**Numerical Investigation of Variable Viscosity and Thermal Conductivity on Natural Convection Flow along a Vertical Flat plate with Heat Conduction**

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ABSTRACT

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| This study numerically investigates the effects of variable viscosity and thermal conductivity on natural convection flow along a vertical flat plate with Joule heating and heat generation. Using the implicit finite difference method, specifically the Crank-Nicolson scheme, this work solves the dimensionless governing equations derived from the two-dimensional, laminar, and unsteady boundary layer equations. The study focuses on viscous incompressible fluids where both thermal conductivity and viscosity are temperature-dependent. Key findings from the research include detailed insights into how these variable properties influence fluid dynamics and heat transfer processes. Notably, increased thermal conductivity results in up to a 15% increase in local Nusselt numbers, indicating enhanced heat transfer rates. Conversely, reductions in viscosity lead to lower skin friction coefficients, suggesting less resistance to flow and potentially higher flow rates. The study also demonstrates variable effects on temperature profiles and velocity fields, where higher thermal conductivity correlates with higher overall temperatures and velocities across the studied range. These results, displayed through comprehensive tables and graphs, provide a significant contribution to the understanding of complex interactions in natural convection flows and offer practical insights for optimizing thermal management systems in engineering applications. Comparisons with existing studies affirm the validity of the numerical approaches used and the relevance of the findings to ongoing research in thermal sciences. |

*Keywords: Steady state condition; variable viscosity; variable thermal conductivity; Heat generation and Joule heating.*

1. INTRODUCTION

A key mechanism in fluid dynamics, natural convection is important for many environmental and engineering activities. Designing efficient thermal management systems and maximizing heat transfer efficiency require an understanding of the complex interactions between variables affecting natural convection flow. The two most important of them, viscosity and thermal conductivity, determine the fluid's behavior and energy transfer properties. Determining how changing viscosity and heat conductivity affect natural convection events has been the subject of a large number of investigations in recent decades. Nevertheless, a distinct and difficult study frontier is presented by the combination of these factors with extra complexity like heat conduction and a vertical flat plate. Prominent scholars have acknowledged the significance of filling this knowledge gap in order to improve our comprehension of heat transport and fluid dynamics. By thoroughly analyzing the effects of varying viscosity and thermal conductivity on natural convection flow on a vertical flat plate, while taking into account the simultaneous influence of heat conduction, this numerical study aims to add to this body of knowledge. This research attempts to shed light on the fundamental principles underlying these intricate events by utilizing sophisticated numerical techniques and verified computational models. The findings of this study have broad ramifications for a number of disciplines, such as environmental sustainability, renewable energy systems, and thermal engineering. The goal of this research is to better understand how changing viscosity and thermal conductivity affect natural convection flow in order to guide the creation of more effective heat transfer technologies and improve our capacity to handle today's energy management and climate resilience issues. This study intends to deepen our understanding of the complex dynamics of fluid flow and heat transfer by means of exacting numerical analysis and careful validation against experimental data, opening the door for further developments in scientific research and engineering practice. The theoretical foundation and computational approach will be covered in detail in the sections that follow. A vertical flat plate with heat conduction and the effects of varying viscosity and thermal conductivity on unconstrained convection flow are significant from different angles. The investigator becomes enamored with the mechanism and fabricates it for their own purpose. In addition to a vertical flat plate, Sarker et al. [1] calculated the effects of varying viscosity and heat conductivity on magneto-hydrodynamic (MHD) natural convection flow. In addition to a vertical flat plate with heat conduction, Alam et al. [2] investigated the effects of pressure stress work and viscous dissipation in natural convection flow. The impact of Joule heating on the association between conduction and magneto-hydrodynamic (MHD) unbound convection flow from a vertical flat plate was investigated by Alim et al. [3]. Rahman et al. [4] provided a vertical flat plate with heat conduction as well as the effect of temperature based on thermal conductivity on magneto-hydrodynamic (MHD) unbound convection flow. The combined effect of viscous dissipation and Joule heating on the association of conduction and unbound convection with a vertical flat plate was calculated by Alim et al. [5]. According to Molla et al. [6], a vertical wavy exterior and a natural convection laminar flow with temperature dependent on viscosity and thermal conductivity appeared. In addition to a vertical flat plate with heat creation, Shafiqul Islam et al. [7] examined the effects of temperature based on thermal conductivity on natural convection flow. In addition to a vertical wavy exterior, Kabir et al. [8] investigated the effects of viscous dissipation on magneto-hydrodynamic (MHD) natural convection flow. Hossain investigates the effects of viscous and Joule heating on magneto-hydrodynamic (MHD) unbound convection flow with varying plate temperature. [9].Soundalgekar et al. [10] compute the finite difference exploration of transitory unbound convection on an isobaric flat plate.Elbashbeshy et al. [11] investigate balanced unbound convection flow with changing viscosity and thermal expansion in conjunction with a vertical plate.Kafoussius et al. [12] investigate numerically the mixed and induced convective laminar boundary layer flow on a vertical isobaric flat plate with temperature dependent on viscosity. Anwar Hossain et al. donate the effect of emanation on unbound convection flow of fluid with varied viscosity from a spongy vertical plate. [13]. In the case of rocky flow, Seddeek calculates the effect of varying viscosity on a magneto-hydrodynamic (MHD) unbound convective flow over a semi-infinite flat plate with an orientated attractive field. G. palani [14]. Numerical studies on vertical plates with varying viscosity and heat conductivity were conducted by Kwang et al. [15].

Significant studies in fluid dynamics have explored the effects of variable properties and environmental conditions on heat transfer. Siattery [16] provided foundational insights into the principles of momentum, energy, and mass transfer. Ockendon and Ockendon [17] focused on variable viscosity flows in different temperature channels, while Elbashbeshy and Dimian [18] explored the impact of radiation on the flow over a wedge with variable viscosity. Seddeek and Abdelmeguid [19] investigated how radiation and thermal diffusivity affect heat transfer over a stretching surface with a variable heat flux. Recent studies have explored the complex effects of physical parameters on heat and mass transfer. Research on the role of surface roughness, like ribs on vertical surfaces, reveals nuanced effects on natural convection, suggesting both benefits and challenges in surface design by E.N. Ahmed and G. Tanda [20]. Studies on variable viscosity and slip effects in vertical channels show significant impacts on transfer rates, enhancing our understanding of fluid dynamics in industrial applications by V. Gosty et al.[21]. Additionally, the influence of solar energy alongside variable density around laminar flows highlights potential improvements in thermal performance for energy systems by T. Maryam et al.[22]. Recent research has explored the complex behaviors of MHD natural convection in cavities by Shafee Ahmad et al. [23], thermal dynamics of flat plates in Martian conditions by Rafael Bardera et al. [24], and the impact of 3D roughness on natural convection along vertical plates by Tse-Yu Chen et al. [25]. Recent studies have focused on the dynamics of MHD nanofluid flows over exponentially accelerated vertical plates, considering the impacts of thermal and mass stratification on heat and mass transfer, which are critical for various engineering applications by Bo Zhao et al. [26], Rupam Shankar Nath and Rudra Kanta Deka [27], T. Aghalya and R. Tamizharasi [28].

Research gaps remain in the comprehensive numerical investigation of how variable viscosity and thermal conductivity impact natural convection flow along vertical flat plates with heat conduction. Current studies often isolate these factors rather than exploring their combined effects, leaving a significant area for further exploration to optimize thermal management in engineering applications.

This job will be based on hypothetical investigation and will develop a logical solution for the fluctuating viscosity and thermal conductivity in natural convection flow across a vertical flat plate in the context of heat conduction. The equations in finite difference fashioned by comparison with purposes and derivatives in designation of the innermost differences between coordinate directions are indicated by the asymptotic of momentum and energy equations in designation of dimensionless coordinates x and y above. These equations' numerical equivalents hasten the development of a computer program for the subject that employs a well-organised implied finite-difference approach.It is known as the Crank-Nicolson scheme. The following data exploration has been represented for velocity outline, temperature outline, local skin friction, local Nusselt number, average skin friction, and average Nusselt number for various parameters, including variable viscosity, based on thermal conductivity, heat formation, joule heating, and Prandtl number.

2. Mathematical analysis

Here, we study the unsteady flow of a viscous incompressible fluid over a semi-infinite vertical plate. As shown in figure 1, the y-axis is pulled perpendicular to the plate at the primary edge, while the x-axis is taken vertically upward with the plate. It is assumed that the principal edge of the plate is where the x-axis originates. All fluid somatic origins are assumed to be continued, with the exception of the fluid viscosity, which varies aggressively with fluid temperature, the thermal conductivity, which varies one-dimensionally with fluid temperature, and the hardness disparity in the frame impose expression in the momentum equation, where the Bossiness estimation is almost new.



Figure 1: Unsteady flow of a viscous incompressible fluid over a semi-infinite vertical plate

The mathematical declaration of the root maintenance code of mass, momentum and energy for the balanced viscous incompressible and electrically conducting flow, after clarifying we get

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |

Where,  and  are the velocity components along with the  and  axis respectively, is the time,  is the temperature of the fluid in the boundary layer and  is the fluid temperature far away from the plate , *g* is the acceleration because of gravity,  is the thermal conductivity of the fluid, is the hardness,  is the particular heat at continuous pressure and  is the variable dynamic co-efficient of viscosity of the fluid. The number of heat formatted or sponged up per unit volume is, *Q0* being a continuous, that can take either positive or negative and the hydrostatic pressure  where, . The origin expression act for the heat formation when *Q0* > 0 and the heat sponging up when *Q0* < 0.  is the thermal conductivity of the fluid depending on the fluid temperature , is the electric conduction and is the attractive area power.

The beginning and boundary conditions are

|  |  |
| --- | --- |
|  for all  at  at  as  | (4) |

On establishing the above dimensionless numbers in equations (1) to (4), we have

|  |  |
| --- | --- |
| , , , , , , ,  | (5) |

Here *l* is the length of the plate,  is the kinematic viscosity, *Gr* is the Grashof number, *Pr* is the Prandtl number. Out of great numbers of shapes of disparity of viscosity and thermal conductivity with dimensionless temperature , that are at hand in the written works. The above shapes are suggest by Stattery[16], Ockendon and Ockendon[17], Elbashbeshy and Ibrahi[18], and Seddeek and Abdelmegguid[19] 

|  |  |
| --- | --- |
|   | (6) |
|   | (7) |

Where  and  attending the viscosity and thermal conductivity disparity parameters respectively, depended on the nature of the fluid. Here  and  are the viscosity and the thermal conductivity at temperature .

The magneto hydrodynamic(MHD) area in the fluid is controlled by the boundary layer equations, that in the dimensionless shape achieved by establishing the dimensionless variables elaborated in (5), can be written the equation of not discontinuity as

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|  | (8) |

Now momentum equation (2) may be minimized by applying the dimensionless modification (5) and (6), we get

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|  | (9) |

Again, the energy equation (3) may be minimized by the following likeness modification (5) and (7), we have

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|  |  |
|  | (10) |

The agree with beginning condition and boundary conditions in a dimensionless shape are as given

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| --- | --- |
|  for all *Y* at y= 0 at *x* = 0 | (11) |

The unbound convective laminar boundary layer flow with variable viscosity and thermal conductivity and an isobaric semi-infinite vertical plate is expressed by equations (8) to (10) with the boundary condition (11).

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Where, , the Prandtl’s number,  is the heat formation parameter, is the joule heating parameter.

The local shear stress in the plate is expounded by

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| --- | --- |
|  |  (12) |

By establishing the dimensionless numbers provided in equations (5) -(6) in (12), we get dimensionless shape of local skin friction and it is provided by

|  |  |
| --- | --- |
|  |  |
|  | (13) |

The integration of equation (13) from  to  provides the average skin friction and it is provided by

|  |  |
| --- | --- |
|  |  (14) |

 The local Nusselt number is expounded by

|  |  |
| --- | --- |
|  |  |
|  | (15) |

The integration of equation (15) from  to  gives the average skin friction and it is provided by

|  |  |
| --- | --- |
|  | (16) |

**2.1 Numerical techniques**

Under the initial and boundary conditions in equation (11) the two-dimensional, non-linear, steady, and connected partial differential equations (8)– (10) are calculated by an assumed Crank-Nicol-son type finite difference method that is fast merging and unconditionally stable. The finite difference equation corresponds with the following equations (8) through (10):

+  (17)





+

  (18)

 

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 +  (19)

The integration field is computed as a rectangle with sides  = 1 and  = 10, where  is y and sits well outside the boundary layers of momentum and energy. After some initial investigation, the extreme of y was determined to be 6, allowing the final two boundary conditions (11) to be modified. In this case, the grid point and u-direction are indicated by the subscript i, the v-direction by the j, and the t-direction by the superscript k.The co-well-ordered  and  appearing in the difference equations are considered continuous in the time of any one-time step. At all grid locations at t = 0, the values of u, v, and T are known from the initial conditions. The following procedure is used to calculate u, v, and T at time level (k + 1) by applying the profits at the prior time level (fe): the tridiagonal method of equations is comprised of the finite difference eq. (18) at each internal crucial point on a specific i-level. As stated by Carnahan et al., the Thomas algorithm calculates a method of equations. [19] Therefore, for a given i at the (k + l) th time level, the profits of T are found at each crucial point. In the same process, find the profits of u at the (k + 1) th time level by applying the profits of T at the (k + 1) th time level in equation (13). Thus, on a specific i-level, the profitability of T and u are well-known. Finally, at each nodal point on a certain i-level at (k + 1) th time level, the profits of v are correctly calculated using eq. (12). For various i-levels, this procedure is repeated. Thus, at every grid point in the rectangular field at the (k + 1) th time phase, the values of T, u, and v are known. They have been solidified at the level  = 0.05,  = 0.25, and time step  = 0.01 after being taken using a few sets of lattice dimensions. This time, the spatial lattice size is reduced by 50% in one direction and then in both directions, with distinct results. It is stated that the results vary to the fourth decimal place when the lattice size is reduced by 50% in both the x and y directions. Therefore, the aforementioned-directed shapes have been investigated as appropriate lattice sizes for calculations. Until the balanced condition is extended, calculations are made. When the perfect difference between the values of u and temperature T at two consecutive time steps is less than at all grid locations, it is assumed that the balanced-condition suspension has been investigated. The local abridge error is *O (Δf2* + *ΔF2* + *ΔAx*) which 0 as *Δt*. *Δx* and *Δy* 0, that proves that the scheme is comfortable. The implied finite difference scheme of the Crank-Nicolson type is also shown to be condition stable for a natural convective flow since there is always a non-negative profit of speed u and a non-positive profit of v. Therefore, safety and suitability verify that the implied finite difference scheme is merging.

3. results and discussion

Heat conduction, often called scattering, is the simple nanoscopic transfer of particle kinetic energy across the interface between two methods. Water is a great heat-conducting fluid because of its high thermal volume and low viscosity. Since oil's liquid temperature is higher than water's, it has been utilized as a commendable alternative to prevent high pressure disturbance. Heat is transferred between the Earth's interior and atmosphere by conduction, radiation, and convection.

When a heated fluid, like water or air, is compelled to flow away from the source of the heat while carrying energy, convection is the process by which heat is transmitted. Convection occurs when hot air expands, loses density, and rises on a heated surface. Because liquid mineral has a low Prandtl's number, heat transmission by atomic thermal conduction is significant in both the flow core and the extremely close-wall layer, even in a fully enhanced turbulent flow.

The above scopes for *λ*, *γ* and *Pr* are examined in the attending research are:

 For air: - 0.6 ≤ *λ* ≤ 0, 0 ≤ *γ* ≤ 6, *Pr =* 0.73

 For water: 0 ≤ *λ* ≤ 0.6, 0 ≤ *γ* ≤ 0.12, 2 ≤ *Pr* ≤ 7.00

To confirm the accuracy of the figures we calculated, we compare our results with the curves generated by G. Palani.Elbashbeshy & Ibrahim and Kwang-Yong Kim for air (Pr = 0.73) with different values. They are shown in Figure 2(a) and Figure 2(b). Excellent agreement exists between our results and those of G. palani. Elbashbeshy & Ibrahim and Kwang-Yong Kim at the steady state. In the early stages of the aforementioned step changes in the wall temperature, the frame imposes have not had enough time to produce a suitable motion in the fluid. As a result, both the velocity factors u and v can be avoided for a little length t. In this first transient regime, the heat conduct is controlled by pure heat conduction for constant viscosity and thermal conductivity. Equation (10)'s outcome is



Therefore, for short periods, it is shown that the temperature profile depends only on time and the natural distance from the wall for a given Prandtl's number and magnetic parameter. The suspensions of equation (15), according to the initial and boundary conditions given in the local Nusellt number, are as follows when Pr = 1.

 (2.16)

Figures 2(a), 2(b), 3(a), 3(b), 4(a), 4(b), 5(a), 5(b), 6(a), 6(b), 7(a), 7(b), 8(a), and 8(b) demonstrate that, for a range of viscosity, thermal conductivity, heat conduction variation parameters, pressure work parameters, and Prandtl's numbers, the temperature and velocity at their transient, temporal maximum, and steady state are opposite the coordinate y at the plate's principal margin, i.e., x = 1.0. After increasing and reaching its maximum value relatively close to the wall (i.e., 0 ≤ *y* ≤8), the fluid velocity monotonically declines to zero as y increases for all time t. Additionally, it is mentioned that as time t increases, the temperature and speed reach a temporary extreme before reaching a constant state.

We deduce from Figures 3(a) and 3(b) that when the variation parameter λ increases with the fixed value of *γ* = 1.00 in air (Pr = 0.73), Jul = 0.80, and Q = 0.50, the velocity profile and temperature profile drop. Figure 3(a) shows that the fluid's velocity increases with time until a brief maximum is reached, after which it moderately decreases until the alternate balanced condition is reached. It is noted that increasing the viscosity variation parameter almost completely decreases the time required to reach the stable condition. From fig. 3(a), it is obvious that velocity u at any vertical plane near to the plate increases as variable viscosity decrease. However, the velocity profile eventually twisted in the opposite direction. Lastly, it intersects the y axis. As *λ* increases, the fluid's temperature decreases, according to fig. 3(b). Additionally, it is tied to distinct times. The numerical values of the variation of transient velocity and temperature profiles for a fixed value of *λ* = 0.3, Q = 0.5, Jul=0.8 ,Pr = 0.73 with the variation of thermal conductivity parameter γ are proved graphically in figs. 4(a) and 4(b). From these figures, it is mentioned that the velocity and temperature distribution in the fluid increases as γ increases for fixed values of λ, Q, Joule and Prandtl’s number. Additionally, it should be noted that temperature and velocity both increase significantly with increasing γ , which suggests that volume flow rate likewise increases with increasing γ . Even during the first brief phase, the impact of temperature and velocity changes due to variations in thermal conductivity is more significant. Also, it is mentioned that the time to reach the temporal extreme and balanced condition reduces with increasing thermal conductivity parameter γ .Equations are used to calculate the numerical values of temperature and variation velocity. Figures 5(a) and 5(b) show the graphical representations of (13) and (14) for various values of Jul with fixed values of Q = 0.5, λ = 0.30, γ = 0.1, and Pr = 0.73. It is made quite evident that as Jul values decrease, it takes less time to achieve the temporal extreme and balanced state. It is clear from fig. 5(a) that when parameter Jul grows, the velocity u at any vertical plane close to the plate increases. However, the velocity profile eventually twisted in the opposite direction. And finally it meets asymptotically.

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**Figures 2(a) and 2(b): Comparison of temperature and velocity profiles Elbashbeshy et al. and G. Palani et al. for different dependent thermal conductivity values with fixed values**

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**Dimensionless velocity and temperature profiles vary with dimensionless y for different viscosity values and steady state conditions with Q=0.5, Jul=0.8, =0.1, and Pr=0.73 (Fig. 3(a) and 3(b)).**

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**Fig: 4(a) and 4(b): Variation of dimensionless velocity profiles and temperature against dimensionless y for various values of thermal conductivity  and steady**

**state condition with Q=0.5, Jul=0.8,  =0.3 and Pr= 0.73**

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**Fig: 5(a) and 5(b): Variation of dimensionless velocity profiles and temperature against dimensionless y for various values of Joule and steady state condition with Q=0.5,=0.3,=0.1 and Pr= 0.73**

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**Fig: 6(a) and 6(b): Variation of dimensionless velocity profiles and temperature against dimensionless y for different values of Prandtl’s number *Pr* and steady state condition with Q=0.5,=0.3,=0.1 and Jul= 0.8**

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**Fig: 7(a) and 7(b): Variation of dimensionless velocity profiles and temperature against dimensionless distance y for different values of heat generation Q and steady state condition with=0.3,=0.1 and *Pr*=0.73 , Jul= 0.**

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**Fig: 8(a) and 8(b): Variation of dimensionless local skin friction and local Nusselt number against dimensionless distance x for different values of Q,,and *Pr* at steady state condition**

As the parameter Jul increases, the fluid's temperature rises, according to fig. 5(b). Additionally, it is tied to distinct times.
Figures 6(a) and 6(b) show how temperature and velocity vary for different Prandtl's numbers (Pr = 0.73) with fixed values of  *γ =*0.1, *λ=* 0.3, Q = 0.5, and Jul = 0.8. The time needed to achieve the balanced state is said to decrease as Pr values rise. Furthermore, the temperature distribution of the fluid is said to diminish as Pr rises.

Figs. 7(a) and 7(b) show transient temperature and velocity variations with heat generation with fixed values Pr=0.73, *λ* = 0.3, Jul=0.8, and *γ* = 0.1.As the fluid's heat-generating Q decreases, the amount of time needed to reach the temporal extreme and balanced condition is said to increase. The numerical data indicates that when the heat-generating Q value decreases, the velocity profile increases. In equations (12), (14), (15), and (16), the derivatives are evaluated using a five-point approximation approach. The integrals are then evaluated using the Newton-Cotes closed integration formula.

Figure 8(a) shows steady state local heat transfer rates in various dimensions for air and water with varying values of variation parameters Q, *λ*, *γ* ,Pr, and Jul=0.8. The local heat transfer rate rises as the thermal conductivity and viscosity parameters do. Pr increases the rate of heat transfer in the immediate vicinity. A larger heat transfer rate can be predicted in the future because of a higher Pr, a thinner thermal boundary layer, and a larger wall temperature differential. Fig. 8 (b) displays the average Nusselt number, variation parameters, and Pr.

The average rate of heat transfer rose as the Prandtl number increased because this increases the rate of heat transfer by speeding up the spatial drop of temperature. As *λ* and *γ* decrease, the average Nusselt number also decreases.

**Table-1: Comparing with G. Palani et al. with our present work Variation of dimensionless average skin friction against dimensionless distance x for different values of *Q, λ,*  and *Pr* at steady state condition.**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | Present | G.Palani.Kwang-Yong Kim  | Present | G.Palani.Kwang-Yong Kim  | Present | G.Palani.Kwang-Yong Kim  |
|  |  |  |  |  |  |
| 0.000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.500 | 0.937118 | 0.937234 | 0.698419 | 0.698543 | 0.172413 | 0.172543 |
| 1.000 | 1.311313 | 1.311432 | 0.951512 | 0.951675 | 0.246665 | 0.246778 |
| 1.500 | 1.478623 | 1.478768 | 1.012406 | 1.012544 | 0.281301 | 0.281432 |
| 2.000 | 1.539530 | 1.539665 | 1.057439 | 1.057554 | 0.337520 | 0.337656 |
| 2.500 | 1.542128 | 1.542276 | 1.058865 | 1.058967 | 0.354601 | 0.354782 |
| 3.000 | 1.545416 | 1.545543 | 1.059411 | 1.059543 | 0.365429 | 0.365543 |
| 3.500 | 1.548541 | 1.548674 | 1.059119 | 1.059221 | 0.367422 | 0.367544 |
| 4.000 | 1.551533 | 1.551657 | 1.065304 | 1.065434 | 0.369599 | 0.369654 |
|  |  |  |  |  |  |  |

**Table-2: Comparing with G.Palani et al. with our present work Variation of dimensionless average Nusselt number against dimensionless distance x for different values of *Q, λ,*  and *Pr* at steady state condition**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | Present | G.Palani.Kwang-Yong Kim | Present | G.Palani.Kwang-Yong Kim | Present | G.Palani.Kwang-Yong Kim |
| *Nuu* | *Nuu* | *Nuu* | *Nuu* | *Nuu* | *Nuu* |
| 0.100 | 1.588001 | 1.588065 | 1.463965 | 1.464051 | 1.247009 | 1.247156 |
| 0.200 | 1.248201 | 1.248318 | 1.124156 | 1.124210 | 1.006375 | 1.006426 |
| 0.300 | 1.082339 | 1.082424 | 0.982812 | 0.982952 | 0.859031 | 0.859158 |
| 0.400 | 0.978536 | 0.978671 | 0.909603 | 0.909735 | 0.810129 | 0.810263 |
| 0.500 | 0.936102 | 0.936178 | 0.855102 | 0.855206 | 0.749724 | 0.749835 |
| 0.600 | 0.899774 | 0.899883 | 0.825006 | 0.825126 | 0.701759 | 0.701835 |
| 0.700 | 0.875902 | 0.876033 | 0.777183 | 0.777299 | 0.672341 | 0.672461 |
| 0.800 | 0.834012 | 0.834137 | 0.753715 | 0.753841 | 0.646003 | 0.646185 |
| 0.900 | 0.816649 | 0.816783 | 0.730207 | 0.730320 | 0.636951 | 0.637094 |
| 1.000 | 0.793309 | 0.793419 | 0.718903 | 0.719023 | 0.625929 | 0.626064 |

Fig: 8(a) shows the average skin friction for different values of *Q, λ,*  ,*Pr* and Jul=0.8 under steady state conditions compared to the work of G.Palani Kwang-YongKim and Elbashbeshy & Ibrahim. The data in Table 1, which can be seen in Fig. 8(a), agrees well with the findings of the current study.

Fig: 8(b) shows the average Nusselt number for different values of *Q, λ,*  ,*Pr* and Jul=0.8 in steady state, comparing our study to those of G.Palani Kwang-Yong Kim and Elbashbeshy & Ibrahim. Table 2, which is derived from Fig 8(b), is in excellent accord with our current findings.

4. Conclusion

This challenge investigates the effects of transposition viscosity and thermal conductivity on heat generation in a laminar natural convection boundary-layer vertical plate work under pressure. The fluid viscosity is intended to differ as an ascending purpose, whereas the thermal conductivity is assumed to be a linear function of temperature. Non-dimension controlling equations are solved using an assumed Crank-Nicolson type finite difference approach. Graphically, a comparison is among the running numerical discovery and before published investigate. It is said that there is a high level of harmony between the two parties. The attending exploration has represented that:

1. The non-dimension fluid velocity increases as the viscosity parameter *λ* increases and the fluid temperature falls. Greater velocity is found in a position close to the wall when the viscosity disparity parameter *λ* is big, following in a greater Nusselt number and minimized skin friction.
2. The fluid velocity, fluid temperature, the non-dimension wall speed gradient, and the non-dimension rate of heat conduct from the plate to the fluid all rise as the thermal conductivity parameter *γ* raises.
3. It has been founded that neglecting the viscosity and thermal conductivity disparities would consequence in important inaccuracies. As a result, we advise that the impacts of changing viscosity and thermal conductivity should be forward according to expect more correct result.
4. When the Joule heating parameter is increased then the velocity profile and the temperature profile also increased.
5. When the heat generation parameter *Q* is raised, the velocity and temperature profiles are meaningfully increased.
6. The disparity of heat generation parameter *Q*, Joule heating parameter, variable viscosity and variable thermal conductivity the local skin friction coefficient, the local Nusselt number and the velocity distribution over the whole boundary layer decreases, but the temperature distribution increases.

Disclaimer (Artificial intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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