Review Form 3

Journal Name:	Asian Research Journal of Mathematics
Manuscript Number:	**Ms_ARJOM_130431**
Title of the Manuscript:	**Optimized hybrid block methods with high efficiency indicies for the solution of first-order ordinary differential equations**
Type of the Article	

Comment [SY1]: Delet

PART 1: Comments

	Reviewer's comment	**Author's Feedback** *(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.	The manuscript introduces a one-step optimized hybrid block method for solving first-order ordinary differential equations. The study effectively integrates optimization techniques to enhance accuracy and efficiency. Numerical results from the method demonstrate improvements compared to the reviewed methods. This contribution is relevant to the modification of existing techniques.	
Is the title of the article suitable? (If not please suggest an alternative title)	The title is suitable but needs a little adjustment. The title should read ''Optimized hybrid block methods with high efficiency for the solution of first-order ordinary differential equations".	
Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.	This article presents optimized hybrid block methods for solving first-order ordinary differential equations. The derivation employed the interpolation and collocation techniques using a three-parameter approximation. The hybrid points were obtained by minimizing the local truncation error of the main method. The obtained schemes were reformulated to reduce the number of function evaluations. The discrete schemes were derived as a by-product of the continuous scheme and used simultaneously to solve first-order initial value problems (IVPs). The schemes are self-starting, consistent, zero-stable, and convergent. The numerical results were compared with some existing techniques and found to be more accurate and efficient.	
Is the manuscript scientifically, correct? Please write here.	The manuscript is well presented.	
Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form.	The references are sufficient and most recently added.	

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Is the language/English quality of the article suitable for scholarly communications?	The English is correct for scholarly communications but needs a little improvement.	
Optional/General comments	Areas for improvement. 1. Introduction: Grammar issues: The introduction needs more improvement and should be grammar-free for smooth flow. 2. Material and methods: Grammar issues: This should read "This section outlines the derivation of a one-step optimized hybrid block method incorporating three optimal points for solving first-order ODEs. To derive the method, we approximate the solution using a polynomial Q(t) expressed as $$x(t) \approx Q(t) = \sum_{j=0}^{k} b_j t^j \,. \qquad (3)$$ where $b_j \in R$ are real unknown coefficients to be determined. The first derivative Q(t) is expressed as $$x'(t) \approx Q'(t) = \sum_{j=0}^{m} j b_j t^{j-1} \,, \qquad (4)$$ where $k = (I + C) - 1$, I and C denote the number of interpolation and collocation points respectively. Let p, q, r be the off-step points such that $0 < p < q < r < 1$. To determine the coefficients of b_j, equation (3) is Interpolated at t_n, and equation (4) is collocated at $t_{n+j}, j = 0, p, q, r, 1$This setup leads to a system of six equations with six unknown coefficients. This system of six equations can be written in matrix form as.....[Put Equation (5)]. Equation (5) is solved to determine the coefficients b_j using the Gaussian Elimination method, which is then substituted into equation (3). The implicit continuous scheme is obtained as.....[Put Equation (6)]. 3. Analysis…… Grammar issues: The authors should improve on the write-up. 4. Results and discussion Grammar issues: The authors should improve on the write-up. The authors should add two more problems by solving linear and nonlinear systems. Since systems of ODEs were mentioned in the introduction. In Tables 1 to 3, I believe the authors are referring to n as the step size. In a standard practice, this should be written as h and not n. Because n is denoted as the grid index. 5. Conclusion Grammar issues: The conclusion needs improvement and should be well-written without grammar. Finally, the manuscript can be accepted if the authors carry out these suggestions.	

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PART 2:

	Reviewer's comment	**Author's comment** *(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)*
Are there ethical issues in this manuscript?	*(If yes, Kindly please write down the ethical issues here in details)*	

Reviewer Details:

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Department, University & Country	**Ibrahim Badamasi Babangida University, Nigeria**