**DESIGN OF GENERALIZED GAMMA** $\overbar{X}$ **AND S CONTROL CHARTS FOR MONITORING INDUSTRIAL PROCESS**

**Abstract**

Shewhart control charts are commonly used to determine if a process is under control. Control limits are essential for monitoring process stability and making smart decisions about which processes in industries require attention and development. Control charts that are successful require accurate control limit specification. The constraints of a Shewhart control chart are predicated on the assumption that the process quality characteristics under consideration may be represented by a symmetrical normal distribution. Because industrial processes are rarely normally distributed in practice, using standard Shewhart Control charts with non-normal process data results in erroneous control limits, leading to wrong conclusions. Consequently, this study proposes utilizing the mean and dispersion of quality parameters that follow generalized gamma distributions to construct control chart limits. A simulation analysis was carried out to evaluate the performance of the proposed charts to existing methodologies. The usefulness of the proposed charts was further validated with process data from the manufacturing industries. The results show that the suggested charts may be used to monitor skewed and reverse-J distributions and give appropriate control chart limits that do not trigger false alarms. As a result, the generalized gamma control chart (GG) approach is better suited for non-normal process data and is strongly recommended for monitoring industrial processes.

**Keywords**: Control limits, Generalized gamma, Industrial process, Non-normal, Quality improvement.

**1.0 Introduction**

Control charts are essential for maintaining the stability and quality of industrial processes. Traditional control charts, such as the Shewhart control chart, presume that process data has a normal distribution. However, in many real-world settings, process data has non-normal features, making typical control charts less effective. To address this constraint, we propose using the generalized gamma distribution, a versatile statistical distribution that can simulate a wide range of process behaviors. A type of generalized gamma distribution that can describe skewed process data would be used to create control charts for monitoring manufacturing and industrial operations.

Several authors have investigated and built control charts based on the underlying distribution to achieve desired false alarm rates. Kantam et al. (2006) developed control charts based on a log-logistic distribution, Subba Rao and Kantam (2008) constructed control charts based on a double exponential distribution, Rao and Sarath Babu (2012) designed control charts for quality characteristics that follow a linear failure rate distribution, and Srinivasa Rao et al. (2015) developed control charts based on Weibull-Pareto percentiles. Adewara and Aako (2018) calculated variable control limits using percentiles of the exponentiated lomax distribution. Ara et al. (2019) constructed a time-between-events chart to track planned time using an exponentially modified Gaussian (EMG) distribution.

Adewara et al. (2020) proposed two control chart techniques for monitoring the process using the two-parameter Gompertz distribution. Aako et al. (2020) used the Marshall-Olkin inverse log-logistic distribution to generate control charts for positively skewed data. Morales and Panza (2022) adapted the exponentially weighted moving averages (EWMA) control chart with dynamic limits to monitor the mean of samples from the skew-normal distribution. Bhat and Patil (2024) proposed a Shewhart type 𝑀̃ control chart based on midrange for monitoring the process location parameter, specifically designed to address non-normality in manufacturing processes.

The fundamental aim of this research project is to construct control charts using the mean and variance of generalized gamma distribution and apply them to industrial processes. We hope to design control charts that are better adapted to handling non-normal process data, hence improving process monitoring accuracy and identifying out-of-control conditions.

**2.0 Methods**

**2.1 Generalized Gamma Distribution (GGD)**

GGD, a three-parameter distribution, introduced by Stacy (1962) is a flexible family which is often suitable for modeling skewed data. Its probability density function is

$f\left(x;κ,ϕ,β\right)=\frac{\left(\frac{β}{κ^{ϕ}}\right)x^{ϕ-1}e^{-\left(\frac{x}{κ}\right)^{β}}}{Γ\left(\frac{ϕ}{β}\right)} ; κ>0,ϕ>0,β>0.$ (1)

where $Γ(.)$ denotes the gamma function.

The cumulative distribution function is

$F\left(x;κ,ϕ,β\right)=\frac{γ\left(\frac{ϕ}{β},\left(\frac{x}{κ}\right)^{β}\right)}{Γ\left(\frac{ϕ}{β}\right)} ; κ>0,ϕ>0,β>0.$ (2)

where $γ(.)$ denotes the lower incomplete gamma function.

Let $X$ has a GGD, the moments of GGD is

$E\left(X^{r}\right)=κ^{r}\frac{Γ\left(\frac{(ϕ+r}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}$ (3)

Hence, the mean of GGD is

$μ\_{GG}=κ \frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}$ (4)

and the variance given by

$σ\_{GG}^{2}=κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right)$ (5)

The maximum likelihood of $\hat{θ}, \hat{α}, \hat{β}$ of parameters $θ, α, β$ of GGD can be obtained by solving the natural log likelihood equation using R software.

**2.2 Proposed Control Charts Based on the GGD**

Control limits for GG control charts are developed analogous to Shewhart-charts based on the distributional properties of GGD.

Let 𝑤 be statistic that measures the quality characteristic, mean and variance of 𝑤 are $μ\_{GG}$ and $σ\_{GG}^{2}$ respectively, then the control limits for mean chart is given as:

$\left\{\begin{array}{c}Upper Control Limit = UCL = μ\_{GG} +kσ\_{GG}\\Central Line = CL = μ\_{GG} \\Lower Control Limit = UCL = μ\_{GG} -kσ\_{GG} \end{array} \right.$ (6)

where *k* = control limits distance from the center, in multiples of the standard deviation of *w*.

Substituting (4) and (5) in (6), we have

$\left\{\begin{array}{c}UCL = κ \frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)} +3\sqrt{κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right) }\\CL = κ \frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)} \\ LCL = κ \frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)} -3\sqrt{κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right) }\end{array}\right.$ (7)

Also, the 3-sigma control limits for S- chart is

$\left\{\begin{array}{c}UCL=C\_{4}σ\_{GG} +3σ\_{GG} \sqrt{1-C\_{4}^{2}}\\CL=C\_{4}σ\_{GG} \\LCL=C\_{4}σ\_{GG} -3σ\_{GG} \sqrt{1-C\_{4}^{2}}\end{array}\right.$ (8)

where $C\_{4} = \left(\frac{2}{n} - 1\right)^{\frac{1}{2}} \left[\frac{Γ\left(\frac{n}{2}\right)}{Γ\left(\frac{n - 1}{2}\right)}\right]$

substituting (4) and (5) in (8), we have

$\left\{\begin{array}{c}UCL=\sqrt{κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right) } \left(C\_{4}+3 \sqrt{1-C\_{4}^{2}}\right) \\CL=C\_{4}\sqrt{κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right) }\\LCL=\sqrt{κ^{2}\left(\frac{Γ\left(\frac{(ϕ+2}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}-\left(\frac{Γ\left(\frac{(ϕ+1}{β}\right)}{Γ\left(\frac{ϕ}{β}\right)}\right)^{2}\right) } \left(C\_{4}-3 \sqrt{1-C\_{4}^{2}}\right) \end{array}\right.$ (9)

**2.3 Performance Evaluation**:

The proposed control charts (GG) were compared to classic Shewhart (SH) and Skewness Correction (SC) control charts using simulation simulations. The Control Limits Interval (CLI) establishes the range within which the process is deemed under control. A lower CLI indicates greater control and sensitivity to identifying process alterations. Number Beyond Limits (NBL) is the number of subgroups that fall outside the control limits. Higher NBL values indicate frequent deviations from the control limits, implying a potentially unstable process or a high susceptibility to changes. While Average Run Length (ARL) is the average number of points shown on a control chart prior to a point indicating that the process is out of control. Lower ARL values indicate that the approach identifies shifts more frequently, which can be valuable for identifying problems early on but may result in more false alarms. These measures are used to assess the efficacy of the new method.

**3.0 Results and Discussion**

**3.1 Simulation Study**

The efficiency of the proposed control charts was assessed using data sets generated from different sample sizes and grouped in subgroups of 3, 4, 5, 7, and 10 using the generalized gamma with parameters ($κ=1.2,ϕ=2.2,β=0.85$), ($κ=0.8,ϕ=1.2,β=0.75$), and ($κ=0.5,ϕ=0.8,β=1.75$). The mean and standard deviation for construction of the proposed charts limits for different samples were determined for the generated data using Equations (4) and (5), respectively. The proposed charts are compared to existing SH and SC methods.

**Table 1:** Descriptive Statistics of Simulated Data Sets 1, 2 and 3

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Data | Parameters | min | median | mean | max | sd | skewness | kurtosis |
| 1 | GG(1.2,2.2,0.85) | 0.0534 | 0.9613 | 0.9862 | 2.3734 | 0.5233 | 0.2830 | 2.3980 |
| 2 | GG(0.8,1.2,0.75) | 0.0012 | 0.3734 | 0.5469 | 3.1863 | 0.5329 | 1.7622 | 6.8188 |
| 3 | GG(0.5,0.8,1.75) | 0.0094 | 0.7136 | 1.1066 | 7.0691 | 1.1313 | 2.1573 | 8.8601 |

Table 1 displays the descriptive statistics of the simulated data sets used to compare the proposed technique against some existing techniques. Data set 1 ranges from 0.0534 to 2.3734, with the median (0.9613) and mean (0.9862) being near together, indicating a reasonably symmetrical distribution around the mean. The positive skewness (0.2830) suggests a somewhat rightward skewed distribution. Data set 2 ranges from 0.0012 to 3.1863, with a median (0.3734) and mean (0.5469) that exhibit slight right skewness. The strong positive skewness (1.7622) indicates a considerable right skew. Data Set 3 has values ranging from 0.0094 to 7.0691, with the mean (1.1066) much higher than the median (0.7136), showing a right-skewed distribution. The very high positive skewness (2.1573) indicates a significant right skew. Figure 1 clearly shows these. The histogram depicted in the graphic displays the frequency distribution of Simulated Data 1, 2, and 3. The histogram of data set 1 is relatively symmetric, but the histograms of data set 2 and 3 are strongly right-skewed. As a result, the data do not follow a normal distribution due to their skewness.

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**Figure 1:** Histogram of Simulated Data sets 1, 2 and 3

**3.1.1 Mean charts for Simulated data**

Tables 2, 3, and 4 include data on the CLI, NBL, and ARL of SH, GG, and SC. Mean charts for various sample sizes (N) and sample numbers (n) for some simulated Generalized Gamma data with changing parameters. In table 2, a decrease in subgroup size (n) leads to a rise in CLI values. The CLI values vary depending on the sample size N. The GG technique produces tighter CLI values, particularly for n = 3, 4, and 5, although all methods demonstrate a similar pattern of diminishing control limits with increasing subgroup size. In all combinations of the two sample sizes utilized (small and big), the NBL is zero. This signifies that no points exceeded the control limits, as is expected from a well-behaved process. Take a second. After a second look at the ARL values, it was discovered that they are infinite for all combinations of N and n, implying that the data do not indicate an out-of-control process. Based on these findings, we may conclude that the monitored process is stable, as no indications of instability are present (NBL = 0 and ARL = ∞).

Table 3 shows that when the number of the subgroups increases, the control limits get narrower. For the same subgroup size n, the CLI stays reasonably stable across a wide range of sample sizes. Smaller sub-group sizes (n=3 and n=4) have more observations beyond the control boundaries than bigger sub-group sizes. In certain circumstances, especially for bigger n, the NBL is 0, indicating that no points exceed the control boundaries. When the NBL is zero, the ARL often has a greater value. An infinite ARL indicates a highly stable process within the control limitations. Table 4 shows that as subgroup size (n) grows, CLI values fall, indicating tighter control limits for bigger subgroups. For different sample sizes (N), the CLI values vary; in rare cases, NBL is greater than zero, indicating that some points fall outside the control limits, but in many cases, no points exceed the limits. For NBL = 0, the ARL values are mostly infinite (∞). The ARL values decrease as the NBL values increase, indicating more frequent out-of-control signals.

In tables 2, 3, and 4, the SH approach has the lowest CLI and ARL, followed by the GG method and finally the SC method. This indicates that the SH technique will generate false alarms when used with non-normal process data. The SC technique would be incapable of detecting any out of control. As a result, the GG technique is more suited to providing insights into process stability and control across a variety of sample and subgroup sizes across three parameter sets.

**Table 2:** SH, GG and SC Mean charts for Simulated data GG (1.2,2.2,0.85)

| GG(1.2,2.2,0.85) | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **N** | **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
| 100 | 3 | 1.9457 | 0 | Inf | 1.7832 | 0 | Inf | 3.0508 | 0 | Inf |
| 4 | 1.6946 | 0 | Inf | 1.5443 | 0 | Inf | 2.7277 | 0 | Inf |
| 5 | 1.4762 | 0 | Inf | 1.3812 | 0 | Inf | 2.4428 | 0 | Inf |
| 7 | 1.2250 | 0 | Inf | 1.1673 | 0 | Inf | 2.1389 | 0 | Inf |
| 10 | 0.9947 | 0 | Inf | 0.9767 | 0 | Inf | 1.8094 | 0 | Inf |
| 200 | 3 | 1.9910 | 0 | Inf | 1.8253 | 0 | Inf | 3.0848 | 0 | Inf |
| 4 | 1.7048 | 0 | Inf | 1.5808 | 0 | Inf | 2.7585 | 0 | Inf |
| 5 | 1.4692 | 0 | Inf | 1.4139 | 0 | Inf | 2.4276 | 0 | Inf |
| 7 | 1.1835 | 0 | Inf | 1.1950 | 0 | Inf | 2.0570 | 0 | Inf |
| 10 | 0.9951 | 0 | Inf | 0.9998 | 0 | Inf | 1.7852 | 0 | Inf |
| 300 | 3 | 1.9567 | 0 | Inf | 1.8252 | 0 | Inf | 3.0275 | 0 | Inf |
| 4 | 1.6895 | 0 | Inf | 1.5807 | 0 | Inf | 2.7397 | 0 | Inf |
| 5 | 1.4613 | 0 | Inf | 1.4138 | 0 | Inf | 2.4064 | 0 | Inf |
| 7 | 1.2036 | 0 | Inf | 1.1949 | 0 | Inf | 2.0793 | 0 | Inf |
| 10 | 0.9988 | 0 | Inf | 0.9997 | 0 | Inf | 1.7789 | 0 | Inf |
| 500 | 3 | 1.8677 | 0 | Inf | 1.7570 | 0 | Inf | 2.8778 | 0 | Inf |
| 4 | 1.6137 | 0 | Inf | 1.5216 | 0 | Inf | 2.6152 | 0 | Inf |
| 5 | 1.3992 | 0 | Inf | 1.3610 | 0 | Inf | 2.3024 | 0 | Inf |
| 7 | 1.1614 | 0 | Inf | 1.1502 | 0 | Inf | 1.9988 | 0 | Inf |
| 10 | 0.9691 | 0 | Inf | 0.9624 | 0 | Inf | 1.7065 | 0 | Inf |

**Table 3:** SH, GG and SC Mean charts for Simulated data GG (0.8,1.2,0.75)

| GG(0.8,1.2,0.75)  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **N** | **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
| 100 | 3 | 1.6775 | 0 | Inf | 1.7879 | 0 | Inf | 2.7350 | 0 | Inf |
| 4 | 1.3579 | 1 | 25.0 | 1.5484 | 1 | 25.0 | 2.3634 | 0 | Inf |
| 5 | 1.1758 | 1 | 20.0 | 1.3849 | 1 | 20.0 | 2.0891 | 0 | Inf |
| 7 | 1.0696 | 0 | Inf | 1.1705 | 0 | Inf | 1.9845 | 0 | Inf |
| 10 | 0.9031 | 0 | Inf | 0.9793 | 0 | Inf | 1.6932 | 0 | Inf |
| 200 | 3 | 1.5299 | 0 | Inf | 1.6435 | 0 | Inf | 2.4615 | 0 | Inf |
| 4 | 1.2723 | 1 | 50.0 | 1.4233 | 1 | 50.0 | 2.1730 | 0 | Inf |
| 5 | 1.1374 | 1 | 40.0 | 1.2730 | 1 | 40.0 | 1.9923 | 0 | Inf |
| 7 | 0.9859 | 0 | Inf | 1.0759 | 0 | Inf | 1.8049 | 0 | Inf |
| 10 | 0.8433 | 0 | Inf | 0.9002 | 0 | Inf | 1.5577 | 0 | Inf |
| 300 | 3 | 1.7574 | 0 | Inf | 1.9379 | 0 | Inf | 2.8220 | 0 | Inf |
| 4 | 1.5006 | 1 | 75.0 | 1.6783 | 0 | Inf | 2.5682 | 0 | Inf |
| 5 | 1.3357 | 1 | 60.0 | 1.5011 | 0 | Inf | 2.3358 | 0 | Inf |
| 7 | 1.1554 | 0 | Inf | 1.2687 | 0 | Inf | 2.1018 | 0 | Inf |
| 10 | 0.9579 | 0 | Inf | 1.0614 | 0 | Inf | 1.7884 | 0 | Inf |
| 500 | 3 | 1.7497 | 1 | 166.0 | 1.8809 | 1 | 166.0 | 2.7929 | 0 | Inf |
| 4 | 1.4891 | 2 | 62.5 | 1.6289 | 1 | 125.0 | 2.5231 | 0 | Inf |
| 5 | 1.3357 | 1 | 100.0 | 1.4570 | 0 | Inf | 2.3130 | 0 | Inf |
| 7 | 1.1343 | 0 | Inf | 1.2314 | 0 | Inf | 2.0445 | 0 | Inf |
| 10 | 0.9725 | 0 | Inf | 1.0302 | 0 | Inf | 1.7754 | 0 | Inf |

**Table 4:** SH, GG and SC Mean charts for Simulated data GG (0.5,0.8,1.75)

| GG(0.5,0.8,1.75)  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **N** | **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
| 100 | 3 | 3.4480 | 1 | 33.0000 | 3.9435 | 0 | Inf | 5.7809 | 0 | Inf |
| 4 | 3.1046 | 1 | 25.0000 | 3.4152 | 1 | 25.0000 | 5.5012 | 0 | Inf |
| 5 | 2.7494 | 0 | Inf | 3.0546 | 0 | Inf | 5.0838 | 0 | Inf |
| 7 | 2.2514 | 1 | 14.0000 | 2.5816 | 1 | 14.0000 | 4.3542 | 0 | Inf |
| 10 | 2.1578 | 0 | Inf | 2.1600 | 0 | Inf | 4.1112 | 0 | Inf |
| 200 | 3 | 3.4792 | 1 | 66.0000 | 3.9148 | 0 | Inf | 5.7631 | 0 | Inf |
| 4 | 3.0927 | 1 | 50.0000 | 3.3904 | 1 | 50.0000 | 5.3444 | 0 | Inf |
| 5 | 2.7439 | 0 | Inf | 3.0324 | 0 | Inf | 4.8681 | 0 | Inf |
| 7 | 2.2914 | 1 | 28.0000 | 2.5629 | 1 | 28.0000 | 4.2506 | 0 | Inf |
| 10 | 2.1244 | 0 | Inf | 2.1442 | 0 | Inf | 3.8973 | 0 | Inf |
| 300 | 3 | 3.4089 | 1 | 100.0000 | 3.9877 | 0 | Inf | 5.6125 | 0 | Inf |
| 4 | 3.0122 | 1 | 75.0000 | 3.4534 | 1 | 75.0000 | 5.1815 | 0 | Inf |
| 5 | 2.7479 | 0 | Inf | 3.0888 | 0 | Inf | 4.8309 | 0 | Inf |
| 7 | 2.3071 | 1 | 42.0000 | 2.6105 | 1 | 42.0000 | 4.2212 | 0 | Inf |
| 10 | 2.1055 | 0 | Inf | 2.1841 | 0 | Inf | 3.8513 | 0 | Inf |
| 500 | 3 | 3.4218 | 3 | 55.3333 | 4.1842 | 1 | 166.0000 | 5.5797 | 1 | 166.0 |
| 4 | 3.0293 | 3 | 41.6667 | 3.6237 | 2 | 62.5000 | 5.1812 | 0 | Inf |
| 5 | 2.7114 | 1 | 100.0000 | 3.2411 | 1 | 100.0000 | 4.7721 | 0 | Inf |
| 7 | 2.3345 | 1 | 71.0000 | 2.7392 | 0 | Inf | 4.2802 | 0 | Inf |
| 10 | 2.0900 | 0 | Inf | 2.2918 | 0 | Inf | 3.8450 | 0 | Inf |

**3.1.2 Standard deviation charts for Simulated data**

Tables 5, 6, and 7 include thorough information on the CLI, NBL, and ARL of SH, GG, and SC Standard deviation charts for various sample sizes (N) and subgroup sizes (n). Table 5 shows that CLI values decrease as subgroup size (n) increases, indicating tighter control limits for larger subgroups. The CLI values vary depending on the sample size (N), but the overall pattern of decreasing CLI as n increases is consistent. For the SH technique, the NBL values are zero. For the GG method, the NBL values indicate that more subgroups fall outside the control limits as sample and subgroup sizes increase, whereas for the SC method, the NBL values are equal 0, but there are occasions when some points are beyond the limits in larger sample and subgroup sizes, implying occasional out-of-control signals. The ARL values for the SH technique are infinite (∞), which is consistent with the NBL values of 0, suggesting no points warning out-of-control circumstances. For the GG technique, ARL values decline as NBL values increase suggesting that out-of-control signals occur more frequently. The ARL values for the SC technique are typically infinite, however they are finite if NBL is larger than zero. As N and n rise, the GG approach predicts that the process becomes less stable, with increasingly frequent out-of-control signals.

For all data sets and methodologies in Table 6, as the sample size grows, the CLI values drop, suggesting more precision and confidence in the estimations. The NBL lowers as sample size increases. In most circumstances, the value is zero. ARL values differ between methodologies and sample sizes. With higher sample sizes, ARL tends to grow, implying that out-of-control circumstances are recognized less frequently or more slowly. The GG approach has lower CLI values and a greater ARL than the other methods. It indicates that the GG approach will detect out-of-control situations faster than the other methods. In Table 7, SH has the lowest CLI, suggesting the greatest sensitivity to process adjustments, whereas GG has a moderate CLI and SC has the highest CLI, indicating the least sensitivity. CLI reduces with increasing subgroup size for all approaches. SH often has greater NBL values, but GG retains lower NBL, suggesting a steadier process. SC constantly displays the lowest or zero NBL, indicating the strongest stability. SH generally has moderate ARL values, suggesting that the process signals more frequently, whereas GG has greater ARL, indicating that the process operates for longer periods before signaling. SC distinguishes itself with limitless ARL in most cases. Larger subgroup sizes produce higher ARL for all approaches, particularly SC. The comparison of the three approaches across subgroup sizes reveals that GG has better sensitivity and fewer variances.

**Table 5:** SH, GG and SC Standard deviation charts for Simulated data GG (1.2,2.2,0.85)

| GG (1.2,2.2,0.85) | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| N | n | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
| 100 | 3 | 1.2874 | 0 | Inf | 1.4308 | 4 | 8.2500 | 2.1648 | 0 | Inf |
| 4 | 1.1703 | 0 | Inf | 1.2008 | 3 | 8.3333 | 1.9998 | 0 | Inf |
| 5 | 1.0782 | 0 | Inf | 1.0538 | 7 | 2.8571 | 1.8638 | 0 | Inf |
| 7 | 0.9403 | 0 | Inf | 0.8714 | 6 | 2.3333 | 1.5967 | 0 | Inf |
| 10 | 0.7689 | 0 | Inf | 0.7173 | 9 | 1.1111 | 1.3288 | 0 | Inf |
| 200 | 3 | 1.3088 | 0 | Inf | 1.4646 | 9 | 7.3333 | 2.2200 | 0 | Inf |
| 4 | 1.1909 | 0 | Inf | 1.2292 | 11 | 4.5455 | 2.0472 | 0 | Inf |
| 5 | 1.0789 | 0 | Inf | 1.0788 | 15 | 2.6667 | 1.8522 | 0 | Inf |
| 7 | 0.9117 | 0 | Inf | 0.8920 | 14 | 2.0000 | 1.5356 | 0 | Inf |
| 10 | 0.7660 | 0 | Inf | 0.7342 | 18 | 1.1111 | 1.3111 | 0 | Inf |
| 300 | 3 | 1.2866 | 0 | Inf | 1.4645 | 14 | 7.1429 | 2.1955 | 0 | Inf |
| 4 | 1.1849 | 0 | Inf | 1.2292 | 18 | 4.1667 | 2.0461 | 0 | Inf |
| 5 | 1.0716 | 0 | Inf | 1.0787 | 23 | 2.6087 | 1.8360 | 0 | Inf |
| 7 | 0.9238 | 0 | Inf | 0.8920 | 20 | 2.1000 | 1.5522 | 1 | 42.0000 |
| 10 | 0.7654 | 0 | Inf | 0.7342 | 25 | 1.2000 | 1.3064 | 1 | 30.0000 |
| 500 | 3 | 1.2242 | 0 | Inf | 1.4098 | 30 | 5.5333 | 2.1318 | 0 | Inf |
| 4 | 1.1324 | 0 | Inf | 1.1833 | 37 | 3.3784 | 1.9934 | 0 | Inf |
| 5 | 1.0267 | 0 | Inf | 1.0384 | 48 | 2.0833 | 1.7567 | 0 | Inf |
| 7 | 0.8895 | 0 | Inf | 0.8587 | 45 | 1.5778 | 1.4921 | 1 | 71.0000 |
| 10 | 0.7356 | 0 | Inf | 0.7068 | 43 | 1.1628 | 1.2533 | 3 | 16.6667 |

**Table 6:** SH, GG and SC Standard deviation charts for Simulated data GG (0.8,1.2,0.75)

| GG(0.8,1.2,0.75)  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **N** | **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
|  |  |  |  |  |  |  |  |  |  |  |
| 100 | 3 | 1.1065 | 1 | 33.0000 | 1.2476 | 0 | Inf | 1.6323 | 0 | Inf |
| 4 | 0.9669 | 0 | Inf | 1.1323 | 0 | Inf | 1.4492 | 0 | Inf |
| 5 | 0.8750 | 0 | Inf | 1.0567 | 0 | Inf | 1.3272 | 0 | Inf |
| 7 | 0.8209 | 0 | Inf | 0.8738 | 0 | Inf | 1.2795 | 0 | Inf |
| 10 | 0.6711 | 0 | Inf | 0.7192 | 0 | Inf | 1.1520 | 0 | Inf |
| 200 | 3 | 1.0108 | 2 | 33.0000 | 1.1544 | 1 | 66.0000 | 1.4863 | 0 | Inf |
| 4 | 0.9039 | 3 | 16.6667 | 1.0485 | 0 | Inf | 1.3399 | 0 | Inf |
| 5 | 0.8498 | 1 | 40.0000 | 0.9713 | 0 | Inf | 1.2551 | 0 | Inf |
| 7 | 0.7624 | 1 | 28.0000 | 0.8032 | 0 | Inf | 1.1711 | 0 | Inf |
| 10 | 0.6321 | 0 | Inf | 0.6611 | 0 | Inf | 1.0670 | 0 | Inf |
| 300 | 3 | 1.1623 | 3 | 33.3333 | 1.3232 | 2 | 50.0000 | 1.6779 | 1 | 100 |
| 4 | 1.0717 | 2 | 37.5000 | 1.1983 | 2 | 37.5000 | 1.5443 | 0 | Inf |
| 5 | 0.9997 | 2 | 30.0000 | 1.1184 | 1 | 60.0000 | 1.4368 | 0 | Inf |
| 7 | 0.8913 | 3 | 14.0000 | 0.9471 | 2 | 21.0000 | 1.3347 | 0 | Inf |
| 10 | 0.7289 | 2 | 15.0000 | 0.7795 | 0 | Inf | 1.2024 | 0 | Inf |
| 500 | 3 | 1.1636 | 4 | 41.5000 | 1.3151 | 3 | 55.3333 | 1.6821 | 1 | 166 |
| 4 | 1.0668 | 2 | 62.5000 | 1.1938 | 2 | 62.5000 | 1.5414 | 0 | Inf |
| 5 | 1.0044 | 3 | 33.3333 | 1.1116 | 1 | 100.0000 | 1.4428 | 0 | Inf |
| 7 | 0.8820 | 3 | 23.6667 | 0.9192 | 2 | 35.5000 | 1.3253 | 0 | Inf |
| 10 | 0.7378 | 1 | 50.0000 | 0.7566 | 1 | 50.0000 | 1.2124 | 0 | Inf |

**Table 7:** SH, GG and SC Standard deviation charts for Simulated data GG (0.5,0.8,1.75)

| GG(0.5,0.8,1.75)  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **N** | **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
| 100 | 3 | 2.2782 | 3 | 11.0000 | 2.7197 | 1 | 33.0000 | 3.4692 | 1 | 33.0000 |
| 4 | 2.1863 | 3 | 8.3333 | 2.4655 | 1 | 25.0000 | 3.2790 | 0 | Inf |
| 5 | 2.0639 | 1 | 20.0000 | 2.3029 | 1 | 20.0000 | 3.0794 | 0 | Inf |
| 7 | 1.7401 | 1 | 14.0000 | 1.9272 | 1 | 14.0000 | 2.7770 | 0 | Inf |
| 10 | 1.5702 | 1 | 10.0000 | 1.5863 | 1 | 10.0000 | 2.6497 | 0 | Inf |
| 200 | 3 | 2.3278 | 6 | 11.0000 | 2.6660 | 3 | 22.0000 | 3.4143 | 1 | 66.0000 |
| 4 | 2.1823 | 5 | 10.0000 | 2.4136 | 2 | 25.0000 | 3.1751 | 0 | Inf |
| 5 | 2.0348 | 3 | 13.3333 | 2.2522 | 1 | 40.0000 | 2.9543 | 0 | Inf |
| 7 | 1.7543 | 2 | 14.0000 | 1.9132 | 1 | 28.0000 | 2.6903 | 0 | Inf |
| 10 | 1.5409 | 1 | 20.0000 | 1.5747 | 1 | 20.0000 | 2.5283 | 0 | Inf |
| 300 | 3 | 2.2898 | 7 | 14.2857 | 2.6826 | 4 | 25.0000 | 3.3341 | 2 | 50.0000 |
| 4 | 2.1392 | 6 | 12.5000 | 2.4255 | 3 | 25.0000 | 3.0971 | 1 | 75.0000 |
| 5 | 2.0436 | 3 | 20.0000 | 2.2611 | 2 | 30.0000 | 2.9254 | 1 | 60.0000 |
| 7 | 1.7660 | 3 | 14.0000 | 1.9488 | 2 | 21.0000 | 2.6647 | 0 | Inf |
| 10 | 1.5459 | 2 | 15.0000 | 1.6040 | 2 | 15.0000 | 2.4967 | 0 | Inf |
| 500 | 3 | 2.2914 | 11 | 15.0909 | 2.7979 | 4 | 41.5000 | 3.3514 | 3 | 55.3333 |
| 4 | 2.1551 | 9 | 13.8889 | 2.5281 | 4 | 31.2500 | 3.1324 | 2 | 62.5000 |
| 5 | 2.0353 | 7 | 14.2857 | 2.3556 | 3 | 33.3333 | 2.9384 | 2 | 50.0000 |
| 7 | 1.8083 | 6 | 11.8333 | 2.0449 | 4 | 17.7500 | 2.7124 | 1 | 71.0000 |
| 10 | 1.5599 | 4 | 12.5000 | 1.6831 | 3 | 16.6667 | 2.5298 | 1 | 50.0000 |

**3.2 Real Data Application**

The values for vol% C4 in propane rundown are obtained from King's Statistics for Process Control Engineers: A Practical Approach (2017). Table 8 summarizes the descriptive statistics for the data set. The data set spans 1.5 to 15.8, with significant variability as seen by the standard deviation of 2.4248. The mean exceeds the median, indicating a right-skewed distribution with a larger right tail. The data distribution is leptokurtic, which means that it has thicker tails and a sharper peak than the normal distribution. This indicates a larger possibility of extreme values than a normal distribution.

**Table 8:** Descriptive Statistics of vol% C4 in Propane Rundown.

| min | median | mean | max | sd | skewness | kurtosis |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| 1.5 | 4.3 | 4.8772 | 15.8 | 2.4248 | 1.2749 | 4.7916 |

Table 9 compares the performance of the SH, GG, and SC approaches for mean charts with different subgroup sizes (n). CLI values for all approaches tend to drop as the size of the subgroup (n) grows. This suggests that bigger subgroups result in tighter control limits, making process monitoring more sensitive. NBL values are low for all subgroup sizes, with slight variation depending on the technique utilized. ARL values are infinite (Inf) in most circumstances, suggesting very stable processes with few out-of-control signals, with the exception of a few examples when NBL is not zero. For the SH approach, the CLI values are relatively high, indicating moderate control tightness; NBL is low, and ARL is high (Inf), indicating steady process control. The CLI values for the GG approach are somewhat lower than SH, indicating tighter control; its NBL value is similar to that of SH, with just a few spots exceeding control limits; and ARL is typically high, as with SH. The SC technique has the greatest CLI values of the three methods, the NBL is often zero, and the ARL is high (Inf). These results imply that the GG approach has slightly tighter control limits than SH, making it suited for processes that need early detection of small shifts. The SC technique is less susceptible to slight shifts.

**Table 9:** SH, GG and SC Mean charts for Real data.

|  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
|  |  |  |  |  |  |  |  |  |  |
| **3** | 8.211585 | 2 | 60.5 | 8.018999 | 2 | 60.5 | 12.816437 | 0 | Inf |
| **4** | 7.187422 | 0 | Inf | 6.944656 | 0 | Inf | 11.712585 | 0 | Inf |
| **5** | 6.519462 | 0 | Inf | 6.211490 | 0 | Inf | 10.778157 | 0 | Inf |
| **7** | 5.530440 | 0 | Inf | 5.249667 | 0 | Inf | 9.384887 | 0 | Inf |
| **10** | 4.638624 | 0 | Inf | 4.392186 | 0 | Inf | 7.887944 | 0 | Inf |

Table 10 compares the SH, GG, and SC approaches for standard deviation control charts with different subgroup sizes (n). For all techniques, the CLI reduces with increasing subgroup size (n). The SH technique offers modest CLI values across various subgroup sizes. The GG approach has somewhat tighter CLIs than SH, but more variations, indicating that it is more susceptible to process changes. The SC technique has the broadest CLIs, which may indicate reduced sensitivity to minor process changes. The NBL varies greatly across methodologies and subgroup sizes. The SH technique yields a relatively low NBL across most subgroup sizes. The GG approach has higher NBL values, especially for smaller subgroup sizes, indicating greater sensitivity and more frequent identification of out-of-control circumstances. The SC technique typically has low NBL values, however there are a few instances (e.g., n=7 and n=10) where deviations are more frequent. ARL is high or infinite across approaches, indicating that processes are generally stable. The SH technique retains high ARL values, indicating fewer false alarms and a steady process. The GG technique produces low ARL values for lower subgroup sizes, indicating its great sensitivity. SC methods frequently have infinite ARL values, suggesting a very stable process but maybe at the expense of detecting small shifts. The results show that the GG technique is best suited for processes where identifying small changes fast is crucial, even if it results in more false alarms. Useful in high-risk procedures where early identification is critical.

**Table 10:** SH, GG and SC Standard deviation charts for Real data.

|  | **SH** | **GG** | **SC** |
| --- | --- | --- | --- |
| **n** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** | **CLI** | **NBL** | **ARL** |
|  |  |  |  |  |  |  |  |  |  |
| **3** | 5.4336 | 2 | 60.5 | 6.4342 | 53 | 2.2830 | 10.0236 | 0 | Inf |
| **4** | 5.0520 | 1 | 91.0 | 5.4003 | 50 | 1.8200 | 9.1080 | 1 | 91.0000 |
| **5** | 4.7851 | 1 | 72.0 | 4.7392 | 44 | 1.6364 | 8.2235 | 0 | Inf |
| **7** | 4.1555 | 0 | Inf | 3.9189 | 38 | 1.3684 | 7.0059 | 6 | 8.6667 |
| **10** | 3.3793 | 0 | Inf | 3.2256 | 33 | 1.0909 | 5.7929 | 9 | 4.0000 |

**4.0 Conclusion**

This study introduced Control Charts based on Generalized Gamma, which provide a unique way to improving control chart approaches for non-normal process data. The mean and variance of GGD were used to calculate the control chart limits. A simulation study was used to assess the performance of the proposed charts in comparison to the SH and SC strategies. The results show that the suggested charts may be used to monitor skewed and reverse-J distributions, give appropriate control chart limits that do not create false alarms, and detect out-of-control events faster than competing control chart approaches. The GG method's sensitivity may be useful for smaller subgroup sizes when early identification of changes is critical. Furthermore, the findings of data collected in real applications reveal that the GG approach provides better sensitivity with less variations. As a result, the offered charts offer realistic answers for companies experiencing unusual process patterns.

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