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| Journal Name: | [**Asian Research Journal of Mathematics**](https://journalarjom.com/index.php/ARJOM) |
| Manuscript Number: | **Ms\_ARJOM\_132564** |
| Title of the Manuscript: | **On Ultrafilters in Matroids: Structural Properties and Metric Considerations** |
| Type of the Article | **Original Research Article** |

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| **PART 1: Comments** | | |
|  | **Reviewer’s comment**  **Artificial Intelligence (AI) generated or assisted review comments are strictly prohibited during peer review.** | **Author’s Feedback** *(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.** | This study discusses the ultrafilter issue in matroid theory and clarifies the mathematical part of this issue and details its metric properties. The connection between matroids and ultrafilters is an intriguing area in applied mathematics and computer science. This study has provided new perspectives not only in the theory of mathematics but also in its application. Thus, it sheds light on future studies. | Many thanks. |
| **Is the title of the article suitable?**  **(If not please suggest an alternative title)** | Yes, the title of the article is suitable. | Many thanks. |
| **Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.** | Although the abstract is focused on the subject, it should be improved a little. The following can be added:  -The basic hypotheses of this study should be mentioned and the purpose of the study should be stated.  -The methods used should be compared with the experimental results.  -And the application areas can be mentioned. | This paper primarily focuses on the theoretical aspects, and applications still need to be explored in future research. We appreciate your insightful comments and have addressed them by including this point in the **Future Tasks** section. Additionally, the **Abstract** has been carefully revised for clarity. |
| **Is the manuscript scientifically, correct? Please write here.** | When we examine the article, it can be seen as theoretically consistent and solid. However, the section in the references cited in the 1.1 Matroid section in the introduction section needs to be detailed. And the classical version of the matroid definition mentioned in the article needs to be detailed in the relationship with independent sets in the standard literature. In addition, giving the history of a related study in an article makes the article more comprehensive.  1.3 Our Contribution section is not detailed. That section needs to be detailed.   1. Definition in this paper section is very incomplete. Only references are given but information is not used.    1. Basic Notation section should have proofs for definitions. But currently there are only definitions in the article.    2. Matroid and Filter section is not explained at all. Information related to the title needs to be added and proofs of the items in the definition discussed in section 2.2 need to be made.   - Conclusion section and future work section of the article are very short and do not include details. More details need to be included. | Many thanks.  The section in the references cited in Section 1.1: Matroid within the introduction needed more detail.  → Revised accordingly.  The Our Contribution section was not detailed enough and required further elaboration.  → Revised accordingly.  Section 2: Basic Notation was expected to include proofs for its definitions. However, it currently only presents the definitions.  → Since this section provides fundamental explanations of set theory, which are generally well understood by readers, we have determined that formal proofs are unnecessary.  Section 2.1: Matroid and Filter lacked explanation, and additional information relevant to the title was required. Furthermore, proofs for the items defined in Section 2.2 were needed.  → To improve clarity, we have restructured the paper by first presenting general definitions before introducing definitions specific to matroids.  The Conclusion section has been revised to provide more specificity. |
| **Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.** | Yes, the references are sufficient and recent. | Many thanks. |
| **Is the language/English quality of the article suitable for scholarly communications?** | Yes, language/English quality of the article is suitable for scholarly communications. | Many thanks. |
| **Optional/General** comments | My general comment about the article is; after some details are made in terms of content, basic information is added and necessary arrangements are made, the similarity rate is checked again and if the similarity rate does not pose any risk in terms of plagiarism within the framework of the journal's regulation rules, I think the article can be published easily. | Many thanks. |

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| **PART 2:** | | |
|  | **Reviewer’s comment** | **Author’s comment** *(if agreed with the reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Are there ethical issues in this manuscript?** | *(If yes, Kindly please write down the ethical issues here in detail)* |  |