Report on the Manuscript Titled:"Some Inequalities via Functional Type Generalization of the Cauchy-Bunyakovsky-Schwartz Inequality"

This paper establishes a novel inequality of the form

$$\left[f^{(n)}(x)\right]^2 \leq k(x) \sum_{k=0}^{m} a_k f^{(m-k)}\left(\frac{p}{r}x+q\right) \sum_{k=0}^{l} b_k f^{(l-k)}\left(\left(\frac{2}{r}-\frac{p}{r}\right)x-q\right).$$

By leveraging a functional-type generalization of the Cauchy-Bunyakovsky-Schwarz inequality, the authors derive new inequalities for the derivatives of a one-parameter deformation of the Gamma function and demonstrate that these results are generalizations of some previous work.

The research paper is divided into three sections:

1. Introduction: This section begins by presenting the formulation of the original Cauchy inequality and offering specific generalizations. Furthermore, it introduces a one-parameter deformation of the classical Gamma function namely v-Gamma function along with its derivatives up to order n, in a clear and concise manner.

   In the main result, The authors derive inequalities involving derivatives of the v-Gamma function by employing a generalization of the Cauchy-Bunyakovsky-Schwarz inequality. Their proof leverages integral substitutions to demonstrate relationships between different expressions of the Gamma function, thus validating the inequalities under specified conditions for parameters x, v, α and β.

   The conclusion contains a concise summary of the result.

Comment

The manuscript presents a novel inequality derived from the Cauchy-Bunyakovsky-Schwarz inequality and applies it to the derivatives of a single-parameter Gamma function (v-Gamma function). The paper is well-structured, clearly demonstrates its main finding, and provides solid evidence and explanations to back it up. These results represent a valuable addition to the analysis of functional spaces, as they generalize previous findings and offer new tools for analyzing these special functions, meriting publication.