 | 105 | 2.6540 |  | 5 | 89 | 1.5278 |  | 5 | 109 | 2.3447 |
| 10 | 110 | 3.1787 |  | 10 | 110 | 2.6075 |  | 10 | 94 | 1.2130 |  | 10 | 114 | 2.2834 |
| 15 | 115 | 3.1502 |  | 15 | 115 | 2.5705 |  |  | 15 | 119 | 2.2368 |
| 20 | 120 | 3.1273 |  | 20 | 120 | 2.5410 |  |  | 20 | 124 | 2.2012 |

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| (6, 6, 5, 5, 4) | | |  | (6, 15, 10, 4, 6) | | |  | (7, 7, 3, 3, 1) | | |  | (7, 7, 4, 4, 2) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 120 | 3.4887 |  | 0 | 264 | 3.7401 |  | 0 | 84 | 2.0134 |  | 0 | 140 | 2.8608 |
| 1 | 121 | 3.4794 |  | 1 | 265 | 3.7360 |  | 1 | 85 | 1.9808 |  | 1 | 141 | 2.8492 |
| 5 | 125 | 3.4467 |  | 5 | 269 | 3.7203 |  | 5 | 89 | 1.8718 |  | 5 | 145 | 2.8079 |
| 10 | 130 | 3.4140 |  | 10 | 274 | 3.7024 |  | 10 | 94 | 1.7823 |  | 10 | 150 | 2.7667 |
| 15 | 135 | 3.3883 |  | 15 | 279 | 3.6862 |  | 15 | 99 | 1.7273 |  | 15 | 155 | 2.7345 |
| 20 | 140 | 3.3678 |  | 20 | 284 | 3.6716 |  | 20 | 104 | 1.6914 |  | 20 | 160 | 2.7091 |

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| (7, 7, 6, 6, 5) | | |  | (7, 21, 6, 2, 1) | | |  | (8, 14, 7, 4, 3) | | |  | (8, 28, 7, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 252 | 4.3220 |  | 0 | 112 | 1.6971 |  | 0 | 256 | 3.0740 |  | 0 | 144 | 1.6515 |
| 1 | 253 | 4.3179 |  | 1 | 113 | 1.6557 |  | 1 | 257 | 3.0667 |  | 1 | 145 | 1.6061 |
| 5 | 257 | 4.3021 |  | 5 | 117 | 1.4252 |  | 5 | 261 | 3.0393 |  |
| 10 | 262 | 4.2847 |  |  | 10 | 266 | 3.0091 |  |
| 15 | 267 | 4.2693 |  |  | 15 | 271 | 2.9829 |  |
| 20 | 272 | 4.2557 |  |  | 20 | 276 | 2.9603 |  |

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| (9, 12, 4, 3, 1) | | |  | (9, 18, 8, 4, 3) | | |  | (9, 12, 8, 6,) | | |  | (9, 36, 8, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 132 | 1.8826 |  | 0 | 324 | 3.0132 |  | 0 | 420 | 4.2153 |  | 0 | 180 | 1.6073 |
| 1 | 133 | 1.8426 |  | 1 | 325 | 3.0059 |  | 1 | 421 | 4.2120 |  | 1 | 181 | 1.5550 |
| 5 | 137 | 1.6957 |  | 5 | 329 | 2.9785 |  | 5 | 425 | 4.1995 |  |
| 10 | 142 | 1.5663 |  | 10 | 334 | 2.9478 |  | 10 | 430 | 4.1853 |  |
| 15 | 147 | 1.4860 |  | 15 | 339 | 2.9206 |  | 15 | 435 | 4.1724 |  |
| 20 | 152 | 1.4329 |  | 20 | 344 | 2.8967 |  | 20 | 440 | 4.1607 |  |

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| (9, 18, 10, 5, 5) | | |  | (10, 15, 6, 4, 2) | | |  | (10, 45, 9, 2, 1) | | |  | (10, 18, 9, 5, 4) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 324 | 3.5057 |  | 0 | 280 | 2.6929 |  | 0 | 220 | 1.5622 |  | 0 | 328 | 3.2766 |
| 1 | 325 | 3.5003 |  | 1 | 281 | 2.6818 |  | 1 | 221 | 1.4958 |  | 1 | 329 | 3.2700 |
| 5 | 329 | 3.4803 |  | 5 | 285 | 2.6404 |  |  | 5 | 333 | 3.2458 |
| 10 | 334 | 3.4580 |  | 10 | 290 | 2.5955 |  |  | 10 | 338 | 3.2193 |
| 15 | 339 | 3.4386 |  | 15 | 295 | 2.5578 |  |  | 15 | 343 | 3.1966 |
| 20 | 344 | 3.4216 |  | 20 | 300 | 2.5263 |  |  | 20 | 348 | 3.1770 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (11, 11, 6, 6, 3) | | |  | (11, 11, 5, 5, 2) | | |  | (11, 55, 15, 3, 3) | | |  | (12, 33, 11, 4, 3) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 396 | 3.6433 |  | 0 | 220 | 2.7203 |  | 0 | 484 | 2.3280 |  | 0 | 576 | 2.8310 |
| 1 | 397 | 3.6382 |  | 1 | 221 | 2.7083 |  | 1 | 485 | 2.3142 |  | 1 | 577 | 2.8228 |
| 5 | 401 | 3.6195 |  | 5 | 225 | 2.6671 |  | 5 | 489 | 2.2581 |  | 5 | 581 | 2.7909 |
| 10 | 406 | 3.5988 |  | 10 | 230 | 2.6277 |  | 10 | 494 | 2.1850 |  | 10 | 586 | 2.7531 |
| 15 | 411 | 3.5809 |  | 15 | 235 | 2.5984 |  | 15 | 499 | 2.1090 |  | 15 | 591 | 2.7180 |
| 20 | 416 | 3.5652 |  | 20 | 240 | 2.5759 |  | 20 | 504 | 2.0306 |  | 20 | 596 | 2.6856 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (12, 22, 11, 6, 5) | | |  | (12, 44, 11, 3, 2) | | |  | (13, 13, 4, 4, 1) | | |  | (13, 26, 12, 6, 5) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 752 | 4.0536 |  | 0 | 400 | 2.0734 |  | 0 | 260 | 2.0363 |  | 0 | 884 | 3.9982 |
| 1 | 753 | 4.0507 |  | 1 | 401 | 2.0490 |  | 1 | 261 | 2.0090 |  | 1 | 885 | 3.9954 |
| 5 | 757 | 4.0397 |  | 5 | 405 | 1.9397 |  | 5 | 265 | 1.9183 |  | 5 | 889 | 3.9844 |
| 10 | 762 | 4.0268 |  | 10 | 410 | 1.7626 |  | 10 | 270 | 1.8397 |  | 10 | 894 | 3.9714 |
| 15 | 767 | 4.0149 |  | 15 | 415 | 1.4584 |  | 15 | 275 | 1.7863 |  | 15 | 899 | 3.9594 |
| 20 | 772 | 4.0039 |  |  | 20 | 280 | 1.7479 |  | 20 | 904 | 3.9481 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (13, 39, 15, 5, 5) | | | | |  | | (15, 15, 7, 7, 3) | | |  | (16, 16, 6, 6, 2) | | |  | (16, 48, 15, 5, 4) | | |
| n0 | N | | a2 | |  | | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 676 | | 3.2818 | |  | | 0 | 1020 | 4.1822 |  | 0 | 576 | 3.0685 |  | 0 | 832 | 2.9474 |
| 1 | 677 | | 3.2764 | |  | | 1 | 1021 | 4.1796 |  | 1 | 577 | 3.0608 |  | 1 | 833 | 2.9391 |
| 5 | 681 | | 3.2555 | |  | | 5 | 1025 | 4.1693 |  | 5 | 581 | 3.0328 |  | 5 | 837 | 2.9071 |
| 10 | 686 | | 3.2315 | |  | | 10 | 1030 | 4.1573 |  | 10 | 586 | 3.0026 |  | 10 | 842 | 2.8697 |
| 15 | 691 | | 3.2095 | |  | | 15 | 1035 | 4.1462 |  | 15 | 591 | 2.9771 |  | 15 | 847 | 2.8354 |
| 20 | 696 | | 3.1894 | |  | | 20 | 1040 | 4.1359 |  | 20 | 596 | 2.9556 |  | 20 | 852 | 2.8041 |
| (16, 20, 5, 4, 1) | | | | | |
| n0 | | N | | a2 | |
| 0 | | 384 | | 1.3470 | |
| 1 | | 385 | | 1.2766 | |
| 5 | | 389 | | 1.0208 | |

**Table 2:** The Variance of the estimated response of SOSRD of Second type using BIBD for different factors 3≤v≤16

[These are SOSRDs of second type with design points]

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (v,b,r,k,λ) | N |  |  |  | c |  |  |  |  |  |  |  |
| (3,3,2,2,1) | 25 | 1.8837 | 0.1600 | 0.6839 | 8.7953 | 0.2132 | 0.0585 | 0.2500 | 0.0625 | -0.0844 | 0.0305 | (0.058488+0.250000d2)σ2 |
| (4,6,3,2,1) | 41 | 1.8185 | 0.0976 | 0.5028 | 8.9679 | 0.1817 | 0.0485 | 0.2499 | 0.0624 | -0.0782 | 0.0310 | (0.048509+0.249900d2)σ2 |
| (5,10,4,2,1) | 61 | 1.7601 | 0.0656 | 0.3967 | 9.2987 | 0.1672 | 0.0413 | 0.2499 | 0.0624 | -0.0760 | 0.0323 | (0.041325+0.249900d2)σ2 |
| (6,15,5,2,1) | 85 | 1.7064 | 0.0471 | 0.3273 | 9.7393 | 0.1587 | 0.0359 | 0.2498 | 0.0619 | -0.0748 | 0.0333 | (0.035945+0.249781d2)σ2 |
| (7,7,3,3,1) | 85 | 1.9808 | 0.0941 | 0.3982 | 7.0986 | 0.1182 | 0.0295 | 0.1250 | 0.0313 | -0.0382 | 0.0108 | (0.029545+0.125023d2)σ2 |
| (8,28,7,2,1) | 145 | 1.6061 | 0.0276 | 0.2425 | 10.8271 | 0.1573 | 0.0284 | 0.2499 | 0.0622 | -0.0775 | 0.0368 | (0.028439+0.249875d2)σ2 |
| (9,12,4,3,1) | 133 | 1.8426 | 0.0602 | 0.3067 | 7.1318 | 0.1064 | 0.0245 | 0.1249 | 0.0311 | -0.0358 | 0.0107 | (0.024515+0.124897d2)σ2 |
| (10,45,9,2,1) | 221 | 1.4958 | 0.0181 | 0.1922 | 12.0030 | 0.1601 | 0.0235 | 0.2500 | 0.0626 | -0.0809 | 0.0398 | (0.023543+0.249994d2)σ2 |
| (11,11,5,5,2) | 221 | 2.7083 | 0.1448 | 0.4374 | 5.9250 | 0.0518 | 0.0103 | 0.0312 | 0.0078 | -0.0098 | 0.0015 | (0.010345+0.031249d2)σ2 |
| (12,44,11,3,2) | 401 | 2.0490 | 0.0399 | 0.2454 | 7.8283 | 0.0655 | 0.0102 | 0.0625 | 0.0157 | -0.0214 | 0.0065 | (0.010162+0.062500d2)σ2 |
| (13,13,4,4,1) | 261 | 2.0090 | 0.0613 | 0.2838 | 6.1612 | 0.0644 | 0.0135 | 0.0625 | 0.0156 | -0.0164 | 0.0035 | (0.013500+0.062503d2)σ2 |
| (15,15,7,7,3) | 1021 | 4.1796 | 0.1881 | 0.4750 | 5.5226 | 0.0125 | 0.0021 | 0.0052 | 0.0013 | -0.0016 | 0.0002 | (0.002062+0.005207d2)σ2 |
| (16,20,5,4,1) | 385 | 1.2766 | 0.0416 | 0.2215 | 5.4570 | 0.0335 | 0.0117 | 0.0624 | 0.0156 | -0.0087 | 0.0016 | (0.011726+0.062438d2)σ2 |

\*For all designs we have taken .