Review Form 3

Journal Name:	**Asian Research Journal of Mathematics**
Manuscript Number:	**Ms_ARJOM_130999**
Title of the Manuscript:	**The Minimum Reduced Sombor Index of Unicyclic Graphs in terms of the Girth**
Type of the Article	**Original Research Article**

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This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty'**, provided the manuscript is scientifically robust and technically sound.
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PART 1: Comments

	Reviewer's comment	**Author's Feedback** *(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.	**This manuscript contributes to the study of topological indices in chemical graph theory by identifying the unicyclic graph with the minimum Reduced Sombor Index (RSO). They proved that** $RSO(G) \geq n\sqrt{2}$, equality holds if and only if g(G)=n.	
Is the title of the article suitable? (If not please suggest an alternative title)	**Yes. The current title, 'The Minimum Reduced Sombor Index of Unicyclic Graphs in terms of the Girth,' is relevant to the study.**	
Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.	**The abstract is clear and comprehensive, effectively summarizing the study's purpose, methodology, and findings. In keywords delete graph invariant, tree, and characterization and add unicyclic graphs, and girth.**	
Is the manuscript scientifically, correct? Please write here.	The manuscript is likely scientifically correct, but further verification of the proofs and a clearer distinction from prior research are necessary.	
Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.	Many references are relevant and recent, particularly those discussing the Sombor index. However, one key reference is missing: the recently published paper **'Ordering Unicyclic Graphs with a Fixed Girth by Sombor Indices' (MATCH, 2024)**, P. Nithya et al, which already determines the first four smallest Reduced Sombor indices for unicyclic graphs. This paper should be cited and discussed in the manuscript to provide proper context and comparison.	
Is the language/English quality of the article suitable for scholarly communications?	"The manuscript is generally well-written and follows a clear scholarly structure. However, there are several grammatical inconsistencies, awkward phrasing, and minor typographical errors that could affect readability. Phrases like *'the study involves in defining and analyzing'* should be revised for grammatical correctness (e.g., *'the study involves defining and analyzing'*). Additionally, some mathematical expressions are not properly integrated into sentences, which may reduce clarity.	
Optional/General comments	**Keywords are irrelevant, remove graph invariant, tree, topological index and add unicyclic graph, girth**	

PART 2:

	Reviewer's comment	**Author's comment** *(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Are there ethical issues in this manuscript?	*(If yes, Kindly please write down the ethical issues here in details)*	

Reviewer Details:

Name:	**P. Nithya**
Department, University & Country	**SRM Institute of Science and Technology, India**