***Original Research Article***

**Second Type of Second Order Slope Rotatable Designs utilizing Balanced Incomplete Block Designs**

**ABSTRACT**

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| --- |
| Kim and Ko proposed second type of second order slope rotatable designs (SOSRD) utilizing central composite designs (CCD) wherein the two digits  denotes the position of star points. In this study, we propose second type of SOSRD utilizing balanced incomplete block designs (BIBD) for 3≤v≤16 (v: factors). In specific cases, the recommended procedure results in fewer design points than second type of SOSRD acquired utilizing CCD of Kim and Ko, Ravikumar and Victorbabu. The first order partial derivative's for estimated second order response variance for factors 3≤v≤16 is also obtained. |

***Keywords:*** *Response surface designs, Second order slope rotatable designs, Second order slope rotatable designs of second type.*

1. **INTRODUCTION**

A response surfaces are set of statistical and mathematical models used to analyze the issues when several explanatory variables have an impact on a response variable. Box and Hunter [1] suggested the rotatability property for exploring response surface models and developed rotatable central composite designs (CCD). Das and Narasimham [6] studied second order rotatable designs (SORD) utilizing balanced incomplete block designs (BIBD). Kim [10] introduced extended CCD with the axial points indicated by two numbers. Victorbabu and Vasundharadevi [26] developed modified quadratic response surface models utilizing BIBD. Jyostna and Victorbabu [8] constructed modified rotatability measure for a quadratic polynomial utilizing BIBD. Jyostna et al. [9] studied modified rotatability measure for quadratic polynomial models utilizing CCD. Chiranjeevi et al. [5] developed second type of SORD utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] constructed second type of SORD utilizing BIBD, pairwise balanced designs (PBD), symmetrical unequal block arrangements (SUBA) with two unequal block sizes respectively. Ravikumar and Victorbabu [17] developed second type of SORD utilizing BIBD with unequal block sizes.

Slope rotatable central composite designs (SRCCD) were first developed by Hader and Park [7]. Victorbabu and Narasimham [22, 23] studied second order slope rotatable designs (SOSRD) utilizing BIBD and pair of incomplete block designs respectively. Victorbabu [19, 20] introduced modified SOSRD utilizing CCD and BIBD respectively. A review was proposed by Victorbabu [21] on SOSRD. Park and Kim [12] developed a measure of slope rotatability for second order response surface experimental designs. Victorbabu and Surekha [24, 25] constructed SOSRD measure utilizing CCD and BIBD. Specifically, Kim and Ko [11] introduced the second type of slope rotatability of CCD for the factors 2≤v≤5 by taking  (‘na’ denotes the number of replications of axial points), where in the two numbers  represent the positions of the star points. Ravikumar and Victorbabu [14] extended the work of Kim and Ko [11] and developed second type of SOSRD utilizing CCD for 6≤v≤17 by taking Ravikumar and Victorbabu [15, 16, 18] studied SRCCD of second type for 2≤v≤17 with , SOSRD of second type utilizing PBD and SUBA with two unequal block sizes respectively.

In this study, we suggest SOSRD of second type utilizing BIBD. The suggested procedure is found to sometimes result in an SOSRD of second type with fewer design points compared to SOSRD of second type utilizing CCD of Kim and Ko [11], Ravikumar and Victorbabu [14]. Specifically for 7, 9 and 13 factors these new designs needs 85, 133 and 261 points whereas corresponding SOSRD of second type utilizing CCD need 93, 165 and 309 design points respectively (including one central point).

1. **STIPULATIONS FOR SECOND ORDER SLOPE ROTATABLE DESIGNS**

A general quadratic polynomial model  for fitting

 (2.1)

In which indicates the sth factor level in the experiment’s wth run (w=1,2,…,N) and ’s are uncorrelated random errors having a mean ‘0’ and variation of . D is then referred to as SOSRD if  with regard to every explanatory variable  is aof the point from the origin (center) of the design.

The general circumstances for SOSRD are given below(cf. [1, 7, 22]).

All moments of odd order are ‘0’. In simple terms when minimum of one odd power X equals zero. i.e;

1. 
2. (i) 

(ii) 

1.  (2.2)

where c, and are constants.

The variances and covariances of the estimated parameters are











 (2.3)

and the remaining covariances disappear.

An inspection of shows an essential condition for the existence of a non singular second order design is

1.  (2.4)

For the second order model

 (2.5)

 (2.6)

The criteria for R.H.S of (2.6) to be a alone (for slope rotatability) is

 (cf. [7]) (2.7)

Simplifying (2.7) using (2.3), we get

1.  (cf. [22]) (2.8)

Therefore A, B, C of (2.2), (2.4) and (2.8) suggest a set of conditions for slope rotatability in any general quadratic model. (cf. [7, 22]).

On simplification of equation (2.6) we get  (2.9)

1. **NEW SECOND TYPE OF SOSRD UTILIZING BALANCED INCOMPLETE BLOCK DESIGNS**

Kim [10] developed second type of rotatable CCD, in which the two digits  are used to denote the star points for 2≤v≤8. Chiranjeevi et al. [5] introduced second type of SORD utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] studied developed second type of SORD utilizing BIBD, PBD and SUBA with two unequal block sizes respectively. Specifically, Kim and Ko [11] constructed second type of slope rotatability of CCD 2≤v≤5 by taking. Ravikumar and Victorbabu [14] extended the results of Kim and Ko [11] and developed SOSRD of second type utilizing CCD for by taking. Ravikumar and Victorbabu [15, 16, 18] studied SRCCD of second type for 2≤v≤17 with , SOSRD of second type utilizing PBD and SUBA with two unequal block sizes respectively.

The design plan of SOSRD of second type using BIBD in which the position of the axial points are indicated by two numbers  and  (≥≥0). Let (v, m, r, k, λ) denote a BIBD, 2t(k) indicate the fractional replicate of  in ±1 levels, wherein no interaction is confounded with fewer than five factors.  represents the central points.

Let  indicates the design points produced from transposed incidence matrix of BIBD. Let are the design points produced from BIBD by multiplication (cf. [13]). We employ the extra set of points like are two axial points sets. Here  indicate the 4v axial points produced from and point sets. Let  indicate the union of design points produced from various point sets and central points represented by. Following the methods of [11], [14] we suggest a method of construction on second type of SOSRD utilizing BIBD as shown in below theorem.

**THEOREM (3.1):**

The design points,  will result in a v-dimensional SOSRD of second type utilizing BIBD in  design points with the following biquadratic equation

 (3.1)

The design exists if the equation mentioned above (3.1) contains at least one positive real root.

c can be obtained from (3.2)

(Evolution of (3.1) is explained in (3.4) below)

**Proof:** Regarding the design points produced from second type of SOSRD utilizing BIBD, simple symmetry stipulations A, B and C of equation (2.2) are true. Since condition A of equation (2.2) is obviously true, condition B and C of (2.2) are also true as follows.

1. (i) 

(ii) 

1.  (3.3)

From B(ii) and C of (3.3), we have .

The result of simplifying equation (2.8) by substituting c, and is

 (3.4)

The biquadratic equation shown in (3.1) is obtained by simplifying (3.4).

**EXAMPLE:**

Construction on second type of SOSRD for 7-factors with the help of a BIBD (v=7, b=7, r=3, k=3, λ=1). The design points,

 will result in a second type of SOSRD utilizing BIBD in N=85 design points with  and .

Here B and C of equation (3.3) are

1. (i) 

(ii) 

1.  (3.5)

From B (ii) and C of equation (3.5), we have 

Substitute c,  and  in equation (2.8) and after simplifying, we obtain the biquadratic equation that follows.



Substitute  in the above equation and on simplification, we get

 (3.6)

(This can be alternatively written directly from equation (3.1) of theorem 3.1)

Only a single positive real root exists in equation (3.6) . The non-singularity criterion D of (2.4) is fulfilled.

In the context of 7-factors, this new approach contains 85 design points, while the corresponding SOSRD of second type obtained utilizing CCD of Ravikumar and Victorbabu [14] need 93 design points. (please see Appendix for v=9, 13 factors)

1. **CONCLUSION**

In this paper, second type of SOSRD utilizing BIBD is suggested. It is observed that the proposed procedure can generate designs with fewer design points than SOSRD of second type acquired utilizing CCD.

The Appendix table 1 gives the appropriate slope rotatability values of the parameter with  for designs using a BIBD and star points and for different number of central points for 3≤v≤16.

Table 2 gives the variance of estimated response of the first order partial derivative of second type SOSRD utilizing BIBD for different factors for 3≤v≤16.

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**APPENDIX**

**Table 1:** Values of a2 for SOSRD of second type using BIBD for 3≤v≤16 with 

[These are SOSRDs of second type with design points ]

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| (3, 3, 2, 2, 1) | | |  | (4, 6, 3, 2, 1) | | |  | (4, 4, 3, 3, 2) | | |  | (5, 10, 4, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 24 | 1.9337 |  | 0 | 40 | 1.8605 |  | 0 | 48 | 2.5563 |  | 0 | 60 | 1.7995 |
| 1 | 25 | 1.8837 |  | 1 | 41 | 1.8185 |  | 1 | 49 | 2.5338 |  | 1 | 61 | 1.7601 |
| 5 | 29 | 1.7346 |  | 5 | 45 | 1.6723 |  | 5 | 53 | 2.4609 |  | 5 | 65 | 1.6044 |
| 10 | 34 | 1.6281 |  | 10 | 50 | 1.5417 |  | 10 | 58 | 2.3989 |  | 10 | 70 | 1.4284 |
| 15 | 39 | 1.5685 |  | 15 | 55 | 1.4583 |  | 15 | 63 | 2.3574 |  | 15 | 75 | 1.2937 |
| 20 | 44 | 1.5319 |  | 20 | 60 | 1.4042 |  | 20 | 68 | 2.3285 |  | 20 | 80 | 1.1982 |

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| (5, 5, 4, 4, 3) | | |  | (5, 10, 6, 3, 3) | | |  | (6, 15, 5, 2, 1) | | |  | (6, 10, 5, 3, 2) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 100 | 3.2614 |  | 0 | 100 | 2.7125 |  | 0 | 84 | 1.7459 |  | 0 | 104 | 2.4233 |
| 1 | 101 | 3.2512 |  | 1 | 101 | 2.6997 |  | 1 | 85 | 1.7064 |  | 1 | 105 | 2.4061 |
| 5 | 105 | 3.2150 |  | 5 | 105 | 2.6540 |  | 5 | 89 | 1.5278 |  | 5 | 109 | 2.3447 |
| 10 | 110 | 3.1787 |  | 10 | 110 | 2.6075 |  | 10 | 94 | 1.2130 |  | 10 | 114 | 2.2834 |
| 15 | 115 | 3.1502 |  | 15 | 115 | 2.5705 |  |  | 15 | 119 | 2.2368 |
| 20 | 120 | 3.1273 |  | 20 | 120 | 2.5410 |  |  | 20 | 124 | 2.2012 |

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| (6, 6, 5, 5, 4) | | |  | (6, 15, 10, 4, 6) | | |  | (7, 7, 3, 3, 1) | | |  | (7, 7, 4, 4, 2) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 120 | 3.4887 |  | 0 | 264 | 3.7401 |  | 0 | 84 | 2.0134 |  | 0 | 140 | 2.8608 |
| 1 | 121 | 3.4794 |  | 1 | 265 | 3.7360 |  | 1 | 85 | 1.9808 |  | 1 | 141 | 2.8492 |
| 5 | 125 | 3.4467 |  | 5 | 269 | 3.7203 |  | 5 | 89 | 1.8718 |  | 5 | 145 | 2.8079 |
| 10 | 130 | 3.4140 |  | 10 | 274 | 3.7024 |  | 10 | 94 | 1.7823 |  | 10 | 150 | 2.7667 |
| 15 | 135 | 3.3883 |  | 15 | 279 | 3.6862 |  | 15 | 99 | 1.7273 |  | 15 | 155 | 2.7345 |
| 20 | 140 | 3.3678 |  | 20 | 284 | 3.6716 |  | 20 | 104 | 1.6914 |  | 20 | 160 | 2.7091 |

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| (7, 7, 6, 6, 5) | | |  | (7, 21, 6, 2, 1) | | |  | (8, 14, 7, 4, 3) | | |  | (8, 28, 7, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 252 | 4.3220 |  | 0 | 112 | 1.6971 |  | 0 | 256 | 3.0740 |  | 0 | 144 | 1.6515 |
| 1 | 253 | 4.3179 |  | 1 | 113 | 1.6557 |  | 1 | 257 | 3.0667 |  | 1 | 145 | 1.6061 |
| 5 | 257 | 4.3021 |  | 5 | 117 | 1.4252 |  | 5 | 261 | 3.0393 |  |
| 10 | 262 | 4.2847 |  |  | 10 | 266 | 3.0091 |  |
| 15 | 267 | 4.2693 |  |  | 15 | 271 | 2.9829 |  |
| 20 | 272 | 4.2557 |  |  | 20 | 276 | 2.9603 |  |

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| (9, 12, 4, 3, 1) | | |  | (9, 18, 8, 4, 3) | | |  | (9, 12, 8, 6,) | | |  | (9, 36, 8, 2, 1) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 132 | 1.8826 |  | 0 | 324 | 3.0132 |  | 0 | 420 | 4.2153 |  | 0 | 180 | 1.6073 |
| 1 | 133 | 1.8426 |  | 1 | 325 | 3.0059 |  | 1 | 421 | 4.2120 |  | 1 | 181 | 1.5550 |
| 5 | 137 | 1.6957 |  | 5 | 329 | 2.9785 |  | 5 | 425 | 4.1995 |  |
| 10 | 142 | 1.5663 |  | 10 | 334 | 2.9478 |  | 10 | 430 | 4.1853 |  |
| 15 | 147 | 1.4860 |  | 15 | 339 | 2.9206 |  | 15 | 435 | 4.1724 |  |
| 20 | 152 | 1.4329 |  | 20 | 344 | 2.8967 |  | 20 | 440 | 4.1607 |  |

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| (9, 18, 10, 5, 5) | | |  | (10, 15, 6, 4, 2) | | |  | (10, 45, 9, 2, 1) | | |  | (10, 18, 9, 5, 4) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 324 | 3.5057 |  | 0 | 280 | 2.6929 |  | 0 | 220 | 1.5622 |  | 0 | 328 | 3.2766 |
| 1 | 325 | 3.5003 |  | 1 | 281 | 2.6818 |  | 1 | 221 | 1.4958 |  | 1 | 329 | 3.2700 |
| 5 | 329 | 3.4803 |  | 5 | 285 | 2.6404 |  |  | 5 | 333 | 3.2458 |
| 10 | 334 | 3.4580 |  | 10 | 290 | 2.5955 |  |  | 10 | 338 | 3.2193 |
| 15 | 339 | 3.4386 |  | 15 | 295 | 2.5578 |  |  | 15 | 343 | 3.1966 |
| 20 | 344 | 3.4216 |  | 20 | 300 | 2.5263 |  |  | 20 | 348 | 3.1770 |

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| (11, 11, 6, 6, 3) | | |  | (11, 11, 5, 5, 2) | | |  | (11, 55, 15, 3, 3) | | |  | (12, 33, 11, 4, 3) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 396 | 3.6433 |  | 0 | 220 | 2.7203 |  | 0 | 484 | 2.3280 |  | 0 | 576 | 2.8310 |
| 1 | 397 | 3.6382 |  | 1 | 221 | 2.7083 |  | 1 | 485 | 2.3142 |  | 1 | 577 | 2.8228 |
| 5 | 401 | 3.6195 |  | 5 | 225 | 2.6671 |  | 5 | 489 | 2.2581 |  | 5 | 581 | 2.7909 |
| 10 | 406 | 3.5988 |  | 10 | 230 | 2.6277 |  | 10 | 494 | 2.1850 |  | 10 | 586 | 2.7531 |
| 15 | 411 | 3.5809 |  | 15 | 235 | 2.5984 |  | 15 | 499 | 2.1090 |  | 15 | 591 | 2.7180 |
| 20 | 416 | 3.5652 |  | 20 | 240 | 2.5759 |  | 20 | 504 | 2.0306 |  | 20 | 596 | 2.6856 |

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| (12, 22, 11, 6, 5) | | |  | (12, 44, 11, 3, 2) | | |  | (13, 13, 4, 4, 1) | | |  | (13, 26, 12, 6, 5) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 752 | 4.0536 |  | 0 | 400 | 2.0734 |  | 0 | 260 | 2.0363 |  | 0 | 884 | 3.9982 |
| 1 | 753 | 4.0507 |  | 1 | 401 | 2.0490 |  | 1 | 261 | 2.0090 |  | 1 | 885 | 3.9954 |
| 5 | 757 | 4.0397 |  | 5 | 405 | 1.9397 |  | 5 | 265 | 1.9183 |  | 5 | 889 | 3.9844 |
| 10 | 762 | 4.0268 |  | 10 | 410 | 1.7626 |  | 10 | 270 | 1.8397 |  | 10 | 894 | 3.9714 |
| 15 | 767 | 4.0149 |  | 15 | 415 | 1.4584 |  | 15 | 275 | 1.7863 |  | 15 | 899 | 3.9594 |
| 20 | 772 | 4.0039 |  |  | 20 | 280 | 1.7479 |  | 20 | 904 | 3.9481 |

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| (13, 39, 15, 5, 5) | | |  | (15, 15, 7, 7, 3) | | |  | (16, 16, 6, 6, 2) | | |  | (16, 48, 15, 5, 4) | | |
| n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |  | n0 | N | a2 |
| 0 | 676 | 3.2818 |  | 0 | 1020 | 4.1822 |  | 0 | 576 | 3.0685 |  | 0 | 832 | 2.9474 |
| 1 | 677 | 3.2764 |  | 1 | 1021 | 4.1796 |  | 1 | 577 | 3.0608 |  | 1 | 833 | 2.9391 |
| 5 | 681 | 3.2555 |  | 5 | 1025 | 4.1693 |  | 5 | 581 | 3.0328 |  | 5 | 837 | 2.9071 |
| 10 | 686 | 3.2315 |  | 10 | 1030 | 4.1573 |  | 10 | 586 | 3.0026 |  | 10 | 842 | 2.8697 |
| 15 | 691 | 3.2095 |  | 15 | 1035 | 4.1462 |  | 15 | 591 | 2.9771 |  | 15 | 847 | 2.8354 |
| 20 | 696 | 3.1894 |  | 20 | 1040 | 4.1359 |  | 20 | 596 | 2.9556 |  | 20 | 852 | 2.8041 |

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| --- | --- | --- |
| (16, 20, 5, 4, 1) | | |
| n0 | N | a2 |
| 0 | 384 | 1.3470 |
| 1 | 385 | 1.2766 |
| 5 | 389 | 1.0208 |

**Table 2:** The Variance of the estimated response of SOSRD of Second type using BIBD for different factors 3≤v≤16

[These are SOSRDs of second type with design points]

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| (v,b,r,k,λ) | N |  |  |  | c |  |  |  |  |  |  |  |
| (3,3,2,2,1) | 25 | 1.8837 | 0.1600 | 0.6839 | 8.7953 | 0.2132 | 0.0585 | 0.2500 | 0.0625 | -0.0844 | 0.0305 | (0.058488+0.250000d2)σ2 |
| (4,6,3,2,1) | 41 | 1.8185 | 0.0976 | 0.5028 | 8.9679 | 0.1817 | 0.0485 | 0.2499 | 0.0624 | -0.0782 | 0.0310 | (0.048509+0.249900d2)σ2 |
| (5,10,4,2,1) | 61 | 1.7601 | 0.0656 | 0.3967 | 9.2987 | 0.1672 | 0.0413 | 0.2499 | 0.0624 | -0.0760 | 0.0323 | (0.041325+0.249900d2)σ2 |
| (6,15,5,2,1) | 85 | 1.7064 | 0.0471 | 0.3273 | 9.7393 | 0.1587 | 0.0359 | 0.2498 | 0.0619 | -0.0748 | 0.0333 | (0.035945+0.249781d2)σ2 |
| (7,7,3,3,1) | 85 | 1.9808 | 0.0941 | 0.3982 | 7.0986 | 0.1182 | 0.0295 | 0.1250 | 0.0313 | -0.0382 | 0.0108 | (0.029545+0.125023d2)σ2 |
| (8,28,7,2,1) | 145 | 1.6061 | 0.0276 | 0.2425 | 10.8271 | 0.1573 | 0.0284 | 0.2499 | 0.0622 | -0.0775 | 0.0368 | (0.028439+0.249875d2)σ2 |
| (9,12,4,3,1) | 133 | 1.8426 | 0.0602 | 0.3067 | 7.1318 | 0.1064 | 0.0245 | 0.1249 | 0.0311 | -0.0358 | 0.0107 | (0.024515+0.124897d2)σ2 |
| (10,45,9,2,1) | 221 | 1.4958 | 0.0181 | 0.1922 | 12.0030 | 0.1601 | 0.0235 | 0.2500 | 0.0626 | -0.0809 | 0.0398 | (0.023543+0.249994d2)σ2 |
| (11,11,5,5,2) | 221 | 2.7083 | 0.1448 | 0.4374 | 5.9250 | 0.0518 | 0.0103 | 0.0312 | 0.0078 | -0.0098 | 0.0015 | (0.010345+0.031249d2)σ2 |
| (12,44,11,3,2) | 401 | 2.0490 | 0.0399 | 0.2454 | 7.8283 | 0.0655 | 0.0102 | 0.0625 | 0.0157 | -0.0214 | 0.0065 | (0.010162+0.062500d2)σ2 |
| (13,13,4,4,1) | 261 | 2.0090 | 0.0613 | 0.2838 | 6.1612 | 0.0644 | 0.0135 | 0.0625 | 0.0156 | -0.0164 | 0.0035 | (0.013500+0.062503d2)σ2 |
| (15,15,7,7,3) | 1021 | 4.1796 | 0.1881 | 0.4750 | 5.5226 | 0.0125 | 0.0021 | 0.0052 | 0.0013 | -0.0016 | 0.0002 | (0.002062+0.005207d2)σ2 |
| (16,20,5,4,1) | 385 | 1.2766 | 0.0416 | 0.2215 | 5.4570 | 0.0335 | 0.0117 | 0.0624 | 0.0156 | -0.0087 | 0.0016 | (0.011726+0.062438d2)σ2 |

\*For all designs we have taken .