Research Article

Mathematical Modeling of Host-Parasitoid Dynamics for Sustainable Pest Management

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ABSTRACT

Aims: The management of agricultural pest populations represents a critical challenge in sustainable crop production, particularly for challenging species like the tomato fruit borer, *Helicoverpa armigera*. The research work aimed in developing and analysing a novel mathematical model describing the interaction between the tomato fruit borer and its egg parasitoid, T*richogramma chilonis*. The main objective of this study to analyze the stability of the proposed system of differential equations.
Model Formulation: The model incorporates stage-structured population dynamics and impulsive control measures, providing insights into biological pest control strategies. In this model the tomato borer is represented by the egg and larval stages, and the parasitoid is considered in terms of the parasitized eggs. This model assumes uniform spatial distribution of both host and parasitoid populations, constant environmental conditions affecting parameters like reproduction and mortality rates, and idealized parasitoid release conditions where the released amount p remains consistent across interventions.
Methodology: We used existing secondary data from research articles' instead of conducting primary field experiments for parameter identification for this mathematical model. The use of secondary data from peer-reviewed literature not only provides validated parameter values that have been rigorously tested and verified by multiple researchers, but also allows for the incorporation of findings from diverse geographical locations and climatic conditions.
Results: Stability analysis identified three equilibrium points: the trivial, host-only E0,coexistence equilibria E1, and E2. The coexistence equilibirium E2 found to be locally stable under optimal parasitoid release intervals and quantities. The equilibrium point E2= (9.14,0.02,5.71) represents a coexistence state in the host-parasitoid dynamics, where the tomato fruit borer's egg density stabilizes at 9.14, the density of parasitized eggs at 0.02, and the larval density stabilizes at 5.71. This equilibrium is analyzed through its associated eigenvalues. This stability provides a reliable basis for implementing successful biological pest control strategies in tomato cultivation, ensuring a resilient and sustainable ecosystem.

Keywords: biological control, impulsive system, tomato fruit borer, stability

1. INTRODUCTION

Tomato (Lycopersicon esculentum) is one of the most commonly and extensively grown vegetables all over the country. This vegetable being a rich source of vitamins and minerals, occupy an important place in the food basket of Indian consumers. One of the major constraints identified in the production is the increasing incidence of insect pests, diseases and nematodes. a most commonly used vegetable by all classes of people is highly susceptible to the fruit borer attack. Tomato fruit borer Helicoverpa armigera, a major pest on developing fruits is responsible for major yield loss. Eggs are laid singly, generally on leaves and flowers in the upper canopy of the plant. The young larvae feeds on the tender foliage and the fourth larval instar bores inside the fruits and eats away the internal content of the fruits. Such fruits are not preferred by consumers. The holes made on the fruits are circular and the larva feeds on keeping the head portion only inside the hole. A female lays approximately 500 eggs.

Biological control is defined as the reduction of pest population by introducing their natural enemies namely predators, parasitoids and pathogens. The parasitoids are species which develop within or on the host and ultimately kill it. Thus parasitoids are commonly reared in the laboratories and periodically released in high density populations as biological control agents. The biological pest control, which is not only eco-friendly but also cheaper and more effective than other insecticides. The main goal of pest control is to maintain the density of the pest population at an equilibrium level below the economic threshold level. In order to increase the profits and to maintain the environmental quality, economic decisions related regarding pests should be appropriately understood and adopted. Hence the mathematical modeling of these agro ecosystems can help to study their dynamics and choose the biological control strategy. Thus a good strategy of biological pest control based on mathematical model to study the interactions between the crop borer and its parasitoids is highly significant in the present world of agriculture.

In this research work, the mathematical model of interaction between the tomato borer, Helicoverpa armigae and its egg parasitoid, Trichogramma chilonis represented by system of impulsive differential equations [2] is considered. The model is developed by the process of biological control of the fruit borer that is carried over by periodically releasing the parasitoids to keep the larvae population at an acceptable low level [3]. We are interested to develop a new mathematical model adequate to represent interactions between the pest and its biological control and to determine the optimum level of release of parasitoids. The dependent variables used in the model are the egg population density of the tomato borer, parasitized egg and the larval population. Also fraction of the eggs the larvae emerged at time, fraction of the parasitized eggs from which the adult parasitoids emerge and the fraction of the larval population which moults into pupal stage at time and the number of parasitoids emerged at time.

The aim of the study is to analyse the stability of the impulsive control system, ie., to maintain the pest population in an equilibirium level below the economic injury level . Relevant computational tools for undergoing numerical simulations for different possible scenarios of biological pest control in Tomato borer based on the mathematical models will be elaborated.

1.1 Brief review of work done

The sustainable control of agricultural pests such as Helicoverpa armigera, the tomato fruit borer, is still a major problem for contemporary farming. In order to parasitize Helicoverpa armigera eggs and inhibit population growth, the application of biological control agents such as egg parasitoids, specifically *Trichogramma chilonis*, has gained significant attention. This review summarizes key studies and advances in modeling pest-parasitoid interactions, with a particular focus on *Helicoverpa armigera* and *Trichogramma chilonis*. The ecological, biological, and operational factors affecting the effectiveness of pest control are also highlighted in this review, which looks at earlier studies on pest-parasitoid dynamics with an emphasis on the creation of mathematical models.

The interaction between Helicoverpa armigera and Trichogramma chilonis. has been used extensively in integrated pest management (IPM) strategies. (Abbas et al., S. 2024) investigated the environmentally friendly use of Trichogramma chilonis against Helicoverpa armigera in tomato crops, showing that targeted releases significantly reduced pest populations. In the same way, (Hussain et. 2015) examined parasitoid's ability to parasitize Helicoverpa armigera, finding that, it was highly effective in controlled settings. Mass rearing and release strategies for Trichogramma species have been optimized in various agricultural systems. (Wang et al. 2014) detailed a comprehensive protocol for large-scale production and application of Trichogramma in cornfields. Landscape diversity is a critical factor in enhancing the biological control potential of Trichogramma chilonis. (Liu et al. 2016) examined the influence of landscape heterogeneity on the parasitism of Helicoverpa armigera eggs in cotton fields. These findings emphasize the need for habitat management strategies that promote biodiversity to support natural enemy populations. (Liu et al. 2024) demonstrated how the incorporation of functional plants could conserve predator populations, repel pests, and enhance biological control in apple orchards.

Mathematical modeling plays a pivotal role in understanding pest-parasitoid interactions and optimizing control strategies. (Akman et al. 2018) developed an impulsive stage-structured IPM model incorporating refuge effects whereas (Liu et al. 2023) proposed pest control switching models that integrate instantaneous and non-instantaneous impulsive effects. (Zhang et al. 2023) further explored predator-prey systems with state-dependent impulsive controls. The key research aspect of (Singh, 2021) is stochastic discrete-time modeling with analysis of cross-correlation functions to study host-parasitoid population dynamics and stability mechanisms Paparella et al. (2016) developed a model to study the biological control of chestnut gall wasps by Torymus sinensis. Although they focus on a different pest, their work illustrates the use of modeling in optimizing parasitoid release strategies and predicting long-term ecological outcomes.

The foraging behavior of parasitoids and the environmental context significantly influence biological control outcomes. (Liu et al. 2018) examined how tomato yellow leaf curl virus infection alters the behavior and parasitism efficiency of the parasitoid Encarsia formosa on Bemisia tabaci.

Thermal requirements and biological parameters also play a crucial role in determining parasitoid efficacy. (Oliveira et al. 2017) investigated the thermal tolerance of T. pretiosum, a species closely related to Trichogramma chilonis .

The integration of various control methods has proven crucial for effective pest management. (Abbas et al. 2015) evaluated the compatibility of insecticides with T. chilonis releases, while (Gulati et al. 2021) formulated mathematical models incorporating chemical treatments. Recent studies by (Rubio et al. 2022) and (Molter et al. 2023) have provided frameworks for multi-parasitoid systems, and (Zevika et al. 2024) emphasized the importance of optimal control strategies.

(Singh et al. 2021) explored fractional models incorporating memory effects, providing a new dimension to the analysis of pest dynamics. (Wang et al. 2024) analyzed pest control models with multiple delays and impulsive effects, offering a sophisticated framework for studying time-dependent interactions.
In recent years, the modern research study by (Agboka et al., 2024; Anam et al., 2024) employs artificial intelligence and ecological modeling to map suitable sites for deploying biological control agent against pests.These findings demonstrate that AI-driven analysis enhances decision-making for sustainable pest management, ensuring high suitability levels for parasitoid introduction.
The integration of biological and mathematical research has significantly advanced our understanding of pest-parasitoid interactions. Studies on Trichogramma chilonis and Helicoverpa armigera have demonstrated the potential for eco-friendly pest management through targeted parasitoid releases. Mathematical models provide invaluable tools for optimizing these strategies, enabling researchers and practitioners to predict outcomes, identify optimal release schedules, and adapt to changing ecological conditions. The existing literature underscores the complexity of pest-parasitoid interactions and the critical role of mathematical modeling in understanding and managing these biological systems

2. MATHEMATICAL MODEL FORMULATION AND METHODOLOGY

The model incorporates stage-structured population dynamics and impulsive control measures, providing insights into biological pest control strategies. We propose a simple mathematical model of interaction between the tomato borer and its egg parasitoid. In this model the tomato fruit borer is represented by the egg and larval stages, and the parasitoid is considered in terms of the parasitized eggs. Consider two main stages of development of the tomato fruit borer helicoverpa armigera*,* the egg and larval stages and there exists only an egg parasitoid Trichogramma chilonis in a common environment. Let $x_1(t)$, the egg density of helicoverpa armigera, $x_2(t)$, the density of parasitized eggs and $x_3(t)$, the larval density of helicoverpa armigera be the state variables representing different population components. Then, by assuming the logistic growth for the egg population of tomato borer, the system dynamics are described by the following set of differential equations:

$$\frac{dx_1}{dt} = \alpha\left(1-\frac{x_1}{k}\right)x_1 - m_1x_1 - \eta_1x_1 - \beta x_1x_2$$

$$\frac{dx_2}{dt} = \beta x_1x_2 - m_2x_2 - \eta_2x_2$$

$$\frac{dx_3}{dt} = \eta_1x_1 - m_2x_3 - \eta_3x_3, \quad t \neq n\tau$$

with impulsive control conditions

$$\boldsymbol{\Delta x_1(t) = 0}$$
$$\boldsymbol{\Delta x_2(t) = p}$$
$$\boldsymbol{\Delta x_3(t) = 0,\ t = n\tau,\ n \in Z}$$

Where the model parameters are described as :

α: net reproduction rate

β: parasitism rate

k: environmental carrying capacity

m_1, m_2, m_3: natural mortality rates of the egg, parasitized egg and larvae populations

η_1: the fraction of the eggs from which the larvae emerge at time *t*

η_2: the fraction of the parasitized eggs from which the adult parasitoids emerge at time *t*

η_3: the fraction of the larvae population which moults into pupal stage at time *t*

p: Release amount of parasitoids

τ: Time interval between releases

2.1 Salient Features of the Model Description

This mathematical model presents a stage-structured representation of host-parasitoid dynamics, specifically focusing on the Helicoverpa armigera and Trichogramma chilonis system. The significant characteristics of this model lies in the integration of density-dependent host population growth through the logistic term $\alpha(1-x_1/k)x_1$, along with explicit tracking of both unparasitized and parasitized egg stages, and subsequent larval development. We included an impulsive control conditions ($\Delta x_2(t) = p$ at $t = n\tau$) that which is useful for the simulation and optimization of periodic parasitoid release strategies. making it particularly valuable for practical Integrated Pest Management (IPM) implementation. The model also includes key biological processes including parasitism ($\beta x_1 x_2$), natural mortality rates (m_1, m_2, m_3), and stage transitions (η_1, η_2, η_3), providing a realistic framework for understanding population dynamics and intervention effects.

2.2 Assumptions made in the formulation of the Model

This model operates under several simplifying assumptions that should be considered when interpreting its predictions. It assumes uniform spatial distribution of both host and parasitoid populations, constant environmental conditions affecting parameters like reproduction and mortality rates, and idealized parasitoid release conditions where the released amount p remains consistent across interventions. These assumptions limit the ability of the model to capture spatial heterogeneity, seasonal variations, and real-world release efficiency variations.

Despite these limitations, the model serves as a valuable tool for determining optimal release intervals (τ) and quantities (p) of parasitoids, predicting population trajectories under different management scenarios, establishing economic control thresholds, and evaluating the cost-effectiveness of biological control strategies. The stage-structured approach particularly enhances its utility in practical applications by allowing managers to track and respond to changes in specific life stages of the pest population.

2.3 Identification of Parameter Values

It was decided to use existing secondary data from research articles' instead of conducting primary field experiments for parameter identification for this mathematical model. Field experiments for Helicoverpa armigera and Trichogramma chilonisinteractions would require extensive time spanning multiple pest generations, substantial resources, and carefully controlled conditions to generate reliable data across different weather patterns and seasonal variations. Also it was understood that parameter estimation through field trials faces significant challenges due to the complex nature of host-parasitoid dynamics, environmental stochasticity, and the difficulty in maintaining consistent observation protocols across multiple life stages. The use of secondary data from peer-reviewed literature not only provides validated parameter values that have been rigorously tested and verified by multiple researchers, but also allows for the incorporation of findings from diverse geographical locations and climatic conditions. This approach ensures model robustness while acknowledging the practical limitations of conducting comprehensive field trials, particularly given the time-sensitive nature of developing effective pest management strategies. The values in Tables 1, 2, and 3 are justified based on their alignment with data reported in previous studies focusing on pest and parasitoid dynamics. The values used for *Helicoverpa armigera* and *Trichogramma chilonis* are justified as they align with biological and ecological parameters reported in relevant studies (S Abbas et al., 2024, Hussain et al., 2015 and Abbas et al., 2015). The net reproduction rate, parasitism rate, and mortality rates are reflective of data specific to pest-parasitoid systems in tomato ecosystems (Liu et al., 2016; Hussain et al., 2015). Stage transition rates, including egg to larval and parasitoid emergence rates, are validated by prior research on developmental timings under similar agroecological conditions (Oliveira et al., 2017). These parameters ensure the model's applicability to *H. armigera* and *T. chilonis* interactions in tomato crop systems.

Table 1: Population Parameters

Parameter	Value	Unit	Description
α	0.42	day^{-1}	Net reproduction rate
β	0.035	day^{-1}	Parasitism rate
k	200	eggs/plant	Environmental carrying capacity

Table 2: Mortality Rates

Parameter	Value	Unit	Description

Parameter	Value	Unit	Description
m_1	0.15	day^{-1}	Natural mortality rate of eggs
m_2	0.12	day^{-1}	Natural mortality rate of parasitized eggs
m_3	0.18	day^{-1}	Natural mortality rate of larvae

Table 3: Stage Transition Rates

Parameter	Value	Unit	Description
η_1	0.25	day^{-1}	Egg to larval transition rate
η_2	0.20	day^{-1}	Parasitoid emergence rate
η_3	0.22	day^{-1}	Larval to pupal transition rate

Table 4: Control Parameters

Parameter	Value	Unit	Description
p	50	parasitoids/release	Release amount
τ	7	days	Time interval between releases

*The above values represent averages under standard field conditions of temperature and rainfall range (25±2°C, 65±5% RH). The values may vary significantly based on local environmental conditions and pest-parasitoid strain characteristics.

3. RESULTS

3.1 Stability Analysis of the Host-Parasitoid System

The system of differential equations incorporated with the parameter values is given by

$$\frac{dx_1}{dt} = 0.42\left(1 - \frac{x_1}{200}\right)x_1 - 0.15x_1 - 0.25x_1 - 0.035x_1x_2 \quad \text{--------------------(1)}$$

$$dx_2/dt = 0.035x_1x_2 - 0.12x_2 - 0.20x_2 \quad \text{------------------------(2)}$$

$$dx_3/dt = 0.25x_1 - 0.18x_3 - 0.22x_3 \quad \text{----------------------------(3)}$$

where $\Delta x_1(t) = 0$, $\Delta x_2(t) = 50$ and $\Delta x_3(t) = 0$, $t = 7n$, $n \in Z$

The stability of the system dynamics is studied by finding the equilibrium points. Equate the above equations to zero to find the equilibrium points of the dynamical biological system.

$$0.42\left(1-\frac{x_1}{200}\right)x_1 - 0.15x_1 - 0.25x_1 - 0.035x_1x_2 = 0 \quad \text{--------(4)}$$

$$0.035x_1x_2 - 0.12x_2 - 0.20x_2 = 0 \quad \text{---------------------------------(5)}$$

$$0.25x_1 - 0.18x_3 - 0.22x_3 = 0 \quad \text{-------------------------------(6)}$$

Solving the system of equations we get the equilibirium points (x_{1*}, x_{2*}, x_{3*})

$$0.42\left(1-\frac{x_{1*}}{200}\right)x_{1*} - 0.15x_{1*} - 0.25x_{1*} - 0.035x_{1*}\,x_{2*} = 0$$

$$0.035\,x_{1*}\,x_{2*} - 0.12x_{2*} - 0.20x_{2*} = 0$$

$$0.25x_{1*} - 0.18x_{3*} - 0.22x_{3*} = 0$$

We used Python code to solve the above complex systems to get the equilibrium values.

Hence we have three equilibirium values for (x_{1*}, x_{2*}, x_{3*}) given by

E_0 = (0.0, 0.0, 0.0), the trivial points

E_1 = (9.52, 0.0, 5.95), the host only state equilibrium and

E_2 = (9.14, 0.02, 5.71), the co-existence equilibrium

The Jacobian matrix at any point (x_1, x_2, x_3) is:

$$J=\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}$$

$$J = \begin{bmatrix} 0.42(1-2x_1/200) - 0.40x_2 + 0.035x_2 & -0.035x_1 & 0 \\ 0.035x_2 & 0.035x_1 - 0.32 & 0 \\ 0.25 & 0 & -0.40 \end{bmatrix}$$

The stability of the dynamic system is fundamentally characterized by the nature of its eigenvalues. Substitute each equilibrium point into the Jacobian matrix to evaluate J at that point.and then compute the eigenvalues of J at each equilibrium. When all eigenvalues possess negative real parts, the system achieves local stability, indicating that small perturbations from the equilibrium point will result in a return to the original state. Conversely, if any eigenvalue exhibits a positive real part, the system becomes unstable, meaning that minor deviations from the equilibrium will lead to progressively larger oscillations or divergence, potentially causing significant system disruption. This eigenvalue analysis provides a critical mathematical framework for understanding the qualitative behavior and long-term dynamics of complex systems across various scientific and engineering domains. The eigen values of the Jacobian matrix J around the three equilibrium points and the stability result based on these eigen values are given in the Table 5.

Table 5: Eigen values around the equilibirium points

Equilibrium points	**Eigen values**	**Result**

E_0 = (0.0, 0.0, 0.0)	(-0.4, 0.02, -0.32)	Unstable
E_1 = (9.52, 0.0, 5.95)	(-0.4, -0.019984, 0.0132)	Unstable
E_2 = (9.14, 0.02, 5.71)	(-0.4, -0.009594, -0.011567)	Stable

3.2 Simulation of the system dynamics

The dynamics of the pest-parasitoid interaction is visualised by 3D phase space and time series graph. A 3D plot shows trajectories in the x_1, x_2, x_3 space, giving insight into the system's attractors or behavior and the time-series graphs show how these variables evolve over time. A phase portrait provides a geometric visualization of how different variables in a system change together over time, offering a comprehensive view of the system's dynamics and stability properties.

The figures 1- 6 show how x_1(t), the egg density of helicoverpa armigera, x_2(t), the density of parasitized eggs and x_3(t), the larval density of helicoverpa armigera evolve over time, allowing us to analyze the system's behavior from different starting points. The figures 7-9 show the phase portrait of the system at three levels of equilibrium points.

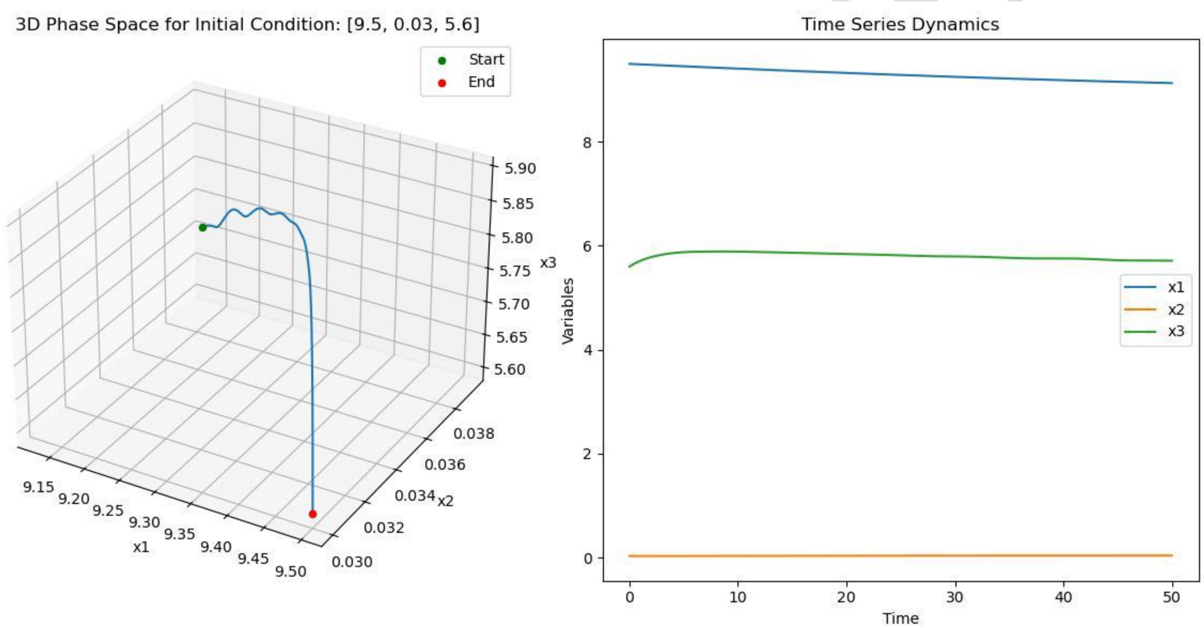

Fig. 1. 3D Phase space and time series dynamics for the initial condition (9.5, .03,5.6)

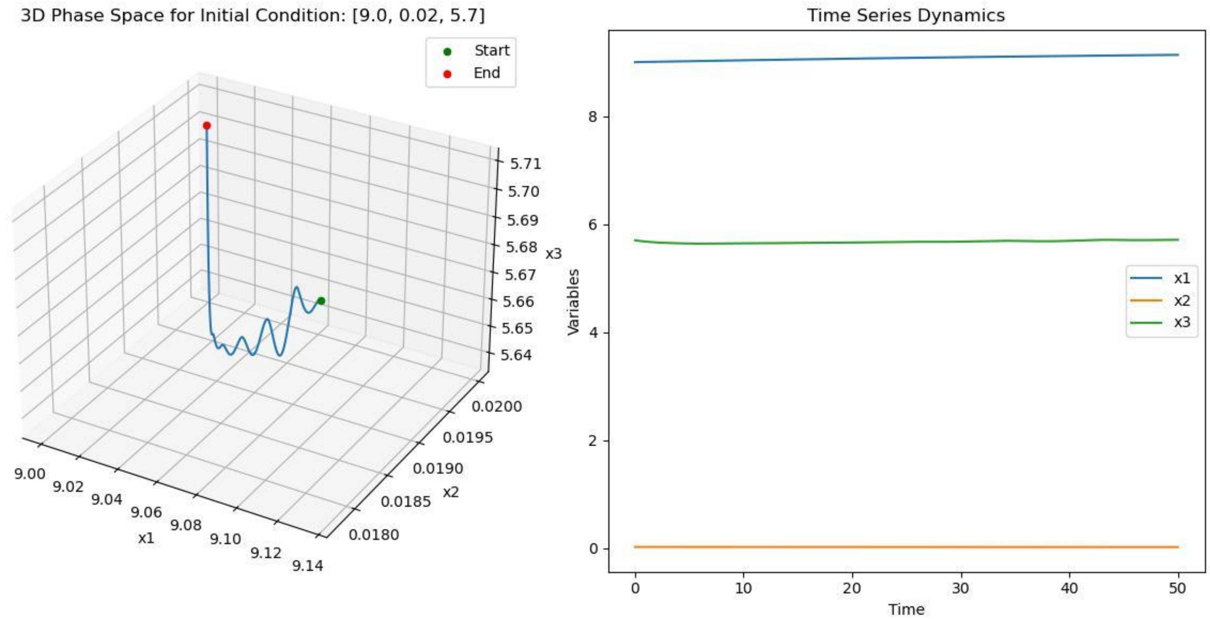

Fig. 2. 3D Phase space and time series dynamics for the initial condition (9.0, .02,5.7)

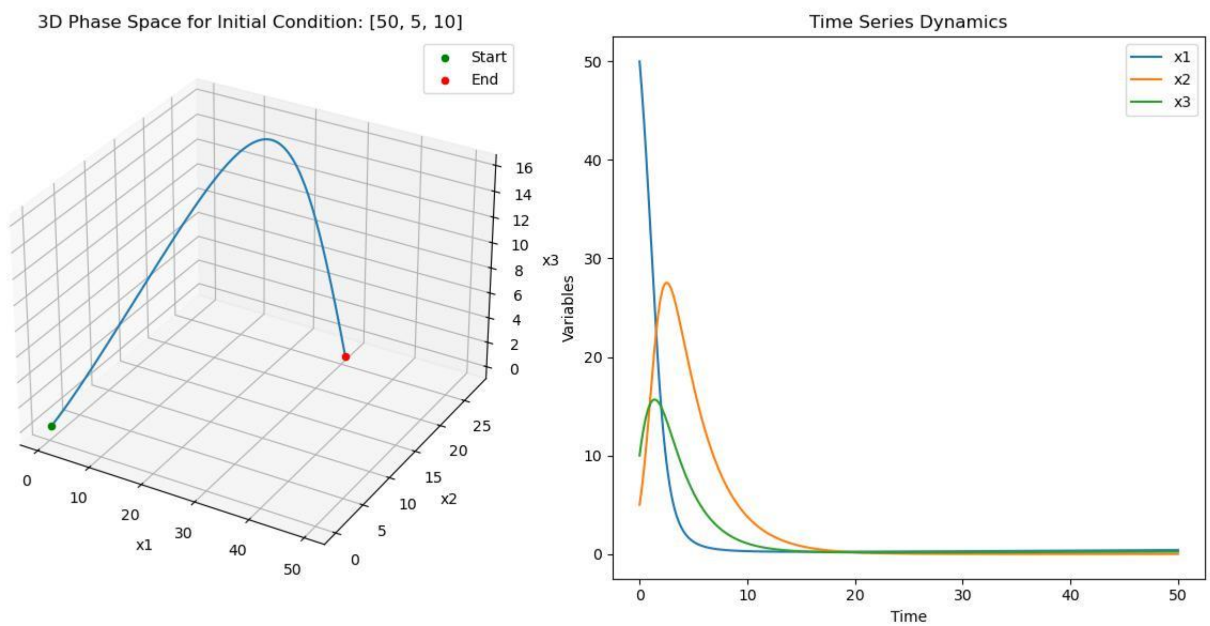

Fig. 3. 3D Phase space and time series dynamics for the initial condition (50, 5,10)

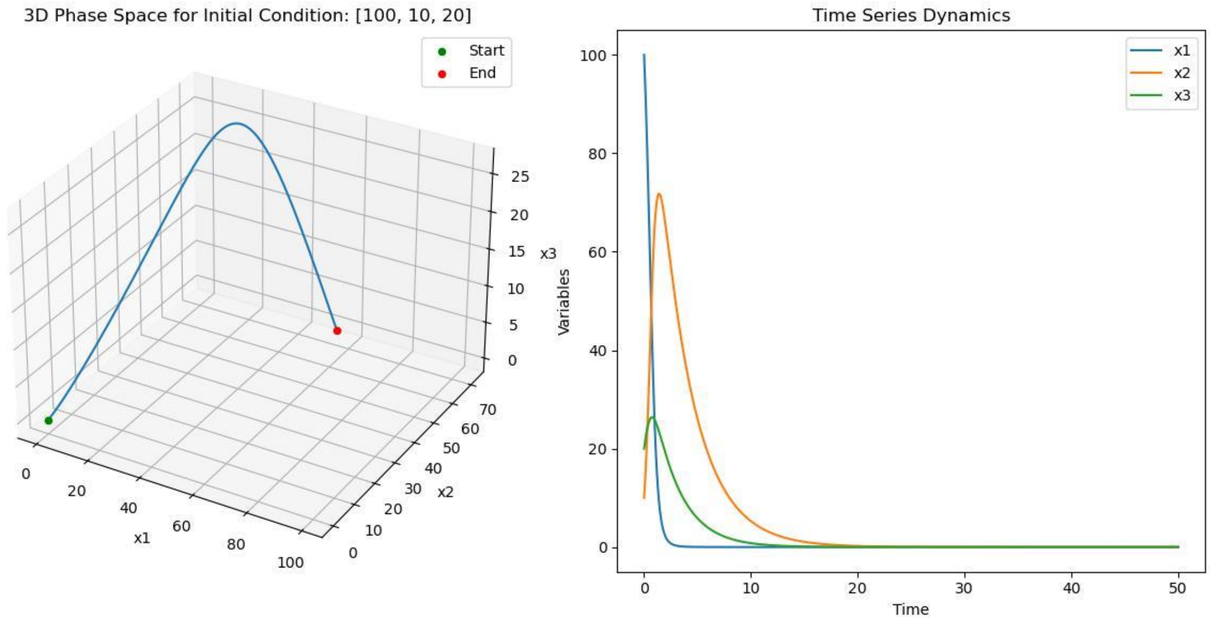

Fig. 4. 3D Phase space and time series dynamics for the initial condition (100,10,20)

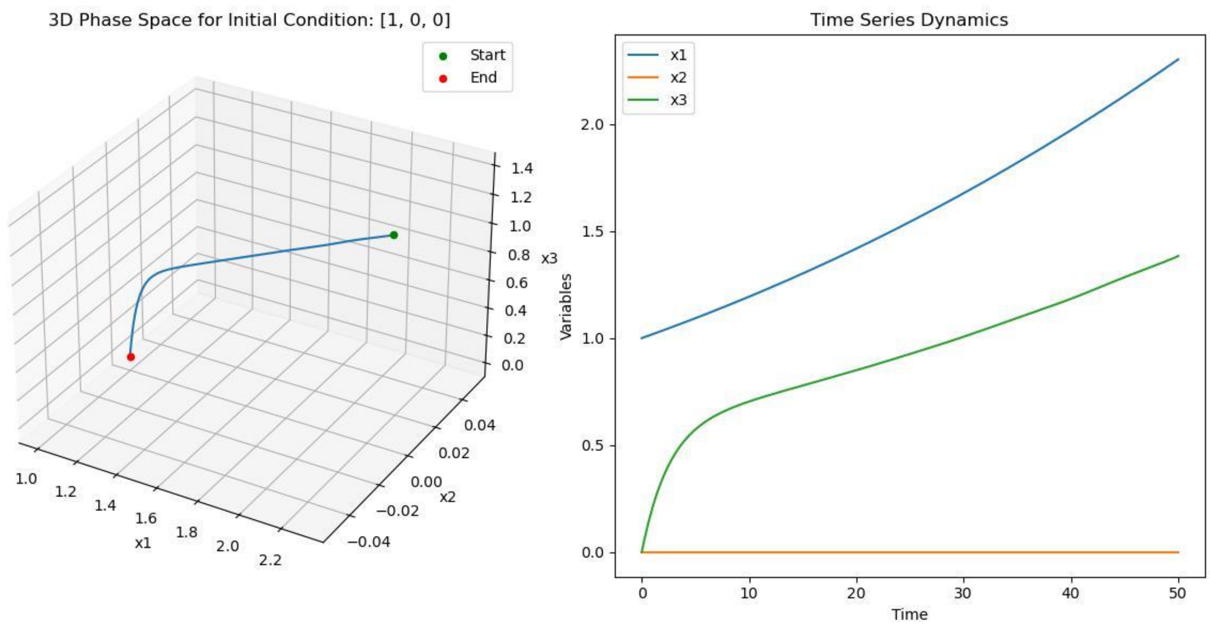

Fig. 5. 3D Phase space and time series dynamics for the initial condition (1, 0,0)

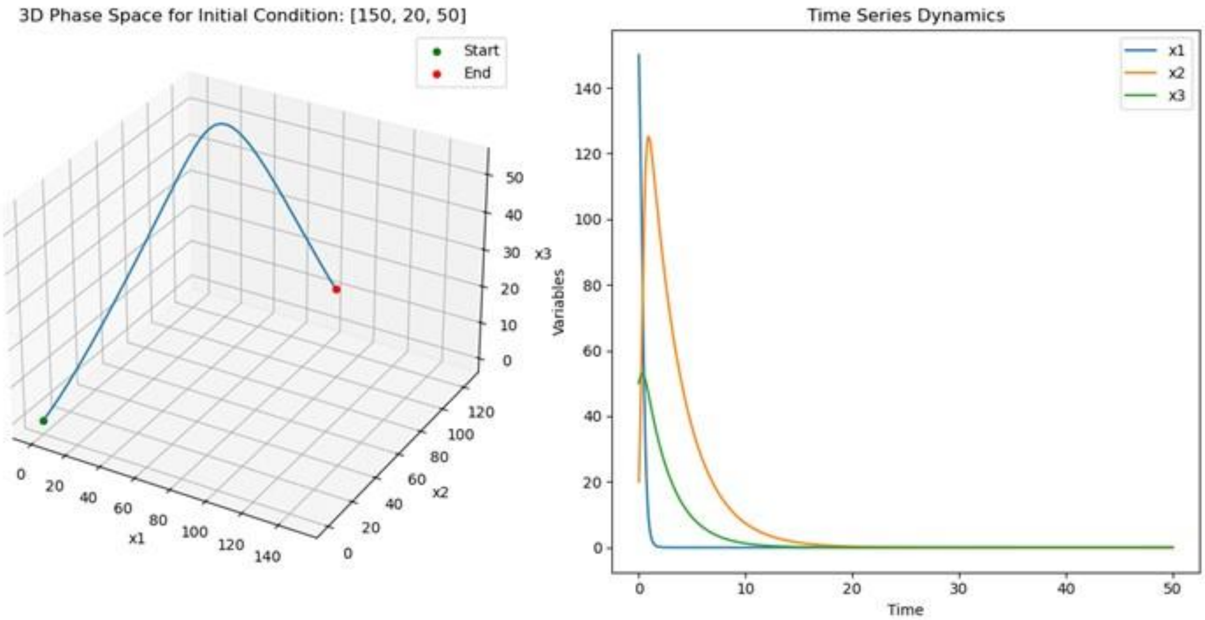

Fig. 6. 3D Phase space and time series dynamics for the initial condition (150,20,50)

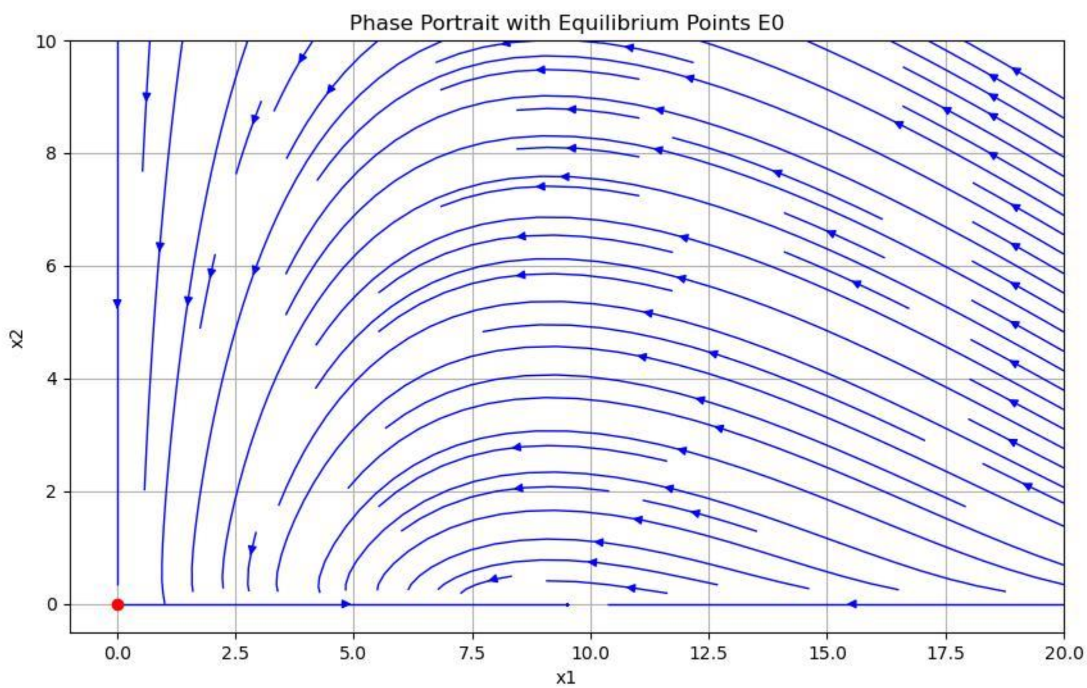

Fig 7. Phase Portraits at the Equilibrium point E_0

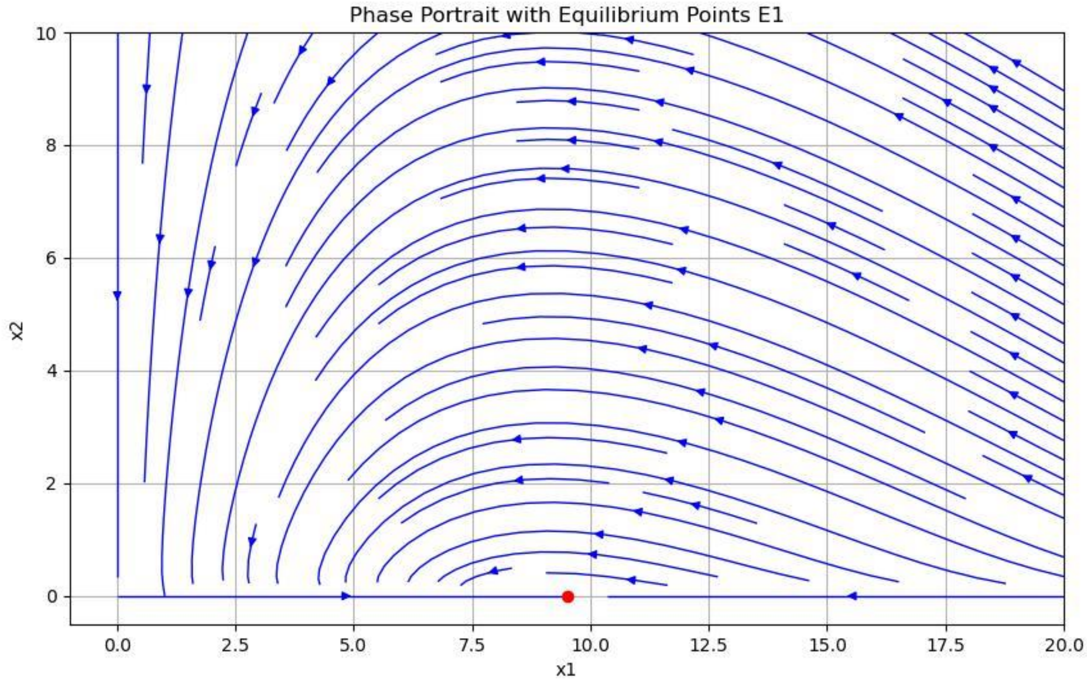

Fig 8. Phase Portraits at the Equilibrium point E_1

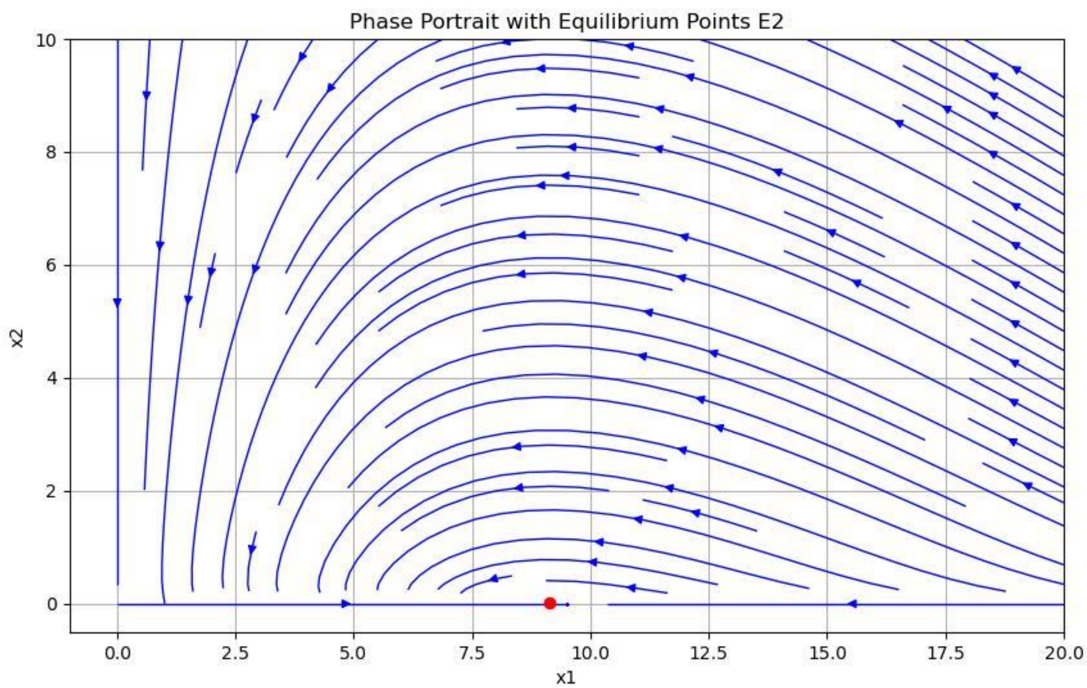

Fig 9. Phase Portraits at the Equilibrium point E_2

3.3 DISCUSSION

The eigenvalue analysis provides critical insights into the stability characteristics of the system at three distinct equilibrium points. Each equilibrium state represents a different ecological scenario, highlighting the potential outcomes of host-parasitoid interactions in pest management.

The zero population state occurs at the equilibrium E_0 where all populations are absent, representing total extinction. The presence of one positive value in the eigenvalues indicates the instability of the system. This result aligns with ecological expectations, as a zero-population state is unlikely to persist in a natural environment. Any small introduction of pest eggs or larvae would drive the system away from this equilibrium, preventing total collapse. This instability is beneficial, ensuring that the system remains ecologically viable under normal conditions.

The equilibrium E_1 (9.52,0,5.95) represents a parasitoid free state where the pests exist in the absence of parasitoid control. The eigenvalues (-0.4, -0.019984, 0.0132) include one positive value, indicating instability. This is a favorable outcome from a pest management perspective, as it suggests that introducing parasitoids into the system will naturally disrupt this pest-only state, potentially leading to better pest control.

The coexistence state occurs at the relevant equilibrium point E_2 (9.14, 0.02, 5.71), where both pest and parasitoid populations coexist. The eigenvalues have all negative real parts, indicating local stability, a crucial finding for pest management. It suggests that when both pests and parasitoids are present at these levels, the system will naturally maintain this balance, even when subjected to small disturbances. This stable coexistence represents an ideal scenario for sustainable pest control.

The progression from E_1 to E_2 shows how the introduction of parasitoids can transform an unstable pest-only system into a stable, balanced ecosystem. The slight reduction in pest population from E_1 to E_2 demonstrates the effectiveness of biological control while maintaining a realistic, non-zero pest population.

This analysis validates the potential of parasitoid-based biological control as a sustainable pest management strategy, offering both theoretical support and practical guidance for implementation in agricultural settings.

4. CONCLUSION

The mathematical model offers a robust framework for understanding host-parasitoid dynamics. The model underscores the importance of regular monitoring and the stability of the system is strongly influenced by natural mortality rates, emphasizing the importance of precise parameter adjustments.

5. RECOMMENDATIONS

Effective implementation of the model requires regular monitoring of field populations to validate its predictions, while adjusting parasitoid release parameters based on optimized model outcomes. To enhance its impact, these strategies should be integrated with existing Integrated Pest Management (IPM) approaches. Future model enhancements should prioritize incorporating environmental variability, spatial components to account for heterogeneity, and validation using field data. Additionally, integrating economic thresholds will boost the model's practical applicability and decision-making relevance, ultimately leading to more effective and sustainable pest management practices.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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