Original Research Article

Supra* Generalized Closed Sets in Supra Bitopological Spaces

Abstract

This paper aims to introduce a new class of sets in the field of supra bitopological space, which is the so-called supra star generalized closed (briefly $S_{\tau_{ij}}$-$^*g$-$closed$) sets. Also, some of the properties of supra* generalized closed sets and its complement are primarily discussed in the results of the study. Moreover, the concepts of supra star generalized closure and supra star generalized interior are introduced.

Keywords: Supra bitopological space, supra closed set, supra open set

2010 Mathematics Subject Classification:

1 Introduction

Topological space is a principal intergrating space that appears in every branch of modern mathematics. Many specialized topological spaces were developed, one of these is the supra topological space denoted as (X, S_τ), which introduced and defined by [4]. The said space has conditions that need to satisfy, such as X and $\varnothing \in S_\tau$, and S_τ is closed under arbitrary union. Over the year, a new concept coined from supra topological spaces is the so-called supra bitopological spaces denoted as X. In 2017, [2] introduced and discussed the concept of supra bitopological spaces and studied S_{τ_i}-interior and S_{τ_i}-closure in supra bitopological spaces. In the same year, [3] studied $S_{\tau_{ij}}$-semi open sets. For further study, the author of this paper is motivated to introduce the new class of sets in supra bitopological space, which is called supra star generalized closed sets. Also, some properties of supra star generalized closed and open sets, supra star generalized closure and supra star generalized interior will be investigate.

2 Preliminary Notes

To aviod redundancy of using notation within this context, $i, j \in \{1, 2\}$ and $i \neq j$ are being used. Throughout this paper X denote the supra bitopological spaces X which no separation axioms are assumed.

Definition 2.1. [4] (X, S_τ) is said to be a supra topological space if it is satisfying these conditions:
(i) $X, \varnothing \in S_\tau$.
(ii) The union of any number of sets in S_τ belongs to S_τ.
Each element $A \in S_\tau$ is called S_τ-$open\ sets$ in (X, S_τ) and the complement of S_τ-open is called S_τ-$closed$.

Definition 2.2. [4] The supra closure of a set A is denoted by S_τ-$cl(A)$ and is defined as S_τ-$cl(A) = \bigcap\{B : B$ is S_τ-$closed$ and $A \subseteq B\}$.

Definition 2.3. [4] The supra interior of a set A is denoted by S_τ-$int(A)$ and is defines as S_τ-$int(A) = \bigcup\{B : B$ is S_τ-$open$ and $B \subseteq A\}$.

Definition 2.4. [1] A subset A of a supra topological space (X, S_τ) is called S_τ-ω-$closed$ if S_τ-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is S_τ-$semi$-$open$ in (X, S_τ). The complement of S_τ-ω-$closed$ is S_τ-ω-$open$.

Definition 2.5. [5] A subset A of supra topological space (X, S_τ) is called a supra star g-closed set (briefly S_τ-$^*g$-$closed$) if S_τ-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is S_τ-ω-$open$ in (X, S_τ).

Definition 2.6. [2] If S_{τ_1} and S_{τ_2} are two supra topologies on a non-empty set X, then the triplet X is said to be a supra bitopological space. Each element of S_{τ_i} is called a supra τ_i-open sets (briefly S_{τ_i}-$open\ sets$) in X. Then the complement of τ_i-open sets are called a supra τ_i-closed sets (briefly S_{τ_i}-$closed\ sets$), for $i \in \{1, 2\}$.

Definition 2.7. [2] The S_{τ_i}-$closure$ of a the set A is denoted by S_{τ_i}-$cl(A)$ and is defined as S_{τ_i}-$cl(A) = \bigcap\{B : B$ is a S_{τ_i}-$closed$ and $A \subseteq B$ for $i \in \{1, 2\}\}$.

Definition 2.8. [2] The S_{τ_i}-$interior$ of a the set A is denoted by S_{τ_i}-$int(A)$ and is defined as S_{τ_i}-$int(A) = \bigcup\{B : B$ is a S_{τ_i}-$open$ and $B \subseteq A$ for $i \in \{1, 2\}\}$.

3 Main Results

3.1 Supra*-g Closed Sets and Supra*-g Open Sets

Definition 3.1. A subset A of a supra bitopological space X is said to be *supra τ_{ij}-star generalized closed* (briefly $S_{\tau_{ij}}$-$^*g$-*closed*) if S_{τ_i}-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is S_{τ_j}-ω-open, where $i, j \in \{1, 2\}$ and $i \neq j$. The complement of $S_{\tau_{ij}}$-$^*g$-closed set is $S_{\tau_{ij}}$-$^*g$-*open set*. The family of all $S_{\tau_{ij}}$-$^*g$-closed (resp. $S_{\tau_{ij}}$-$^*g$-open) $sets$ of X is denoted by $S_{\tau_{ij}}$-$^*g$-$C(X)$ (resp. $S_{\tau_{ij}}$-$^*g$-$O(X)$).

Definition 3.2. A subset A of a supra bitopological space X is supra τ_{ij}-star generalized open (briefly $S_{\tau_{ij}}$-$^*g$-open) if its complement is $S_{\tau_{ij}}$-$^*g$-closed set.

Example 3.1. *Let $X = \{1, 2, 3, 4\}$, consider the following supra bitopological spaces on X,*
$S_{\tau_1} = \{X, \varnothing, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 3, 4\}, \{2, 3, 4\}\}$*; and*
$S_{\tau_2} = \{X, \varnothing, \{1\}, \{3\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$.
Then,
S_{τ_1}*-closed* $= \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{2, 3, 4\}\}$*; and*
S_{τ_2}*-closed* $= \{X, \varnothing, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 4\}, \{2, 3, 4\}\}$.
Now, S_{τ_2}*-*ω*-open* $= \{X, \varnothing, \{1\}, \{3\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\}$.
Suppose $A = \{1, 4\}$, where $\{1, 4\}$ is a subset of $\{1, 2, 4\}$, and X in U, which is $U = S_{\tau_2}$-ω-open. This implies that S_{τ_1}-$cl(A) = S_{\tau_1}$-$cl(\{1, 4\}) = \{1, 4\}$. So, $\{1, 4\} \subseteq \{1, 2, 4\}$ and X. Hence, $\{1, 4\}$ is a $S_{\tau_{12}}$-$^*g$-closed set. The family of all $S_{\tau_{12}}$-$^*g$-closed sets and its complements are,

$S_{\tau_{12}}$-$^{*}g$-$C(X)$ $= \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$; *and*
$S_{\tau_{12}}$-$^{*}g$-$O(X)$ $= \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$.

Theorem 3.2. *A subset A of a supra bitopological space X is said to be supra τ_{ij}-star generalized open (briefly $S_{\tau_{ij}}$-$^{*}g$-open) if and only if $F \subseteq S_{\tau_i}$-$int(A)$ whenever $F \subseteq A$ and F is a S_{τ_j}-ω-closed.*

Proof. Suppose that A is $S_{\tau_{ij}}$-$^{*}g$-open. Let $F \subseteq A$ and F be S_{τ_j}-ω-closed. Then F^c is S_{τ_j}-ω-open and $A^c \subseteq F^c$. Since A is $S_{\tau_{ij}}$-$^{*}g$-open, A^c is $S_{\tau_{ij}}$-$^{*}g$-closed, by the Definition 3.1, S_{τ_i}-$cl(A^c) \subseteq F^c$ and note that S_{τ_i}-$cl(A^c) = (S_{\tau_i}$-$int(A))^c$. Hence, $F \subseteq S_{\tau_i}$-$int(A)$.

Conversely, suppose that $F \subseteq S_{\tau_i}$-$int(A)$ where F is a S_{τ_j}-ω-closed and $F \subseteq A$. Then $A^c \subseteq F^c$, where F^c is S_{τ_j}-ω-open. Since $F \subseteq S_{\tau_i}$-$int(A)$, it follows $(S_{\tau_i}$-$int(A))^c \subseteq F^c$ that is S_{τ_i}-$cl(A^c) \subseteq F^c$ since S_{τ_i}-$cl(A^c) = (S_{\tau_i}$-$int(A))^c$. Thus A^c is $S_{\tau_{ij}}$-$^{*}g$-closed and A is $S_{\tau_{ij}}$-$^{*}g$-open. □

Remark 3.1. *The union of two $S_{\tau_{ij}}$-$^{*}g$-closed (resp. $S_{\tau_{ij}}$-$^{*}g$-open) sets need not be $S_{\tau_{ij}}$-$^{*}g$-closed (resp. $S_{\tau_{ij}}$-$^{*}g$-open) as seen from the following example.*

Example 3.3. *Consider $S_{\tau_{12}}$-$^{*}g$-$C(X) = \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$. Suppose $A = \{2\}$ and $B = \{1,4\}$. Now, $A \cup B = \{2\} \cup \{1,4\} = \{1,2,4\}$, implies $\{1,2,4\} \notin S_{\tau_{ij}}$-$^{*}g$-closed set.*

Consider $S_{\tau_{12}}$-$^{*}g$-$O(X) = \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$. Suppose $A = \{4\}$ and $B = \{1,2\}$. Now, $A \cup B = \{4\} \cup \{1,2\} = \{1,2,4\}$, implies that $\{1,2,4\} \notin S_{\tau_{ij}}$-$^{*}g$-open set.

Remark 3.2. *The intersection of two $S_{\tau_{ij}}$-$^{*}g$-closed (resp. $S_{\tau_{ij}}$-$^{*}g$-open) sets need not be $S_{\tau_{ij}}$-$^{*}g$-closed (resp. $S_{\tau_{ij}}$-$^{*}g$-open) as seen from the following example.*

Example 3.4. *Consider $S_{\tau_{12}}$-$^{*}g$-$C(X) = \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$. Suppose $A = \{2,3\}$ and $B = \{3,4\}$. Now, $A \cap B = \{2,3\} \cap \{3,4\} = \{3\}$. Hence, $\{3\} \notin S_{\tau_{12}}$-$^{*}g$-closed set.*

Consider $S_{\tau_{12}}$-$^{*}g$-$O(X) = \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$. Suppose $A = \{1,2,3\}$ and $B = \{1,3,4\}$. Now, $A \cap B = \{1,2,3\} \cap \{1,3,4\} = \{3\}$. Thus, $\{3\} \notin S_{\tau_{12}}$-$^{*}g$-open set.

Remark 3.3. *$S_{\tau_{12}}$-$^{*}g$-$C(X)$ (resp. $S_{\tau_{12}}$-$^{*}g$-$O(X)$) is generally not equal to $S_{\tau_{21}}$-$^{*}g$-$C(X)$ (resp. $S_{\tau_{12}}$-$^{*}g$-$O(X)$) as can be seen from the following example.*

Example 3.5. *Consider,*
$S_{\tau_{12}}$-$^{*}g$-$C(X)$ $= \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$; *and*
$S_{\tau_{21}}$-$^{*}g$-$C(X)$ $= \{X, \varnothing, \{1\}, \{3\}, \{4\}, \{1,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$.
Thus, $S_{\tau_{12}}$-$^{*}g$-$C(X) \neq S_{\tau_{21}}$-$^{*}g$-$C(X)$. Also,
$S_{\tau_{12}}$-$^{*}g$-$O(X)$ $= \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$; *and*
$S_{\tau_{21}}$-$^{*}g$-$O(X)$ $= \{X, \varnothing, \{1\}, \{3\}, \{4\}, \{1,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$.
Hence, $S_{\tau_{12}}$-$^{*}g$-$O(X) \neq S_{\tau_{21}}$-$^{*}g$-$O(X)$.

Theorem 3.6. *Let A be a supra bitopological space X. If A is $S_{\tau_{ij}}$-$^{*}g$-closed, then S_{τ_i}-$cl(A) \setminus A$ contains no non empty S_{τ_j}-ω-closed set.*

Proof. Let A be $S_{\tau_{ij}}$-$^{*}g$-closed. Suppose F be S_{τ_j}-ω-closed set and S_{τ_i}-$cl(A) \setminus A$ contains F where $F \neq \varnothing$. That is $F \subseteq S_{\tau_i}$-$cl(A) \setminus A$, where $F \neq \varnothing$. Note that S_{τ_i}-$cl(A) \setminus A = S_{\tau_i}$-$cl(A) \cap A^c$. This implies $F \subseteq S_{\tau_i}$-$cl(A) \cap A^c$. Thus, $F \subseteq S_{\tau_i}$-$cl(A)$ and $F \subseteq A^c$. Since A is $S_{\tau_{ij}}$-$^{*}g$-closed, by Definition 3.1, S_{τ_i}-$cl(A) \subseteq F^c$, where F^c be S_{τ_j}-ω-open and $A \subseteq F^c$, then $F \subseteq (S_{\tau_i}$-$cl(A))^c$. Now, $F \subseteq S_{\tau_i}$-$cl(A) \cap (S_{\tau_i}$-$cl(A))^c = \varnothing$. It follows that $F = \varnothing$, which is a contradiction. Therefore, S_{τ_i}-$cl(A) \setminus A$ contains no non empty S_{τ_j}-ω-closed set. □

Remark 3.4. *The converse of the above Theorem 3.6 is not true as seen from the following example.*

Example 3.7. *Consider* $S_{\tau_{12}}$-$^*g$-$C(X) = \{X, \varnothing, \{1\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}\}$. *And,* S_{τ_2}-ω-*open* $= \{X, \varnothing, \{1\}, \{3\}, \{1,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}\}$. *Suppose* $A = \{1,2\}$, *then* S_{τ_1}-$cl(A) \setminus A = S_{\tau_i}$-$cl(\{1,2\}) \setminus \{1,2\} = \{1,2,3\} \setminus \{1,2\} = \{3\}$ *which does not contain any non empty* S_{τ_2}-ω-*closed set. Hence,* $A \notin S_{\tau_{12}}$-$^*g$-*closed.*

Theorem 3.8. *If subset* A *is* S_{τ_j}-ω-*closed set, then* S_{τ_i}-$cl(A) \setminus A$ *is* S_{τ_j}-ω-*open.*

Proof. Let A be S_{τ_j}-ω-closed set. Let $F \subseteq S_{\tau_i}$-$cl(A) \setminus A$ where F is S_{τ_j}-ω-closed set. Since A is S_{τ_j}-ω-closed, we have S_{τ_i}-$cl(A) \setminus A$ does not contain non empty S_{τ_j}-ω-closed, by Theorem 3.6. Consequently, $F = \varnothing$. Therefore, $F = \varnothing \subseteq S_{\tau_i}$-$cl(A) \setminus A$, and so $F = \varnothing \subseteq S_{\tau_i}$-$int(S_{\tau_i}$-$cl(A) \setminus A)$. Hence, S_{τ_i}-$cl(A) \setminus A$ is S_{τ_j}-ω-open. ☐

Theorem 3.9. *If* A *is* $S_{\tau_{ij}}$-$^*g$-*closed set of* X *such that* $A \subseteq B \subseteq S_{\tau_i}$-$cl(A)$, *then* B *is a* $S_{\tau_{ij}}$-$^*g$-*closed set of* X.

Proof. Suppose that A is $S_{\tau_{ij}}$-$^*g$-closed set and $A \subseteq B \subseteq S_{\tau_i}$-$cl(A)$. Let $B \subseteq U$ and U is S_{τ_j}-ω-open. Given $A \subseteq B$, the $A \subseteq U$. Since A is $S_{\tau_{ij}}$-$^*g$-closed, we have S_{τ_i}-$cl(A) \subseteq U$. Since $B \subseteq S_{\tau_i}$-$cl(A)$, then S_{τ_i}-$cl(B) \subseteq S_{\tau_i}$-$cl(S_{\tau_i}$-$cl(A)) = S_{\tau_i}$-$cl(A) \subseteq U$, implies S_{τ_i}-$cl(B) \subseteq U$. Therefore, B is $S_{\tau_{ij}}$-$^*g$-closed set. ☐

Theorem 3.10. *Let* A *and* B *be subset of supra bitopological space* X *such that* S_{τ_i}-$int(A) \subseteq B \subseteq A$. *If* A *is a* $S_{\tau_{ij}}$-$^*g$-*open set, then* B *is* $S_{\tau_{ij}}$-$^*g$-*open set.*

Proof. Let A be $S_{\tau_{ij}}$-$^*g$-open. Let U be a S_{τ_j}-ω-closed such that $U \subseteq B$. Since $U \subseteq B$ and $B \subseteq A$, we have $U \subseteq A$. by assumption, $U \subseteq S_{\tau_i}$-$int(A)$. Since S_{τ_i}-$int(A) \subseteq B$, we have S_{τ_i}-$int(S_{\tau_i}$-$int(A)) \subseteq S_{\tau_i}$-$int(B)$. Therefore, S_{τ_i}-$int(A) \subseteq S_{\tau_i}$-$int(B)$. Consequently, $U \subseteq S_{\tau_i}$-$int(B)$. Hence, B is $S_{\tau_{ij}}$-$^*g$-open. ☐

3.2 Supra*-g Closure

Definition 3.3. Let X be a supra bitopological space and $A \subseteq X$. An element $x \in X$ is called $S_{\tau_{ij}}$-$^*g$-$adherent$ to A if $V \cap A \neq \varnothing$ for every $S_{\tau_{ij}}$-$^*g$-open set V containing x. The set of all $S_{\tau_{ij}}$-$^*g$-$adherent\ points\ of\ A$ is called the $S_{\tau_{ij}}$-$^*g$-$closure\ of\ A$ denoted by $S_{\tau_{ij}}$-$^*g$-$cl(A)$.

Theorem 3.11. *Let* X *be a supra bitopological space and* $A \subseteq X$, *then* $S_{\tau_{ij}}$-$^*g$-$cl(A) = \bigcap\{F : F$ *is* $S_{\tau_{ij}}$-$^*g$-$closed$ *and* $A \subseteq F\}$.

Proof. Let A be a subset of a supra bitopological space. Suppose $x \in S_{\tau_{ij}}$-$^*g$-$cl(A)$. Then $V \cap A \neq \varnothing$ for every $S_{\tau_{ij}}$-$^*g$-open set V containing x. Suppose $x \notin \bigcap\{F : F$ is $S_{\tau_{ij}}$-$^*g$-closed and $A \subseteq F\}$. Then $x \notin F$ for some $S_{\tau_{ij}}$-$^*g$-closed set F and so $x \in F^c$ for some $S_{\tau_{ij}}$-$^*g$-open F^c. Thus $F^c \cap A = \varnothing$ for some $S_{\tau_{ij}}$-$^*g$-open F^c containing x, which is a contradiction. Hence, $x \in \bigcap\{F : F$ is $S_{\tau_{ij}}$-$^*g$-closed and $A \subseteq F\}$. Consequently, $S_{\tau_{ij}}$-$^*g$-$cl(A) \subseteq \bigcap\{F : F$ is $S_{\tau_{ij}}$-$^*g$-closed and $A \subseteq F\}$. Next, ley $y \in \bigcap\{F : F$ is $S_{\tau_{ij}}$-$^*g$-closed and $A \subseteq F\}$. Then $y \in F$ for all $S_{\tau_{ij}}$-$^*g$-closed set such that $A \subseteq F$. Suppose $y \notin S_{\tau_{ij}}$-$^*g$-$cl(A)$. Then $V \cap A = \varnothing$ for some $S_{\tau_{ij}}$-$^*g$-closed set V containing y. Hence V^c is $S_{\tau_{ij}}$-$^*g$-closed set such that $A \subseteq V^c$ and $y \notin V^c$, a contradiction. Thus $y \in S_{\tau_{ij}}$-$^*g$-$cl(A)$. ☐

Lemma 3.12. *Let* X *be a supra bitopological space. The following properties hold:*
(i) $A \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A)$; *and*
(ii) *If* $A \subseteq B$, *then* $S_{\tau_{ij}}$-$^*g$-$cl(A) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(B)$

Proof. Let $A, B \subseteq X$.

(i) Let $x \in A$ and suppose $x \notin S_{\tau_{ij}}$-$^*g$-$cl(A)$. Then there exists a $S_{\tau_{ij}}$-$^*g$-open set V containing x such that $V \cap A = \varnothing$,this is a contradiction since $x \in A$. Thus, $x \in S_{\tau_{ij}}$-$^*g$-$cl(A)$.

(ii) Let $x \in S_{\tau_{ij}}$-$^*g$-$cl(A)$. Thus, for all $S_{\tau_{ij}}$-$^*g$-open set V containing x, $V \cap A \neq \varnothing$. Since $A \subseteq B$, $\varnothing \neq V \cap A \subseteq V \cap B$, and so $V \cap B \neq \varnothing$ for every V such that $x \in V$. Therefore x is $S_{\tau_{ij}}$-$^*g$-$cl(B)$. □

Theorem 3.13. *Let A and B be subsets of a supra bitopological space X. Then the following properties hold:*

(i) $S_{\tau_{ij}}$-$^*g$-$cl(A) \cup S_{\tau_{ij}}$-$^*g$-$cl(B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A \cup B)$*; and*

(ii) $S_{\tau_{ij}}$-$^*g$-$cl(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A) \cap S_{\tau_{ij}}$-$^*g$-$cl(B)$

Proof. Let $A, B \subseteq X$

(i) Note that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Lemma 3.12 (ii), it follows that $S_{\tau_{ij}}$-$^*g$-$cl(A) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A \cup B)$ and $S_{\tau_{ij}}$-$^*g$-$cl(B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A \cup B)$. Thus, $S_{\tau_{ij}}$-$^*g$-$cl(A) \cup S_{\tau_{ij}}$-$^*g$-$cl(B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A \cup B)$.

(ii) Note that $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Then $S_{\tau_{ij}}$-$^*g$-$cl(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A)$ and $S_{\tau_{ij}}$-$^*g$-$cl(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(B)$, by Lemma 3.12 (ii). Thus, $S_{\tau_{ij}}$-$^*g$-$cl(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A) \cap S_{\tau_{ij}}$-$^*g$-$cl(B)$. □

Theorem 3.14. *Let X be a supra bitopological space. If A is $S_{\tau_{ij}}$-$^*g$-closed, then $S_{\tau_{ij}}$-$^*g$-$cl(A) = A$.*

Proof. Let A be $S_{\tau_{ij}}$-$^*g$-closed. Note that by Lemma 3.12 (i), $A \subseteq S_{\tau_{ij}}$-$^*g$-$cl(A)$. It suffices to show that $S_{\tau_{ij}}$-$^*g$-$cl(A) \subseteq A$. Suppose $x \in S_{\tau_{ij}}$-$^*g$-$cl(A)$. Then for all $S_{\tau_{ij}}$-$^*g$-open set V containing x, $V \cap A \neq \varnothing$. Suppose on the contrary $x \notin A$. Then $x \in A^c$ where A^c is $S_{\tau_{ij}}$-$^*g$-open set and $A^c \cap A = \varnothing$. This is a contradiction since $x \in S_{\tau_{ij}}$-$^*g$-$cl(A)$. Thus, $x \in A$, consequently $S_{\tau_{ij}}$-$^*g$-$cl(A) \subseteq A$. Therefore, $S_{\tau_{ij}}$-$^*g$-$cl(A) = A$. □

Proposition 3.1. *X and $\varnothing$ are both $S_{\tau_{ij}}$-$^*g$-closed sets.*

Proof. Let U be S_{τ_j}-ω-open such that $X \subseteq U$. Note that X is the only S_{τ_j}-ω-open containing X. Now, $S_{\tau_{ij}}$-$^*g$-$cl(X) \subseteq X$, and so X is $S_{\tau_{ij}}$-$^*g$-closed.

Additionally, let U be S_{τ_j}-ω-open such that $\varnothing \subseteq U$. Note that $\varnothing$ is the subset of all S_{τ_j}-ω-open $sets$. Now, $S_{\tau_{ij}}$-$^*g$-$cl(\varnothing) = \varnothing \subseteq U$. Therefore, $\varnothing$ is also a $S_{\tau_{ij}}$-$^*g$-closed. □

Theorem 3.14 and Proposition 3.1, support the claim of the remrk below.

Remark 3.5. *If a subset A of supra bitopological space, then*

(i) $S_{\tau_{ij}}$-$^*g$-$cl(X) = X$*; and*

(ii) $S_{\tau_{ij}}$-$^*g$-$cl(\varnothing) = \varnothing$.

3.3 Supra*-g Interior

Definition 3.4. Let X be a supra bitopological space and $A \subseteq X$. An element $x \in A$ is called $S_{\tau_{ij}}$-$^*g$-$interior\ point\ of\ A$ if there exists an $S_{\tau_{ij}}$-$^*g$-open set G such that $x \in G \subseteq A$. The set of all $S_{\tau_{ij}}$-$^*g$-$interior\ points\ of\ A$ is called the $S_{\tau_{ij}}$-$^*g$-$interior\ point\ of\ A$ denoted by $S_{\tau_{ij}}$-$^*g$-$int(A)$.

Theorem 3.15. *Let X be a supra bitopological space and $A \subseteq X$. Then $S_{\tau_{ij}}$-$^*g$-$int(A) = \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$.*

Proof. Let $A \subseteq X$. Suppose $x \in S_{\tau_{ij}}$-$^*g$-$int(A)$. Then x is a $S_{\tau_{ij}}$-$^*g$-$interior\ of\ A$. Then there exists a $S_{\tau_{ij}}$-$^*g$-open set G such that $x \in G \subseteq A$. Hence, $x \in \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$. Thus, $S_{\tau_{ij}}$-$^*g$-$int(A) = \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$. Suppose, $y \in \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$. Then there exists G which is $S_{\tau_{ij}}$-$^*g$-open such that $y \in G \subseteq A$. Hence, by Definition 3.4, $y \in S_{\tau_{ij}}$-$^*g$-$int(A)$. □

Lemma 3.16. *Let A and B be subsets of a supra bitopological space X. The following properties hold:*

(i) $S_{\tau_{ij}}$-$^*g$-$int(A) \subseteq A$*; and*

(ii) *If $A \subseteq B$, then $S_{\tau_{ij}}$-$^*g$-$int(A) \subseteq S_{\tau_{ij}}$-$^*g$-$int(B)$.*

Proof. Let $A, B \subseteq X$. Then,

(i) Let $x \in S_{\tau_{ij}}$-$^*g$-$int(A)$. Then by Theorem 3.15, $x \in \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$. It means that $x \in G$ for some $S_{\tau_{ij}}$-$^*g$-open set G such that $G \subseteq A$ and so $x \in A$. Hence, $S_{\tau_{ij}}$-$^*g$-$int(A) \subseteq A$.

(ii) Let $x \in S_{\tau_{ij}}$-$^*g$-$int(A)$. Thus, for some $S_{\tau_{ij}}$-$^*g$-open set G containing x, $G \subseteq A$. Since $G \subseteq A \subseteq B$, then $G \subseteq B$. So $x \in G \subseteq B$, where G is $S_{\tau_{ij}}$-$^*g$-open, it follows that $x \in S_{\tau_{ij}}$-$^*g$-$int(B)$. Therefore, $S_{\tau_{ij}}$-$^*g$-$int(A) \subseteq S_{\tau_{ij}}$-$^*g$-$int(B)$. ☐

Theorem 3.17. *Let A and B be subsets of a supra bitopological space X. Then the following properties hold:*

(i) $S_{\tau_{ij}}$-$^*g$-$int(A) \cup S_{\tau_{ij}}$-$^*g$-$int(B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A \cup B)$*; and*

(ii) $S_{\tau_{ij}}$-$^*g$-$int(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A) \cap S_{\tau_{ij}}$-$^*g$-$int(B)$.

Proof. Let $A, B \subseteq X$.

(i) Note that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By Lemma 3.16 (ii), it follows that $S_{\tau_{ij}}$-$^*g$-$int(A) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A \cup B)$ and $S_{\tau_{ij}}$-$^*g$-$int(B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A \cup B)$. Thus, $S_{\tau_{ij}}$-$^*g$-$int(A) \cup S_{\tau_{ij}}$-$^*g$-$int(B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A \cup B)$.

(ii) Note that $A \cap B \subseteq A$ and $A \cap B \subseteq B$. By Lemma 3.16 (ii), it follows that $S_{\tau_{ij}}$-$^*g$-$int(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A)$ and $S_{\tau_{ij}}$-$^*g$-$int(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(B)$. Thus, $S_{\tau_{ij}}$-$^*g$-$int(A \cap B) \subseteq S_{\tau_{ij}}$-$^*g$-$int(A) \cap S_{\tau_{ij}}$-$^*g$-$int(B)$. ☐

Theorem 3.18. *Let X be a supra bitopological space and $A \subseteq X$. If A is $S_{\tau_{ij}}$-$^*g$-open, then $A = S_{\tau_{ij}}$-$^*g$-$int(A)$.*

Proof. Let A be a $S_{\tau_{ij}}$-$^*g$-open set. Then,$A \in \bigcup\{G : G$ is $S_{\tau_{ij}}$-$^*g$-open and $G \subseteq A\}$. Since every member in the collection is $S_{\tau_{ij}}$-$^*g$-open and A is in collection, it follows that the union of this collection is A. Thus, by Lemma 3.16, $A = S_{\tau_{ij}}$-$^*g$-$int(A)$ ☐

Proposition 3.1 supports the claim of remrk 3.6 (i) as complement of $S_{\tau_{ij}}$-$^*g$-closed set. By Theorem 3.18, the remrk 3.6 (ii) and (iii) hold.

Remark 3.6. *If a subset A be a supra bitopological space, then,*

(i) *X and $\varnothing$ are both $S_{\tau_{ij}}$-$^*g$-open set s;*

(ii) $S_{\tau_{ij}}$-$^*g$-$int(X) = X$*; and*

(iii) $S_{\tau_{ij}}$-$^*g$-$int(\varnothing) = \varnothing$.

4 CONCLUSIONS

In this study, the researchers defined and investigated the concept of the supra-star generalized closed (briefly $S_{\tau_{ij}}$-$^*g$-closed) sets in supra bitopological space. In addition, some properties of $S_{\tau_{ij}}$-$^*g$-closed set and $S_{\tau_{ij}}$-$^*g$-open set are established. The result shows that the union and intersection of the two $S_{\tau_{ij}}$-$^*g$-closed set s (resp. $S_{\tau_{ij}}$-$^*g$-open sets) need not to be a $S_{\tau_{ij}}$-$^*g$-closed (resp. $S_{\tau_{ij}}$-$^*g$-open). Also, the researcher finds out that $S_{\tau_{12}}$-$^*g$-$C(X)$ (resp. $S_{\tau_{12}}$-$^*g$-$O(X)$) is generally not equal to $S_{\tau_{21}}$-$^*g$-$C(X)$ (resp. $S_{\tau_{21}}$-$^*g$-$O(X)$). Moreover, some properties of supra star generalized closure (briefly $S_{\tau_{ij}}$-$^*g$-$cl(A)$) and supra star generalized interior (briefly $S_{\tau_{ij}}$-$^*g$-$int(A)$) are analyzed and established. In general, the researchers obtained the desired objectives.

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