**Modelling Interest Rate Return in Nigeria based on Asymmetric Power Autoregressive Conditional Heteroscedasticity**

**ABSTRACT**

*This study modeled the Nigerian commercial bank interest rates using Generalized Autoregressive Conditional Heteroscedasticity (GARCH). The data spanning January 1997 to December 2023 was sourced from the Central Bank of Nigeria (CBN) statistical bulletin. The time plot of the original and return series initially followed a trend and metamorphosed, which caused it to become stationary. The variables were stationary at lag one, according to the results of the series' Augmented Dickey-Fuller test. To simulate the interest rate return series, the study used both symmetric and asymmetric (GARCH) models, which capture characteristics of financial time-series data such as volatility clustering and leverage impact. Asymmetric power autoregressive conditional heteroscedasticity (APARCH (1,1)), however, was chosen as the best model for the return series after six models were estimated for the conditional variance. Ultimately, the asymmetric APARCH model was the most suitable model to estimate the interest rate volatility. The model's post-estimation revealed that its distribution was normal.*

**Keyword***s:* Interest Rate, APARCH, GARCH, Volatility, and Leverage Effect*.*

* 1. **Background to the Study**

Autoregressive conditional heteroskedasticity (ARCH) models measure volatility and forecast it into the future. ARCH models are dynamic, meaning they respond to changes in the data. ARCH models are used by financial institutions to model asset risks over different holding periods. Volatility problems arising in financial markets, which have become more pronounce in more than twenty years back, are especially visible cases of large interest rate fluctuations. This has been of major concern to developing nations and emerging market economies. Deposit money banks, as opposed to traditional savings accounts, safeguard investors' funds and enable them to generate better returns; these returns are known as interest. Sunday (2012) stated that interest is a payment that a borrower makes to a lender in exchange for using money that has been put in the bank for a predetermined amount of time. He went on to say that interest is the payment made to those who contribute the money needed to buy capital goods. It was discovered that the set price paid or charged fluctuates over time, regardless of the amount or frequency of the payment made against the total amount of money deposited in the bank for a specific amount of time. It is observed to be very low at times and high at others. Sunday (2012) also disclosed that investors benefit from low interest rates on money deposited in banks, as high interest rates put depositors in danger of losing money.

 Besides, an accurate volatility model and forecasting can allow a more appropriate estimation of the value at risk. Therefore, it becomes necessary to develop an appropriate model for modelling shocks in interest rate levies in a developing country like Nigeria. According to Volodymyr (2002), the theoretical approach to modelling interest rates differs according to countries. Some interest rate models developed and tested in one country may not be appropriate when they are applied in a transitory or inflationary economy. Nonetheless, it has long been held since the initial reaction to the global financial crisis that interest rates respond more sharply to fluctuations in volatility. This fact necessitates more precise volatility modelling. Furthermore, a more precise evaluation of the value at risk can be made possible by reliable volatility modelling and forecasting. As a result, it becomes essential to create a suitable model for simulating interest rate shocks in developing nations like Nigeria. Volodymyr (2002) asserts that the theoretical framework for interest rate modelling varies by nation. Certain interest rate models that have been studied and established in one nation might not be suitable for use in an economy that is experiencing inflation or transitory conditions.

It is also observed that even when economists apply different models or approaches in the same country with similar hard economic conditions, the outcomes may differ.

.

2.0 **LITERATURE REVIEW**

This study's statistical theoretical approach was based on the 1986 Anderson and Bollerslev model and the 1982 Engle model. Iulian (2012) claims that before the scientific community focused on heteroskedasticity and how it affects forecasting and investment decisions, researchers typically measured financial asset volatility using the Auto-regressive Integrated Moving Average (ARIMA) model, which was created by Box and Jenkins (1976). Additionally, implied volatility was calculated using the option pricing Black and Scholes (1973) formulae. These methods relied on the false assumption that the time series of financial returns would always have a constant variance. The "stylised facts" (Cont, 2001) of financial returns, such as leptokurtosis, volatility clustering, intermittency, fat tails, and leverage, were therefore not captured by the techniques.

Engle (1982) created the Autoregressive Conditional Heteroskedasticity (ARCH) model, which considers variance to be reliant on previous squared errors, to account for the empirical finding that variance varies over time and might be interpreted as depending on past values. Iulian (2012) added that although Eagle's model successfully solved the aforementioned problems with financial time series, estimating its coefficients proved to be challenging. A generalization of the ARCH model was presented by Bollerslev (1986), in which variance is dependent on both the squared residuals and their historical values at the same time.

It should be noted that, in contrast to other models, the maximum likelihood approach is used by ARCH and GARCH to estimate coefficients rather than the sample standard deviation. The scientific community has given the modelling of financial time series with ARCH and GARCH models a great deal of attention. Bollerslev et al. (1992) compiled a list of approximately 300 pertinent references at the time, but since then, more significant research has been done in this area.Deebom and Essi (2017) stated that the 1982 Engle model, also known as the ARCH model, can be considered an alternative model to the standard deviation model. It calculates the linear of the square of the disturbance in the recent past along with the conditional variance of the disturbance term extracted from an ARMA process. This theory is relevant to the study since the study's goal is to capture volatility, which is solely determined by using a model that calculates the variables' mean and variance. However, several problems generally revolve around the optional lag length of the variable when employing the ARCH model to describe the mean and variance of financial time series.As a result, choosing the ideal lag frequently causes over-parameterization, which is an issue for users of the ARCH model. As per Rynbery's (2000) findings, over-parameterization might result in huge values of new lags in the model when adding more lags. In response to the aforementioned information, Anderson and Bollerslev (1986) proposed an extension to the Autoregressive Conditional Heteroskedasticity model known as the Autoregressive Moving Average (ARMA) model. The primary reason for introducing this model was to address the parsimonious situation in the error term of a linear regression model.A more comprehensive approach to the ARMA model was created as a step forward. The Generalised Auto-regressive Conditional Heteroskedasticity (GARCH) was the term used to describe this. The conditional variance of a variable as a function of its lagged value and the disturbance term in a linear regression model was modelled using this model, known as the GARCH model. The symmetric and asymmetric effects of volatility in macroeconomic variables can be measured with great benefit using this GARCH model. This can also be used to gauge the volatility of interest rates and the macroeconomic effects on the economy.

**3.0 METHODOLOGY**

3.1 **Conditional Mean Model**

**3.1.1 ARMA (p,q)**

Both an AR and an MA procedure are involved in the movement of the Nigerian Naira (NN) to the US Dollar (USD). Consequently, ARMA (p,q), where p denotes the autoregressive term order and q denotes the moving-average term order, may be expressed as

**= + + (1)**

An autoregressive moving average model of orders p and q, denoted ARMA (p, q), is stated to be followed by a series {yt}. The model is stationary and invertible due to the constants βj and α\_(i), and {εt} is a white noise process.

**3.2.0 Volatility Models**

There are two primary categories of volatility modelling techniques: symmetric models and asymmetric models. While the asymmetric model's negative and positive size shocks have differing effects on future volatility, the symmetric model's conditional variance solely depends on the asset's magnitude and not its sign. Brook, Chris (2008).

**3.2.1 Symmetric Models**

**2.2.2 ARCH (1)**

The conditional variance of a series is modelled using the autoregressive conditional heteroscedastic (ARCH) model. This approach is frequently applied to explain variations that rise and decrease. Assume that the variance of a series y\_t is to be modelled. The ARCH (1) model states that the variance of y\_t depends on Yt-1 at time t.

The ARCH model is represented mathematically as follows.

**+ (2)**

We impose the constraints that ≥ 0 and ≥ 0 to avoid negative variance

**3.2.3. GARCH (1, 1) Model**

The variance at time t is modelled by the generalised autoregressive conditional heteroscedastic model, which permits the conditional variance to depend on prior lags. The model uses the value of the past squared observation and past variance. The replicas calculate the amount that a volatility shock from today influences the volatility in the following period. Determines how quickly this effect changes over time. Below is an illustration of a GARCH (1, 1) model.

**+ + (3)**

This is a GARCH (1, 1) model.Is known as the conditional variancesince it is a one-period ahead estimate for the variance.

**32.4 GARCH- M (1,1)**

In the financial markets, a high level of risk is thought to yield a high return. One could think about the GARCH IN MEAN model in these kinds of situations. The model permits a sequence's condition mean to rely on its conditional variance.

The following is the model.

**= + + (4)**

**=**

**( 0, )**

**= + + (5)**

Where are constant, if is positive the return is also positive related to volatility

**3.3.0. Asymmetric Models**

It is necessary to discuss the ASYMMETRIC GARCH model because negative shocks, or bad news, typically have a greater impact on volatility than positive shocks. To that aim, we limited our research to the most well-known asymmetric GARCH models, including EGARCH, TS-GARCH, and APARCH.

**3.3.1. EGARCH Model:**

The model is known as exponential GARCH (EGARCH). Compared to the pure GARCH specification, the model has several advantages. First, even if the parameters are negative, (σ\_i^2) will still be positive because the log (σ\_i^2) is simulated. Therefore, the model parameters do not require the imposition of non-negativity requirements in an unnatural manner. Second, asymmetries are permitted by the EGARCH formulation since the model has the following representation if the link between volatility and returns is negative, γ will be negative:

**Log= + + (6)**

Where is leverage effect co-efficient. (If it indicates the presence of leverage effect).

Note that when is positive there is good news, when is negative there is bad news

**3.3.2. Ts GARCH Model:**

Another GARCH method that is capable of modelling leverage effects is the Threshold GARCH

(TGARCH) model, which has the following form:

**= + + + (7)**

**Where =**

is the leverage effects coefficient. (if  it indicates the presence of leverage effect). That is depending on whether is above or below the threshold value of zero,

 has different effects on conditional variance when is positive.

**3.3.3 APARCH (1,1)**

The Asymmetric Power ARCH (APARCH) model can handle both power transformation of the variance and asymmetric effects. The following is its specification for the conditional variance.

**= + - ) + (8)**

Where = , the parameter (assumed positive and ranging between 1 and 2)

**4.0 RESULTS**

 

**Figure 1** The time plot at levels and of Returns on Interest Rate

The presence of a trend in a series will make it not to remain stationary. The first difference's time plot demonstrates a stationary process and provides indications of clustering in the interest rate returns series time plot.

**Table 1: Summary of Descriptive Statistic for Interest Rate**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Mean** | **medium** | **maximum** | **mini** | **Std. dev** | **skewness** | **kurtosis** | **prob** | **sum** | **observe** |
| 5.16 |  4.03 |  19.38 | 1.33 | 3.8 | 3.8 | 1.8 |  0.00 | 1672.3 | 324 |

Similarly, the descriptive test was applied to the study's variable, interest rate returns. The descriptive statistics for the series are shown in Table (1), along with the mean, median, maximum, minimum, standard deviation, skewness, kurtosis, and Jarque-Bera test statistic. The variable's sample mean is positive, indicating a positive mean return; nevertheless, the return series' matching standard deviation is considerable. Furthermore, the series' skewness statistics are positive. This indicates a high increase (rightward tilt) in the series. One of the prevalent features of financial time series data is an indication that is skewed to the right. Additionally, the 5.88 kurtosis values indicate the existence of a fat tail. At 306.4977, the Jarque-Bera (J-B) test statistic demonstrates statistical significance.

**Table 2: STATIONARITY TEST**

|  |  |  |
| --- | --- | --- |
|  | Raw series | Return series |
| Critical value  | ADF test statistics (-3.382) | ADF test statistics (-16.2) |
| 1% | -3.45 | -3.45 |
| 5% | -2.867 | -2.867 |
| 10% | -2.57 | -2.57 |

Stationarity tests were performed on the data to prevent false regression. The unit root on variables was tested using the Augmented Dickey-Fuller (ADF) test; Table 2 displays the outcome of Since the series is non-stationary, the results of the Augmented Dickey-Fuller (ADF) test at the level and first differences and probability values in brackets indicate the presence of a unit root. The probability values (p-values) at the level are less than 0.05 (p-values >0.05) in the variable. Gujarati (2003)

**Table 3: Testing for the Presence of an ARCH Effect**

|  |  |
| --- | --- |
| **Estimator** | **Lag 1** |
| F-statistic  | 0.027 |
| Prob F(1,237) | 0.869 |
| n\*R2 | 0.027 |
| X2(1,1) | 0.869 |

**4.1 Test for Heteroscedasticity**

Nonetheless, the residual derived from the return series' test for heteroscedasticity (the ARCH effect) is displayed in Table (3). An ongoing, permanent ARCH effect was found after the return series was tested for it. The chi-squares test statistics (0.8689) correlate to a higher value than the F-statistic (0.027). This suggests that ARCH effects are present in the return series.

.

**Table 4:** Model E**stimation for Symmetric and Asymmetric Models of Return on Interest Rate**

|  |  |  |
| --- | --- | --- |
| **Models/****Parameter.** | **Symmetric GARCH** | **Asymmetric GARCH** |
| **ARCH (1)** | **GARCH (1,1)** | **GARCH-M (1,1)** | **APARCH (1,1)** | **TS GARCH (1,1)** | **GARCH (1,1)** |
| C | 101.39(0.000) | 30.276(0.000) | 0.28.939(0.000) | **0.087****(0.000)** | 9205.931(0.9637) | 0.0019(0.000) |
|  | 0.253(0.0042) | 0.624(0.000) | 0.714(0.000) | **0.244****(0.000)** | 0.997(0.000) | 0.465(0.000) |
|  |  | 0.375(0.000) | 0.356(0.000) | **0.9268****(0.000)** | 0.4215(0.1155) | 0.616(0.000) |
|  |  |  |  | **0.876****(0.000)** | 0.169(0.323) | -0.124(0.000) |
|  |  |  |  | **0.5278****(0.000)** | 0.7672(0.000) | 0.822(0.000) |
|  |  | 1.001 | 1.07 | **1.17** | 1.4 | 1.081 |
|  | -1227.7 | -1184.89 | -1184.095 | **-1160.21** | **-**1184.617 | -1163.325 |
| AIC | 7.61 | 7.36 | 7.35 | **7.22** | 7.366 | 7.234 |
| SC | 7.63 | 7.408 | 7.403 | **7.29** | 7.42 | 7.29 |
| Obs | 324 | 324 | 324 | **324** | 324 | 324 |

Using the information criteria and log-likelihood function, six models were estimated for the interest rate return: three symmetric models and three asymmetric models. At lag 1, every variable was stationary. The asymmetric model comprises APARCH, TS-GARCH, and EGARCH, whereas the symmetric model contains the ARCH, GARCH, and GARCH-M models. Table 4 indicates that all of the return series' models have a coefficient of the ARCH model that is greater than zero (α\_0>0), indicating extremely high volatility. The financial mark is more risky when volatility is high, and the positive news is lower when volatility is low.

.



**Figure 2 Quantile –Quantile plot of standardized residual fitted from APARCH (1,1)**

**Model return on interest rate**

5.8 **Quantile –Quantile plot of standardized residual fitted**

Strong evidence of a normal distribution may be seen in the standardised residuals plot's normal quantile-quantile plots. It is seen that the lines of normalcy and quantile lie exactly on top of one another.



# Table 5: Correllogramme of Standardized Residuals Square

**4 .2 Correlogram of APARCH Model**

Since the lag decreased gradually and the lags of the ACF and PACF all fell within the same level non-lag cut across the level, the correlogram of the square residual demonstrated the existence of the ARCH effect as the value of the ACF, and PACF.

**5.0 Conclusion**

The research model uses Nigerian commercial bank interest rate data from 1997 to 2023. The information was taken from the Nigerian Central Bank website. The application of several Garch models to model the interest rate among Nigerians was the main emphasis of this study. A rigorous analysis of the time plot showed that the series' movement was erratic. The unit root was tested for each of the variables using the Augmented Dickey-Fuller (ADF) test. Since the series is non-stationary, the unit root was found because of the ADF test at the level and first differences and probability values in brackets, the probability values (p-values) at a level are less than 0.05 (p-values >0.05) in the variables.

For interest rates, the univariate specification of generalized autoregressive conditional heteroscedasticity (GARCH) models was utilized. These models included symmetric and asymmetric models that capture features of a financial series, such as leverage impact and volatility clustering. Based on the minimal AIC and SC information criterion (AIC=-7.22 SC= 7.29), the APARCH model is determined to be the best model when comparing the symmetric and asymmetric models using the information criteria. The sum of the coefficients on the lagged squared error and lagged conditional variance is one (1), and the coefficients on the lagged squared residual and lagged conditional variance term in the conditional variance equation are both highly statistically significant.

A common illustration of GARCH models for return series on financial assets is this one. This suggests that there will be very persistent shocks to the condition variance.

On the conditional variance, however, good and bad news have different effects: good news affects α, whereas bad news affects (α+β). The good news has an impact of 0.244 and the bad news has an impact of 1.17 in the APARCH model. λ has a positive and statistically significant value. This suggests that higher risk, as indicated by a higher conditional variance, raises the interest rate mean return.

**COMPETING INTERESTS DISCLAIMER:**

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Option 2:

Author(s) hereby declare that generative AI technologies such as Large Language Models, etc. have been used during the writing or editing of manuscripts. This explanation will include the name, version, model, and source of the generative AI technology and as well as all input prompts provided to the generative AI technology

Details of the AI usage are given below:

1.

2.

3.

**REFERENCES**

Atoi, M. (2014). Testing Volatility in Nigeria Stock Marketing using Generalized Autoregressive Conditional Heteroskedastic Model. *Central Bank of Nigeria Journal of Applied Statistics,* 3(5), 2- 65.

Bencik, M. (2009).The Equilibrium Interest Rate Theoretic Concepts and Application Research Department National Bank of Slovakia.

Bollersehev, T. (1986). Generalized Auto-Regressive Conditional Heteroskedasticity. *Journal of Econometrics,* 31, 307-321.

Bollerslev *et al*. (1992).7 GARCH Models of Volatility Handbook of Statistics 14,209-240.

Box, G. E. P. & Jenkins, G.M. (1976). Time series analysis: „Forecasting and control, Holden-Day, San Francisco.

Brooks, C. (2006). Introductory Econometrics for Finance. Second Edition, Cambridge University Press.

Central Bank of Nigeria (2024), Retrieved from http://www.Statistics. [cbn.gov.ng/statsonline](http://www.Statistics.cbn.gov.ng/statsonline)16th

Cont, R. (2001). Empirical Properties of Asset Returns: Stylized Facts And Statistical Issues. *Quantitative Finance,* 1( 4),223–236.

Deebom, Z.D. & Essi, I.D. (2017). Modelling price Volatility of Nigeria Crude Oil Markets using GARCH Model: 1987–2017.*International Journal of Applied Science & Mathematical Theory,* 4(3), 23.

Engle, R. F (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Various of UK Inflation. *Economical* 4 (50 ),987-1008.

[Heston](https://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=38940) S, L. & [Nandi](https://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=16277), S. (1998 ).A Closed-Form GARCH Option Pricing Model. *International Journal of Business and Management Review*, 1 (3), 35-55.

Hussainatu A.& Muftaudeen O.O. (2011). Macroeconomic Policy and Agricultural Output in Nigeria Implication for Food Security. *American Journal of Economics,4(2),99-113*

Heston, S. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, 6(3), 327-343.

Iulian, P. *(*2012*),* Using Garch-in-Mean Model to Investigate Volatility and Persistence at Different

Volodymyr, K. (2002). Interest Rates And Their Role In The Economy During Transition. The Problem of High-Interest Rates. Case of Ukraine. A thesis submitted in partial fulfilment of the requirements for the degree of Master of Arts in Economics