**Estimating Activity Duration in PERT Using the Burr XII Distribution**

**Abstract**

The problem of parameter elicitation has restricted project managers to the use of few known and sometimes unrealistic input activity distributions in Project Evaluation and Review Technique (PERT). In this paper, the problem of parameter elicitation in PERT with Burr XII activity duration distribution is addressed using classical quantile estimation approach. The proposed method uses any two extreme quantiles alongside the median quantile to develop a procedure for the estimation of activity duration in PERT-Burr XII. Some desirable properties that qualify Burr XII distribution as input duration distribution are also outlined. An adequacy check was demonstrated by comparing the absolute percent errors of the mean estimates from the proposed method with the PERT-Beta and other approximations from the PERT-Beta family. The proposed method presents a relatively simple and accurate procedure for parameter elicitation in PERT. The procedure is therefore recommended for wide application in PERT especially where heavily tailed and right skewed data are involved.

**Keywords and Phrases**: PERT, Stochastic Activity Network, Burr XII Distribution, Project Completion Time, Quantile Estimation.

**2020 Mathematical Subject Classification:** 90B15

**1. INTRODUCTION**

Project Evaluation and Review Technique (PERT) is considered as a simple but reliable tool for the estimation of project completion time [1]. This has led to the wide application of PERT in both construction and management engineering [2-6]. Prominent among the PERT scheme is the PERT-Beta which assumes that the project activity time follows the generalized beta distribution with mean and variance approximated as and ; where , and are optimistic, most likely and pessimistic times respectively. These approximations place some restrictions to a particular distribution in the beta family that is near normal [8]. Earlier works have shown that some real activity duration data deviate from the PERT-Beta postulation [9-11]. Consequently, there has been growing interest in further modification and/or refinement of PERT-Beta to accommodate greater likelihood of extreme events by introducing other activity duration distributions [12-18]. The introduction of simulation in PERT analysis and also the gains in computational ease through the use of computer packages have encouraged the use of other distributional forms to improve on the robustness of PERT procedure [19-24]. For a review of activity duration distributions in PERT interested readers are referred to [26]. In spite of the wide acceptability of PERT and its flexibility in terms of the choice of activity duration distributions, experts are often restricted to the use of few known and somewhat unrealistic activity duration distributions owing to difficulty in parameter elicitation of most proposed PERT procedures. It was demonstrated that a heavy tail distribution from the Burr family of distributions could approximate the duration of activities during borehole drilling projects [25]. However, as in some other cases, the procedure for the elicitation of Burr XII activity duration parameters was not presented in the work. In this paper, the problem of parameter elicitation of Burr XII activity duration in PERT is addressed using the classical quantile estimation approach. The Burr XII distribution belongs to the Burr System of twelve continuous distributions [27]. The Burr system of twelve continuous distributions were a solution to a differential equation that resulted to the Burr system of distributions (). Among these, the Burr XII distribution appears to be the most elaborate and hence popular in application [28]. The Burr XII distribution is suitable for the description of both right skewed and heavy tailed data and has been parameterized in various forms to accommodate extreme events [29].

The remaining part of the paper is structured as follows: in Section 2 we present Burr XII activity duration distribution and parameter elicitation and outline the justification for the choice of Burr XII distribution as an input activity duration distribution in PERT. An illustration of the proposed PERT procedure is presented in Section 3; estimates from the proposed method are compared with those obtained using other well-known procedures and then insightful conclusion is drawn in Section 4.

**2. METHODS**

**2.1 2 -Parameter Burr XII Activity Duration Distribution**

A random variable is said to be Burr (XII) with shape parameters and , if the probability density function is given as;

; , , (1)

The cumulative distribution function of X is

; , , (2)

The reliability function is given as;

; , , (3)

The moment about the origin is

;

Hence, the 1st moment of is

; (4)

and the variance is

(5)

Burr(XII) has a unique mode at

; if (6)

and L shape otherwise.

**2.2 Quantile Estimation for 2-Parameter Burr XII Activity Duration Distribution**

Consider a continuous distribution function which approximate the activity duration, the quantile of the distribution denoted by is defined as

(7)

In other words, it is the smallest number that satisfy [30].

Let and represent the lower and upper expert judgment quantiles of Burr XII activity duration with CDF as in Equation (2) and also considering Equation (7), and are obtained as follows [30]:

(8)

and

(9)

Substituting into Equation 8 and Equation 9 yields

(10)

(11)

Note that Equations (10) and (11) are expressed in terms of the parameters of Burr XII distribution which are unknown. Assume , , ) and) are known quantities which are obtained from the expert as judgmental estimates, then and can calculated. Solving Equations 10 and 11 simultaneously provide estimators for the computation of the parameters of the activity duration distribution, viz;

(12)

and

(13)

where is the root of

(14)

Observe that is unknown in Equation (14) which could be evaluated using appropriate iterative procedure. In this paper the solution Equation (14) was obtained with the help of maplesoft. Given the values of and one can easily calculate the values of the mean, variance, and mode using Equations (4), (5), and (6) respectively.

**2.3 3-Parameter Bur XII Activity Duration Distribution**

If a scale parameter, is introduced into equation (1), the result is a 3 Parameter Burr XII distribution with probability density function,

; , , , (15)

The cumulative density function is

; , , , (16)

the mean and variance are respectively;

; (17)

and

(19)

**2.4 Quantile Estimation for 3-Parameter Burr XII Distribution**

Let , and represent the lower, median and upper expert judgment percentiles of a 3 parameter Burr XII activity duration with scale parameter , and shape parameters and . From Equation (16)

(19)

To estimate, and the median quantile is introduced, such that we have the same number of equations as the unknown. Consequently, the lower, median and upper quantiles of a 3 Parameter Burr XII distribution are respectively

(20)

(21)

and

(22)

Note that the choice of the median quantile is based on the fact that the median as a measure of location is not affected by outliers in the data; hence it is well suited for positively skewed distribution. To estimate, and , Equations 20, 21 and 22 were solved simultaneously to obtain the following results:

(23)

(24)

(25)

where is the root of

(26)

Again, Equation 26 can easily be solved using appropriate iterative technique. Given the solution of Equation 26 the values of ,, and are then computed using Equations 23, 24, and 25 respectively. The values of, , and can then be used to compute the activity mean and variance using Equations 17 and 18 respectively. Recall that we need the values of , , with corresponding probabilities or , or and or to compute , and . These quantities can be supplied by the expert as judgmental estimates or chosen by the project manager based on previous knowledge.

One advantage of using the classical quantile technique to estimate the parameters of Burr XII distribution is that the user has the liberty to choose whatever quantiles that are considered appropriate. However, there is need to be guided by the fact that the assessments of extreme quantiles like 0.0 and 1.0, 0.01 and 0.99 are difficult and almost unrealistic, since these extremes has to do with the rare events. Quantiles like 0.10 and 0.90, 0.05 and 0.95 are found to be more reliable and easy to assess based on experimentation.

**2.5 Justification for the Use of Burr XII Distribution**

The PERT-Beta suggested the beta probability distribution as the activity time distribution based on the following properties: (i) unimodality (ii) continuity (iii) possession of positive range of values (iv) robustness in shape; having the asymmetric and symmetric properties [32]. In contrast, the Burr XII distribution is a continuous distribution which has positive range of values. It has a defined mode at , if . Burr XII distribution can fit a wide range of empirical data because it possesses a broad range of skewness and kurtosis [33, 34]. Particularly, Burr XII region covers sections of the areas corresponding to the Pearson Type I, IV, and VI on the Skewness-Kurtosis plane. Some sections of normal, exponential, logistic, extreme value and log-normal distributions are also covered. The Burr XII distribution can effectively model positively skewed data which is a common feature of most activity duration data [35, 36]. Burr XII distribution has relationship with some well-known distributions used in modeling event data. For instance, the Weibull, Exponential, and Pareto type 1, use in the analysis of events data are special limiting cases of the Burr XII [33]. If equals 1, in Equation 2, Burr XII becomes the log-logistic distribution. Hence, Burr XII is often termed the generalized log-logistic distribution. The log-logistic distribution is an important distribution that is use to model event data and has similar shape with log normal distribution which has been used as an approximation to activity duration [37]. Elsewhere, it has been demonstrated that Burr XII could be used to model log-normal random variable [38]. In the case of simulation, Burr XII distribution has advantage over the log-normal because the cumulative density function of Burr XII can be presented in a closed form [39].The Burr distribution well suited for modelling evets data with heavy tailed feature [40].

**3. ILLUSTRATION**

As an illustration, we refer to a data set on water borehole drilling projects from 20 drilling sites Ref [25]. The borehole drilling project was divided into 9 activities and all 20 water boreholes were drilled at 100 meters’ depth using air drilling method with the descriptive statistics presented in Table 1. It could be observed that the coefficients of skewness for all activities are positive, ranging from 0.6991 to 1.843. This supports the general opinion that most activity durations are positively skewed Refs [35, 36].

**Table 1: Descriptive Statistics of Bore Hole Drilling Activities**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | Description | Mean | Median | Mode | Variance | Skewness | 5th Percentile | 95th Percentile |
| A | Site Survey | 32.95 | 31 | 30 | 22.3658 | 1.4976 | 31.00 | 44.85 |
| B | Analysis of data | 63.1 | 61 | 60 | 34.7263 | 1.7203 | 56.10 | 79.75 |
| C | Drilling | 315.55 | 305.5 | 300 | 811.8394 | 0.9950 | 280.00 | 379.00 |
| D | Casing | 30.75 | 30 | 30 | 3.25 | 1.1940 | 28.05 | 35.00 |
| E | Gravel Packing | 16.55 | 16 | 15 | 6.7868 | 1.6038 | 14.00 | 23.85 |
| F | Connection of pipes to pump | 202.25 | 185 | 180 | 1782.8289 | 1.3595 | 150.50 | 300.00 |
| G | Connection of cables to pump | 23.95 | 24 | 25 | 7.8395 | 1.0320 | 20.00 | 31.80 |
| H | Lowering of pump into the well | 3 | 3 | 3 | 0.6316 | 0.6991 | 2 | 4.95 |
| I | Plumbing | 103.7 | 100 | 100 | 113.8 | 1.0126 | 90.00 | 129.50 |

Table 2 presents the estimate of parameters of the 3P-Burr XII distribution for 9 activities using simulation and quantile estimation approaches. For more information on the simulation approach interested readers are referred to [24]. The procedure for quantile estimation is as follows: the 5th, 50th, and 95th percentiles were obtained from the data for each of the activities[[1]](#footnote-1). These values with their corresponding probabilities were substituted into Equations (19), (20), and (21) and solved simultaneous to obtain the quantile estimates. A comparison of the qauntile and simulated estimates was done using the plots of Burr XII PDFs in Figures 1-9. From Figures 1-9, it could be observed that there is no marked distinction between the simulated and classical quantile estimates. The quantile estimates were then used to obtain the mean of each activity duration. Estimates of the mean for each activity duration using the Classical PERT otherwise known as PERT-Beta and some of its modifications [14, 17] were also obtained. The results are presented in Table 3.

**Table 2: Estimates of Burr XII Parameters**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activities | Simulated (Sample Size=5000) | | | Quantile Estimation Method | | |
|  |  |  |  |  |  |
| A | 57.868 | 0.1475 | 29.1770 | 56.2578 | 0.1521 | 29.1984 |
| B | 59.563 | 0.2176 | 58.5180 | 58.7081 | 0.2208 | 58.5313 |
| C | 48.194 | 0.2596 | 292.5600 | 47.5893 | 0.2628 | 292.6152 |
| D | 57.427 | 0.3623 | 29.511 | 56.7209 | 0.3665 | 29.5162 |
| E | 50.815 | 0.1331 | 14.185 | 49.0159 | 0.1385 | 14.2006 |
| F | 23.202 | 0.2555 | 170.59 | 22.9115 | 0.2586 | 170.6633 |
| G | 17.308 | 0.898 | 23.568 | 17.0418 | 0.9197 | 23.6264 |
| H | 6.1298 | 1.4502 | 3.1856 | 6.0630 | 1.4768 | 3.2017 |
| I | 35.784 | 0.3064 | 95.331 | 35.3249 | 0.3102 | 95.3581 |



**Figure 1: PDF of Burr XII for Activity A Figure 2: PDF of Burr XII for Activity B**



**Figure 3: PDF of Burr XII for Activity C Figure 4: PDF of Burr XII for Activity D**



**Figure 5: PDF of Burr XII for Activity E Figure 6: PDF of Burr XII for Activity F**



**Figure 7: PDF of Burr XII for Activity G Figure 8: PDF of Burr XII for Activity H**



**Figure 9: PDF of Burr XII for Activity I**

**Table 3: Mean Estimates**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Activity | Proposed  Method | Classical  PERT | Shankar &  Sireesha | Gollenko-Ginzburg | Simulated |
| A | 32.945 | 32.1667 | 32.4074 | 32 | 32.938 |
| B | 63.112 | 62.6667 | 62.9630 | 62.4615 | 63.099 |
| C | 315.81 | 310 | 311.1111 | 309.2308 | 315.79 |
| D | 30.759 | 30.5000 | 30.5556 | 30.4615 | 30.756 |
| E | 16.594 | 16.3333 | 16.4815 | 16.2308 | 16.591 |
| F | 202.66 | 195 | 196.6664 | 193.8462 | 202 |
| G | 23.966 | 25.3333 | 25.3704 | 25.3077 | 23.96 |
| H | 3.0057 | 3.1667 | 3.1852 | 3 | 3.0051 |
| I | 103.81 | 103.3333 | 103.7037 | 103.0769 | 103.8 |

In Table 4, the absolute percentage error of mean estimates using the four methods are presented. This was computed using the formula:

**Table 4: Absolute Percentage Error on Mean Estimates**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Activity | Proposed Method | Classical  PERT | Shankar & Sireesha | Gollenko-Ginzburg |
| A | 0.0213 | 2.3417 | 1.6109 | 2.8478 |
| B | 0.0206 | 0.6851 | 0.2155 | 1.0103 |
| C | 0.0063 | 1.8335 | 1.4817 | 2.0711 |
| D | 0.0098 | 0.8324 | 0.6516 | 0.9575 |
| E | 0.0181 | 1.5533 | 0.66 | 2.1711 |
| F | 0.3267 | 3.4654 | 2.6404 | 4.0365 |
| G | 0.0250 | 5.7316 | 5.8865 | 5.6248 |
| H | 0.0200 | 5.3775 | 5.9931 | 0.1697 |
| I | 0.0096 | 0.4496 | 0.0928 | 0.6966 |

From Table 4, it could be observed that the proposed method presents the least absolute percentage error on mean estimates as compared to the other three methods. For instance, the highest and the lowest absolute error on mean using the proposed method is 0.3267 and 0.0063 respectively. The highest and lowest absolute error on mean estimates from classical PERT is 5.775 and 0.4496 respectively. The highest and lowest absolute error from Shankar & Sireesha approximation is 5.9931 and 0.0928 respectively. From Gollenko-Ginzburg method the highest and lowest absolute error on mean are 5.6248 and 0.1697 respectively. On the average, the percentage error for mean estimates using the proposed method, classical PERT, Shankar & Sireesha and Gollenko-Ginzburg are 0.0508 0.0345, 2.4745 0.6593, 2.1369 0.7651, and 2.17620.5878 respectively. These results imply that the proposed method is most accurate in estimating activity duration as compared to the Classical PERT, Shankar & Sireesha, and Gollenko-Ginzburg techniques based on the data in use.

**5 Conclusion**

In this paper, the problem of parameter elicitation in PERT with Burr XII activity duration distribution was addressed using the classical quantile approach. Results obtained from the proposed method were compared with some existing approximate methods. Findings from the illustration give room for one to believe that a wrong choice of an input activity duration distribution may adversely affect the estimates of activity duration. The elicitation procedure presented in this paper is relatively simple, hence it will promote the use of Burr XII distribution as an input activity duration distribution in PERT. It is recommended that more empirical research should be carried out to compare the performance of the proposed method with other existing methods that use other probability distributions as input activity distributions. The illustration was limited to a dataset which is approximately Burr distributed. Researchers should explore the performance of the proposed model with the use of other datasets that deviate significantly from the Burr family of distributions.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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1. Note that the quantile estimates could as well be obtained as judgmental estimates with associated probability values when there are no real data. [↑](#footnote-ref-1)