Original Research Article

Three New Approaches to estimating Energy Losses in Stepped Spillways with the Channel slope of 8.9^{o}.

ABSTRACT

A stepped spillway is a hydraulic structure built at storage and detention dams to discharge flood water that cannot be safely kept in the reservoir. It was created to minimize the kinetic energy that would have otherwise produced dangerous scour at the natural river bed beneath the spillway. They discharge this energy in floodwater using their stepping nature. Several studies in the literature show the detrimental consequences of falling water's kinetic energy on the river bed underneath the structure. Only a handful of these studies, however, have evaluated the impact of energy losses caused by stepped spillways with channel slope of 8.9^{o}. As a result, there are gaps in the rules and recommendations for designers of stepped spillways with a channel slopes of 8.9^{o}. Additionally, the existing models for estimating energy losses in stepped spillways with channels of all slopes contain a parameter, the friction factor, f, which is difficult to estimate with certainty, thereby to leading their subjective provision by the designers involved in stepped spillways design.

The goal of this study is not only to provide designers with design recommendations and information for stepped spillways with a channel slope of 8.9^{o}, but also to eliminate the 'troublesome' frictional factor; f. Using phase-detection intrusive probes, air-water flow tests were carried out in transitional and skimming flows on a stepped spillway with channel slope of 8.9° in a large facility. Three new expressions for evaluating energy losses in stepped spillways with slopes of 8.9^{o} are developed. In terms of energy dissipation, the data from the new models compared well with the measured data, with high coefficients of correlation that range between 0.87 and 1.00. All of the measured data and the estimated data are in good agreement. The models are simple to use.

Keywords: *Stepped Spillway, Energy Dissipation, Nappe Flow, Skimming Flow*

LIST OF SYMBOLS

The following symbols are used in this report:

C void fraction defined as the volume of air per unit volume of air and water; it is also called air concentration or local air content;

D_H - hydraulic diameter (m);

d_w - equivalent clear water flow depth (m);

d_c - critical flow depth (m): ;

g gravity constant (m/s2);

H - total head (m);

H_{dam} - dam height (m);

H_{max} - maximum upstream head (m) above chute toe:

$H_{max} = H_{dam} + 3/2 \times d_c$;

H_{res} - residual head (m);

h - vertical step height (m);

l - horizontal step length (m);

q_w - water discharge per unit width (m^2/s);

Re - Reynolds number defined in terms of the hydraulic diameter: $Re = \rho_w \times U_w \times D_H / \mu_w$;

U_w - mean flow velocity (m/s): $U_w = q_w/d$;

U_w - average mean flow velocity (m/s) for stepped spillway

W - channel width (m);

Y_{90} - characteristic depth (m) where the void fraction is 90%;
D - distance (m) measured normal to the invert (or channel bed);
ΔH - total head loss (m): ΔH = Hmax – Hres;
θ - angle between pseudo-bottom formed by the step edges and the horizontal;

Subscript
c - critical flow conditions;
max - maximum value;
mean - mean signal component;
w - water properties;

1. INTRODUCTION

Stepped Spillway, an innovative approach to water flow management, is a type of hydraulic structure constructed next to dams, reservoirs, and other water containment systems to manage the flow of water in a controlled and safe manner. These spillways are designed with a series of steps or cascades on their upward face that allow water to flow down in a safe, efficiently, and more orderly manner thereby reducing the velocity of flow and lowing the risk of flooding downstream, the danger of erosion, and minimizing the forces acting on the structure. The main aim of a stepped spillway is to dissipate the kinetic energy of flowing water making it one of the most efficient designs for water flow management.

Stepped spillways can be made from concrete, rock, or even metal, depending on the environmental conditions and the specific design requirements. Concrete is commonly used due to its durability, while rocks are often used in natural or rural settings. They have a visually attractive scheme that look like natural waterfalls. In certain cases, this feature can improve the visual value of the dam or reservoir, making it a more beautiful feature for visitors and tourists. They rely on gravity and natural aeration, requiring fewer external interventions or mechanical energy systems, thereby making beneficial for agricultural application such as irrigation and in wastewater treatment.

Typically, the steps of stepped spillway are designed to be steep enough to break the water flow into smaller flows, but mild enough to prevent extreme turbulence. The step geometry (its size, slope, and number) plays a major role in design component of spillways. As the flow cascades over the steps down the spillway, the growing boundary layer that started at the crest of the chute will reach the free surface where air will be entrained into the flow (Fig 1). The entrained air further reduces energy and helps to prevent erosion.

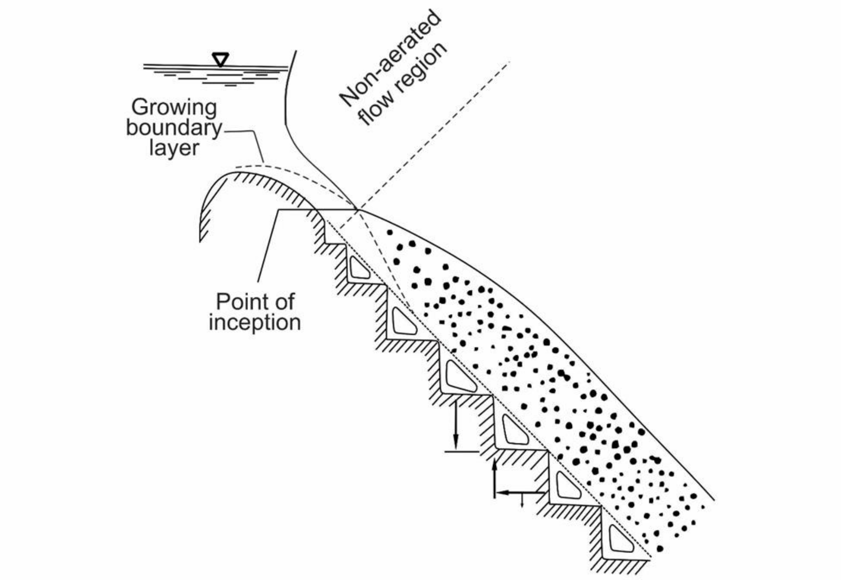

Figure 1. Skimming flow regime - Sorensen (1985)

Unlike traditional smooth-faced spillways, which depends on the smooth surfaces to direct water flow, stepped spillways integrate stepped surface where the water flows over succeeding steps. The design looks like natural waterfalls or steps in hilly topography, which reduces the velocity of water flow with substantial reduction in its erosive energy; this considerable energy dissipation and reduced velocity do not only lead to the use of small-sized basins at the end of the spillway as well as the prevention of erosive damage at the base of the structure and the natural river channel below, but also result to lower maintenance costs and fewer repairs.

Stepped spillways are an effective and efficient design used in many hydraulic structures to manage the flow of water. They provide numerous benefits, including energy dissipation, reduced erosion, and lower maintenance costs, while also offering aesthetic appeal. While they may require higher initial investment and careful design, their long-term benefits make them an attractive choice for managing water flow in dams, reservoirs, and other flood control systems. As the need for sustainable water management systems continues to grow, the stepped spillway is likely to become an increasingly popular choice in civil engineering projects.

Many researchers have investigated the energy loss in stepped spillways with the channel slope of 26.6^o and below, only a handful of these studies, however, have evaluated the impact of energy losses caused by stepped spillways with channel slope of 8.9^o. As a result, there are gaps in the rules. Additionally, the existing models for estimating energy dissipation in stepped spillways contain a parameter, f, the friction factor, the value which is difficult to estimate.
Hence, the aim of this study to investigate the energy dissipation in a flat stepped spillway with a channel slope of 8.9^o and develop models that do not contain, the ‘troublesome’ friction factor, f, to predict energy losses in it.

Flows over stepped spillways are classified into three types, each with its complicated flow that is determined by the flow rate and step geometry: nappe, transition, and skimming flow regimes (Gonzalez and Chanson, 2007).
The skimming flow regime occurs for big discharges, and it is distinguished as a cohesive stream with huge recirculation vortices between the mainstream and the steps (Chanson, 1996) (Figures 3 and 4).
The transition flow is identified by the substantial spray and splashing near the free surface and occurs at the intermediate flow rates (Gonzalez and Chanson, 2007).
The nappe flow regime occurs for modest discharges and appears as a free-falling nappe, with water bouncing from one-step to the next in a succession of small free falls (Chanson, 1996)(Figure 2).

The mode of energy dissipation for each of these three flow regime is distinct and they are estimated by Chanson as follow:

i. Energy Dissipation at Nappe Flow Regime

In a nappe flow regime with a fully developed hydraulic jump, the head loss at any intermediate step equates the energy loss. The total head loss, ΔH, along the spillway equals the maximum head available, Hmax, and the residual heat, H_1, at the bottom of the spillway (Chanson) [11]

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{3}{2} + \frac{H_{dam}}{d_c}} \quad \textit{ungated spillway} \qquad (1)$$

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\frac{d_1}{d_c} + \frac{1}{2}\left(\frac{d_c}{d_1}\right)^2}{\frac{H_{max} + H_o}{d_c}} \quad \textit{gated spillway} \qquad (2)$$

where H_{dam} is the dam height and H_0 is the reservoir free surface elevation above the spillway crest. The residual energy is dissipated at the toe of the spillway by a hydraulic jump in the dissipation basin.

For an un-gated spillway, the maximum head available and the dam height are related by:

$$\Delta H = H_{max} - H_{res} \quad (3)$$

$$H_{max} = H_{dam} + 1.5d_c \quad (4)$$

For a gated spillway, the maximum head available and the dam height are related by:

$$H_{max} = H_{dam} + H_o \quad (5)$$

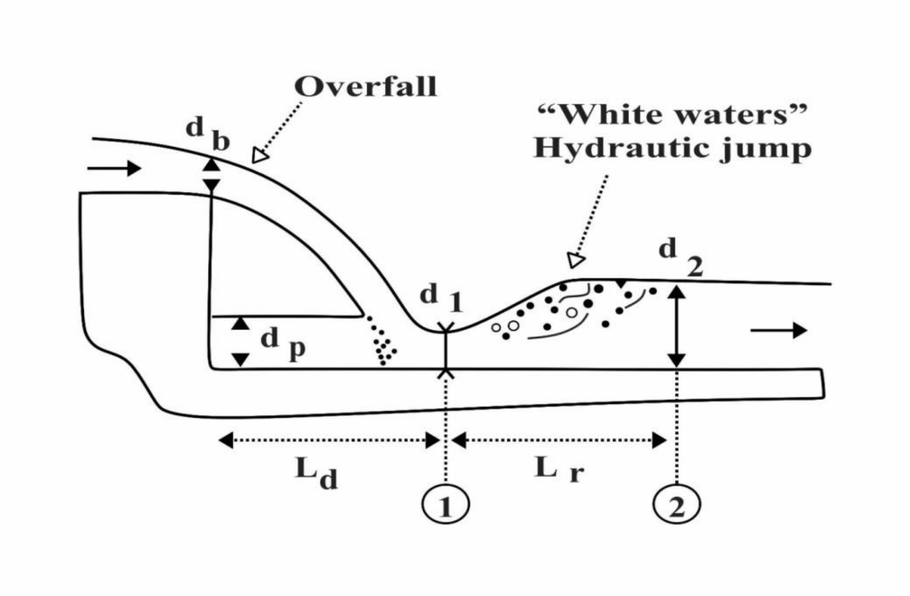

Figure 2: Nappe flow regime (Flow at a drop structure)

ii. Energy Dissipation at the Skimming Flow Regime

The energy dissipated occurred to keep stable depression vortices. If uniform flow conditions are reached downstream of the spillway, the energy loss could be calculated as follows [10] (Fig. 4):

Eq (6) is formulated for spillway slope with θ = 52 (degrees). Friction factor, f = 0.3 and f = 1.30 represent the average flow resistance on smooth spillways and stepped spillways, respectively,

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{d_w}{d_c}\right)\cos\theta + \frac{1}{2}\left(\frac{d_c}{d_w}\right)^2}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \quad (6)$$

Where E is the kinetic energy correction coefficient, θ is the dam slope in degrees
dw is the clear water depth, U_{avg} is the average velocity, the total head loss may be rewritten in terms of the friction factor, f, the spillway slope, θ, in degree, the critical depth, dc, and the dam height, H_{dam}:

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8sin\theta}\right)^{1/3} cos\theta + \frac{E}{2}\left(\frac{f}{8sin\theta}\right)^{-2/3}}{\frac{H_{dam}}{d_c} + \frac{3}{2}} \quad (7)$$

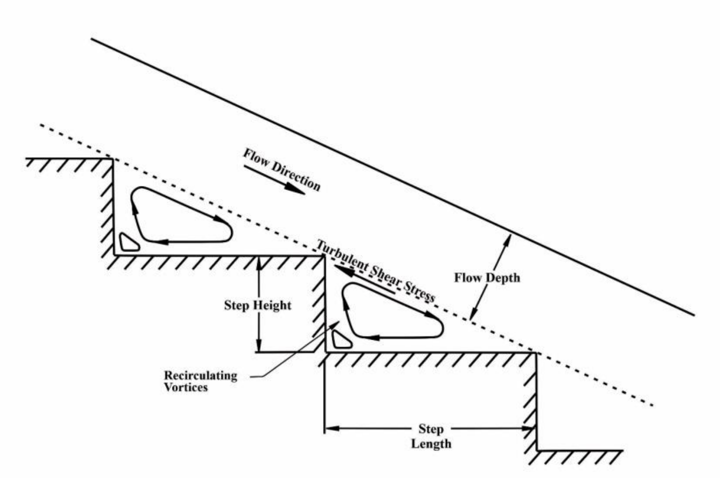

Figure 3: Skimming flow regime with uniform flow conditions

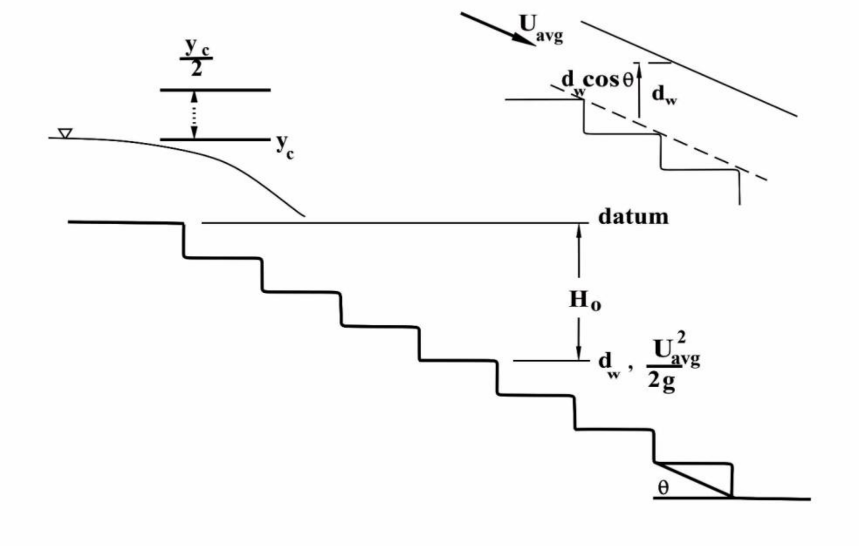

Figure 4: Arrangement of the spillway with the definition of the variables

2. MATERIAL AND METHODS

The National Water Institute's large-scale stepped spillway models with a channel slope of 8.9° was used for the experimental study.
The facility was 12 meters long and was made up of an input tank that continuously delivered an upstream water head. A 0.5 m wide uncontrolled broad-crested weir was used to let water into the experimental test area. The stepped spillway had 10 No steps with a step height of 0.10 m and a step length of 0.20 m; Perspex was used for the channel walls and PVC for the steps.
A huge upstream intake basin with dimensions of 3.0 m 2.5 m and a depth of 1.6 m provided steady flow rates. A 1.0 m long smooth sidewall convergent with a 4:1 contraction ratio provided a smooth inflow. A broad-crested weir with a height of 1 m, width of 0.52 m, crest length of 1.01 m 1.01 m, and an upstream rounded corner controlled the flow in the test portion.

The experimental facilities were large scale to minimize scale effects affecting the microscopic air–water flow processes in high-velocity free-surface flows [15]. Scale effect is a term used to describe slight distortions that are introduced by ignoring secondary forces such as viscous forces, surface tension in stepped spillway models. Viscous forces and surface tension in most open-channel

applications are deemed negligible, but in highly air- entrained flows like those expected in stepped spillways, these forces are more dominating and cannot be simply ignored. Scale effects in modeling stepped spillways have been thoroughly established by Boes (2000), Chanson (2002), Boes and Hager (2003a), and Takahashi et al. (2005). In stepped spillway models, scale effects are most typically linked with scales less than 10:1. For minimizing scale effects, see Chanson (2002), Boes and Hager (2003a), and Takahashi et al. (2006). According to Chanson (2002), a model scale of 10:1 or greater is recommended, and Boes and Hager (2003a) recommend a minimum Reynolds number of 105 and a minimum Weber number of 100. Takahashi et al. (2006) urge that Froude, Reynolds, and Morton similarity be satisfied when simulating strongly air entrained flow, but they acknowledge that this can only be done at full scale. Although researchers have not established an agreement on the boundaries to reduce scale effects in physical models of stepped spillways, some information is available. The use of traditional mono-phase flow instrumentation in high-velocity air-water flows is not possible due to the three-dimensional air-water flow with enormous volumes of air-water.
Because substantial quantities of air are entrained at the air-water interface, using a Dall Tube flow meter, or V-notch for flow rates, Prandtl-Pitot for flow velocities, or point gauge for clear water flow depth to get air-water flow attributes is impractical [11, 12].

A double-tip conductivity probe was used to conduct the experiments. Air-water flow studies were carried out with conductivity phase-detection intrusive probes at all step edges downstream of the inception site of air entrainment for all stepped sizes. The sensor diameters of the double-tip conductivity probes were 0.13 mm and 0.25 mm, and they were sampled for 45 seconds at a frequency of 20 kHz per sensor. Typically, the probe was placed at step edges in the air-water flow zone.

All measurements lasted 45 seconds at a sampling rate of 20 kHz per probe tip. The conductivity probe's basis is based on the differing resistance of air and water, which provides an immediate voltage signal. A single sensor's signal can be analyzed using a threshold technique to determine the time averaged local air concentration or void fraction C, the number of air-to-water voltage shifts expressed as bubble count rate F, and the air bubble and water droplet chord diameters.
The trials were carried out for a wide variety of discharges at numerous step edges downstream of the free surface aeration inception point.

B. Formulation of the Models

The authors analyzed about 500 with complete data to formulate energy dissipation models that govern transition and skimming flow over a wide range of operating conditions.
In modeling, it is necessary to determine the values of the parameters that can fit the model of the system it shall describe [1]. By the least square method, the best fit curve for this study was as:

$$\frac{\Delta H}{H_{max}} = \left[\alpha_o \frac{Nh}{y_c}\right]^{\alpha_1} N^{\alpha_2} h^{\alpha_2} \theta^{\alpha_3} \quad (8)$$

Where
$\frac{\Delta H}{H_{max}}$ is the energy loss ratio,
H_{max} is the maximum available height,
N is the number of spillway steps,
h is the height of the spillway steps,
θ is the spillway channel slope.

They used a portion of the measured data sets and multiple regression analysis to solve Equation (8), which yielded the values of the constant α_o along with the coefficients α_1, α_2, α_3, and α_4, which are then substituted back to give the developed models in (9 to 11).

C. Model Verification

The author used the remaining data sets, known as verification data sets, to evaluate the models' performance (interpolation).
If the model describes verification data well, then the model describes the real system and this is known the interpolation aspect [28]

3. RESULTS AND DISCUSSIONS

The Developed Models for the Nappe/Transition/Skimming Flow Regime

The discharges had transition and skimming flow rates of $0.035 \leq q_w \leq 0.234 m^2/s$ for the spillways with $\theta = 8.9^o$ with Reynolds numbers of $1.4 \times 10^5 \leq Re \leq 9.3 \times 10^5$

The authors developed the following 3 No Models to predict the energy losses in stepped spillways with channel slope of 8.9º. They, thereafter, them plotted them vis-à-vis the measured data sets in Figures 5 through 16.

Model - 1: $\Delta H/H_{max} = (1.5\,Nh/d_c)^{0.12} N^{-0.006} h^{0.02} \theta^{-0.32}$ (9)
Model - 2: $\Delta H/H_{max} = (0.05\,Nh/d_c)^{0.29} N^{0.20} h^{-0.17} \theta^{-0.30}$ and (10)
Model - 3: $\Delta H/H_{max} = (0.25\,Nh/d_c)^{0.36} N^{-0.09} h^{-0.24} \theta^{-0.09}$ (11)

MODEL - 1: $\Delta H/H_{max} = (1.5\,Nh/d_c)^{0.12} N^{-0.006} h^{0.02} \theta^{-0.32}$
The charts using the measured data sets (Run 1 to Run 4) and the developed **Model – 1** (9) are displayed in figures 5 through 8. The coefficients of correlation range from 0.89 to 1.00.

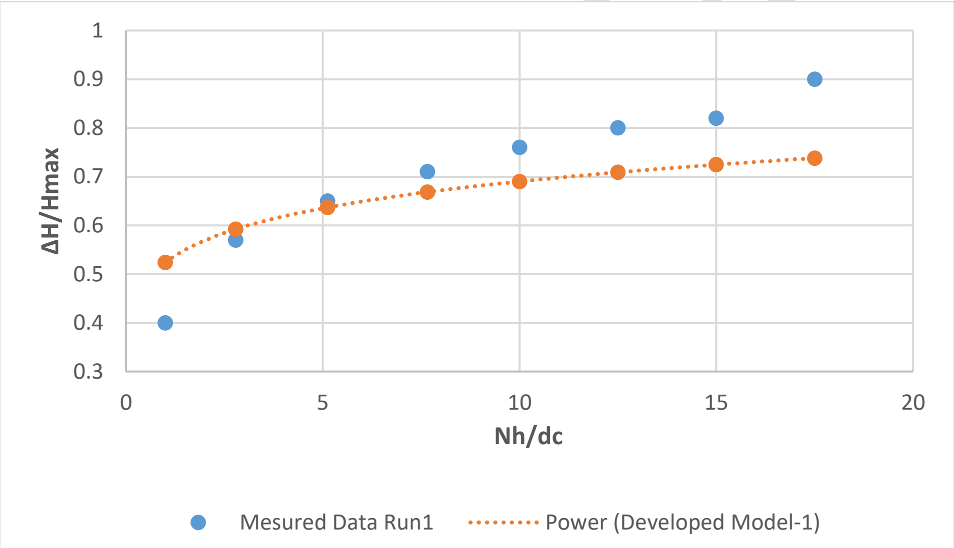

Figure 5: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,
The coefficient of correlation is 0.99.

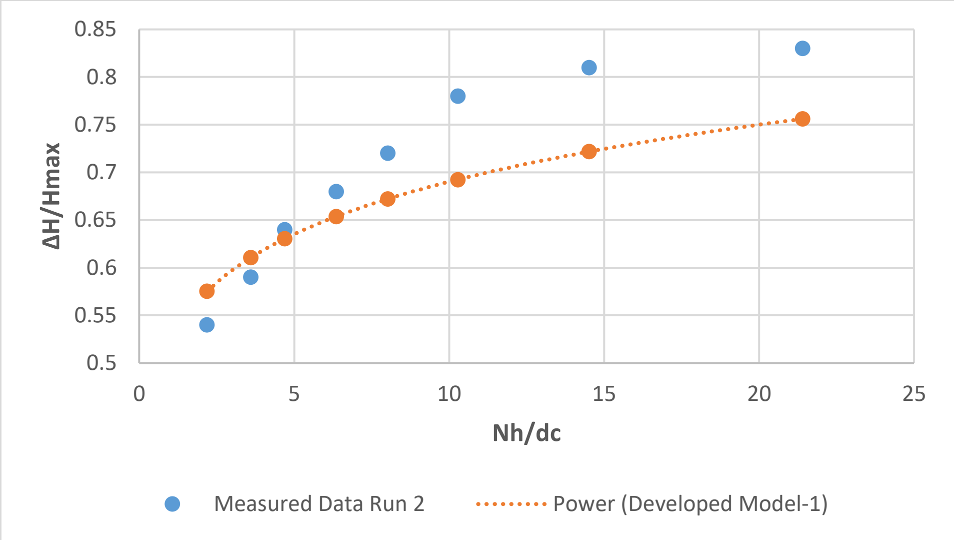

Figure 6: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00.
The coefficient of correlation is 0.99.

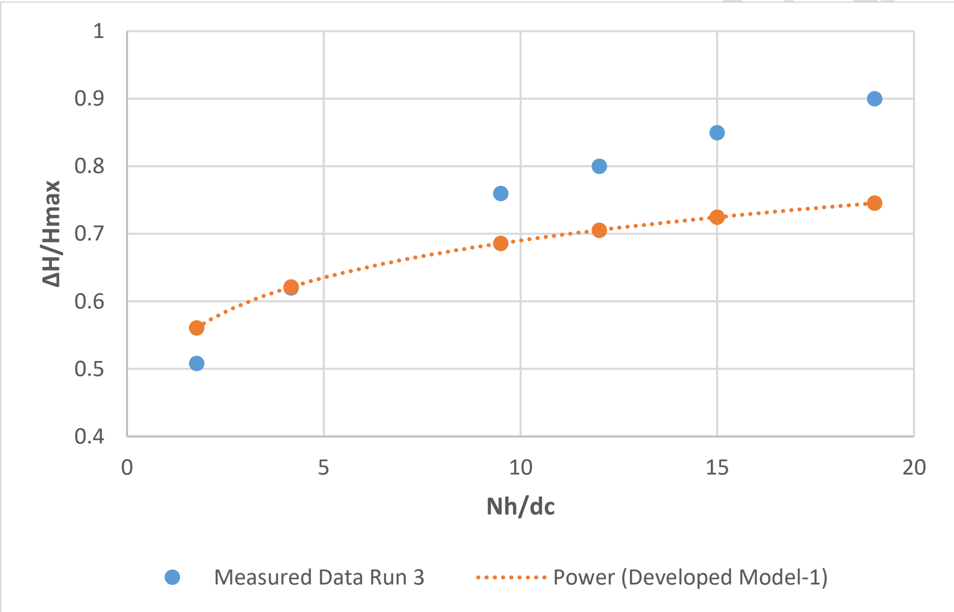

Figure 7: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00.
The coefficient of correlation is 1.00.

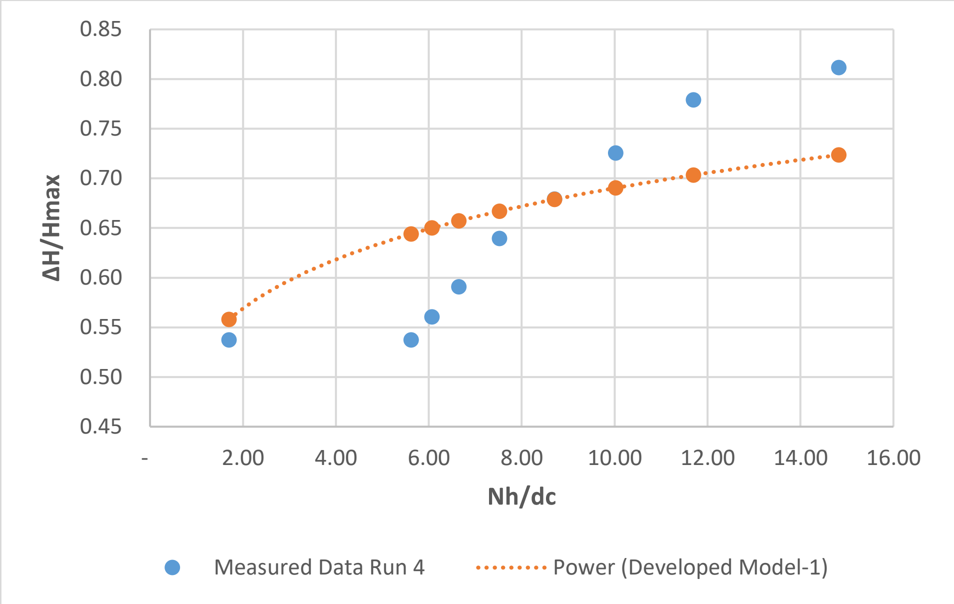

Figure 8: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00.
The coefficient of correlation is 0.87.

Model - 2: $\Delta H/H_{max} = (0.05\,Nh/d_c)^{0.29}N^{0.20}h^{-0.17}\theta^{-0.30}$
The charts using the measured data sets (Run 1 to Run 4) and the developed **Model – 2** (10) are displayed in figures 9 through 12. The coefficients of correlation range from 0.87 to 0.99.

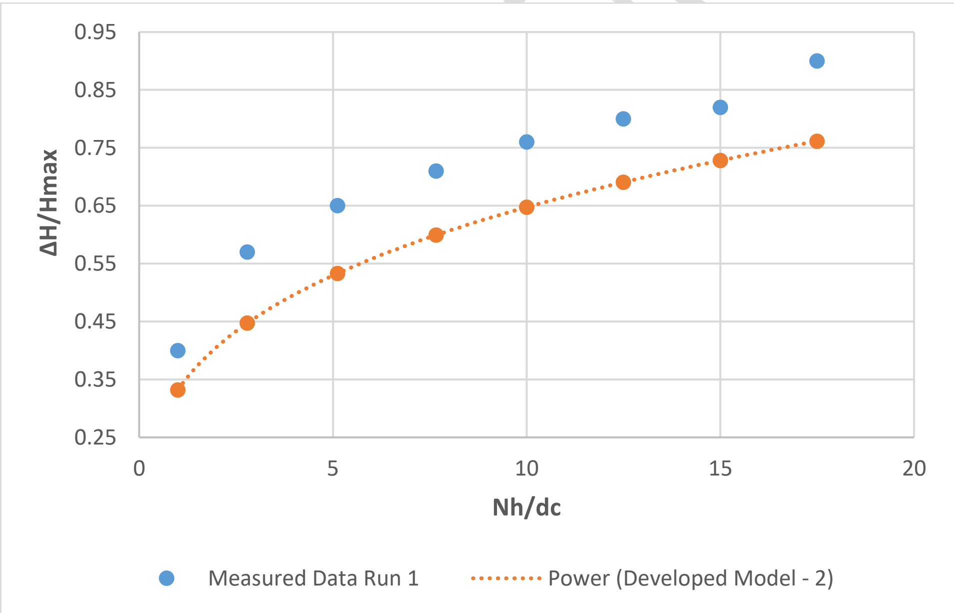

Figure 9: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00.
The coefficient of correlation is 0.99.

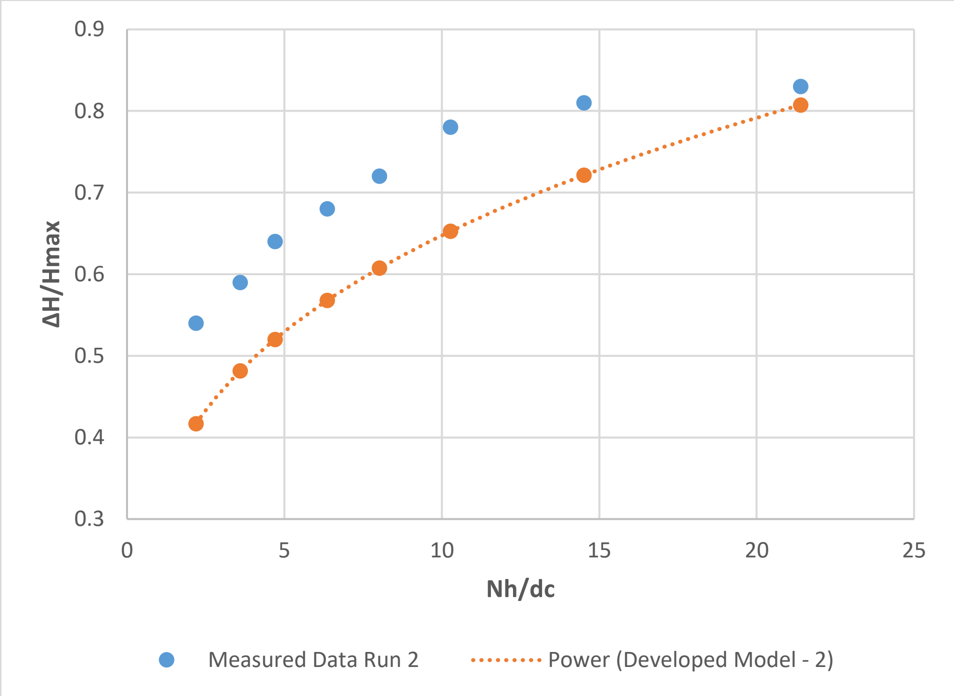

Figure 10: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00. The coefficient of correlation is 0.98.

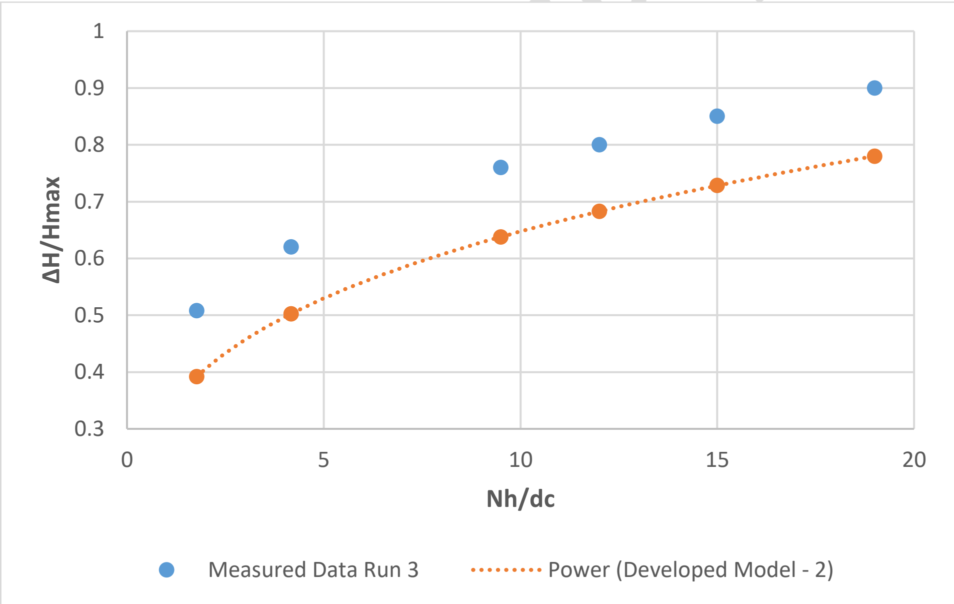

Figure 11: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00, The coefficient of correlation is 1.0.

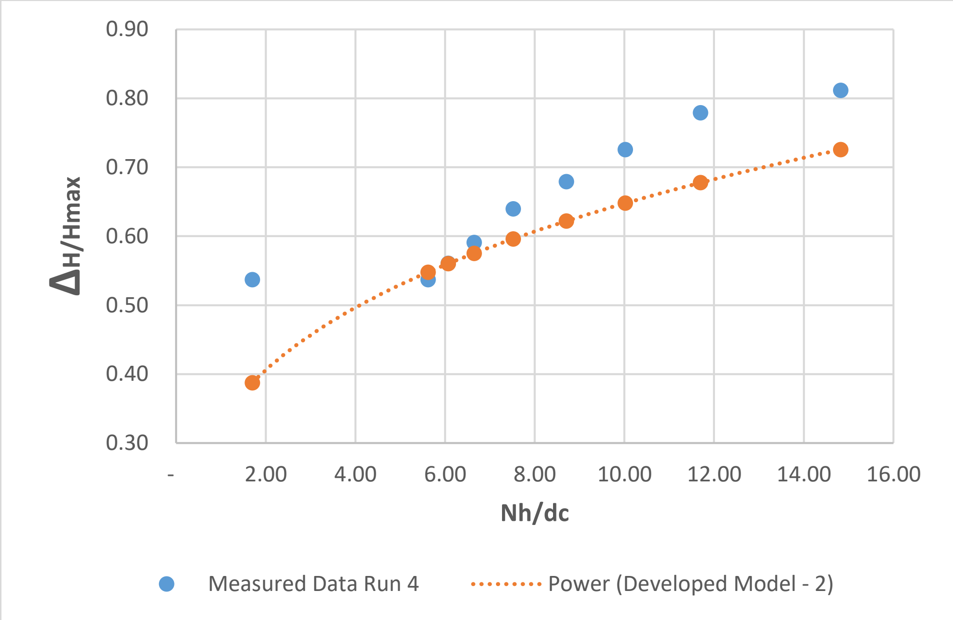

Figure 12: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,.The coefficient of correlation is 0.87.

MODEL - 3: $\Delta H/H_{max} = (0.25\,Nh/d_c)^{0.36} N^{-0.09} h^{-0.24} \theta^{-0.09}$

The charts using the measured data sets (Run 1 to Run 4) and the developed **Model – 3** (11) are displayed in figures 13 through 16. The coefficients of correlation range from 0.88 to 0.99.

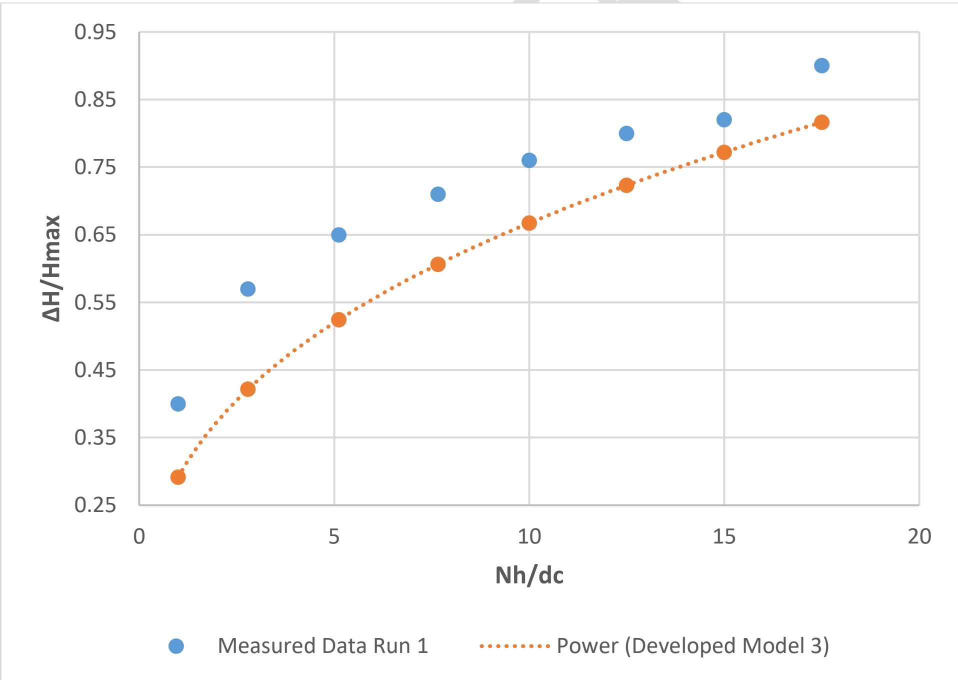

Figure 13: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,
The coefficient of correlation is 0.99.

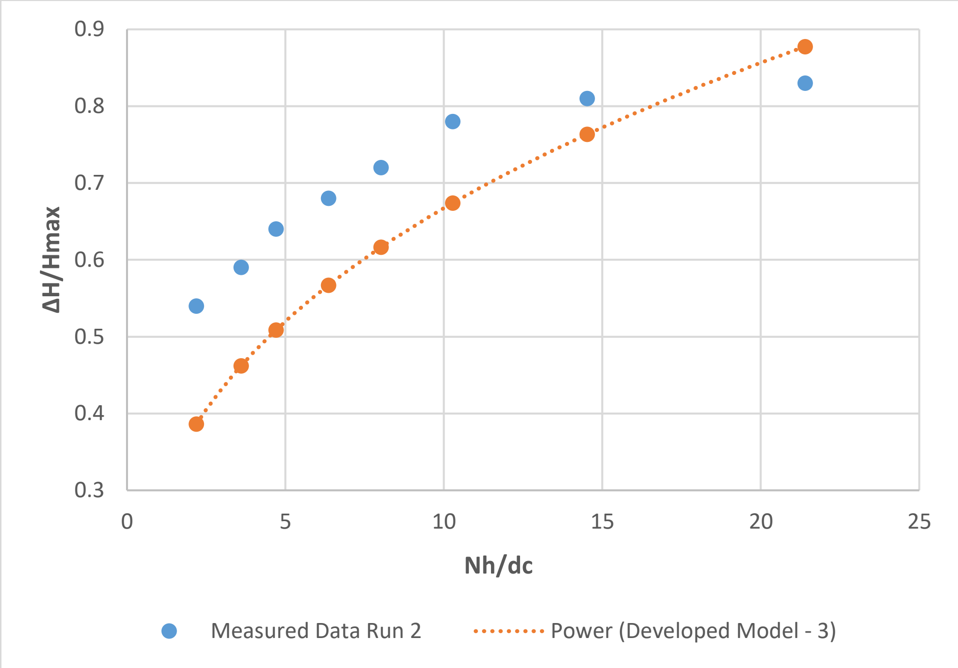

Figure 14: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,
The coefficient of correlation is 0.97.

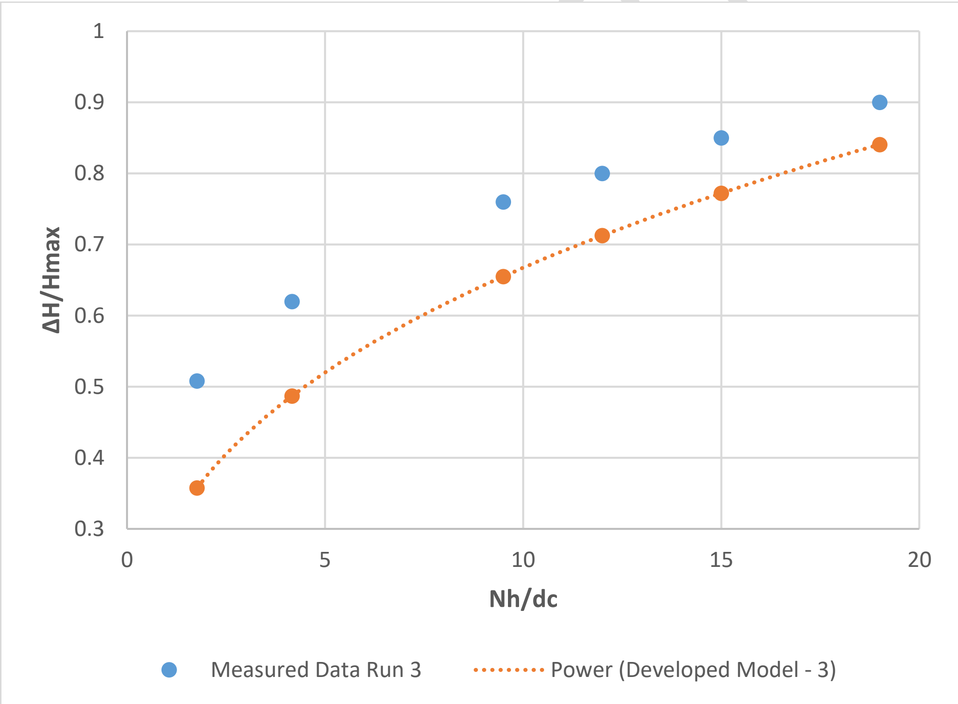

Figure 15: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,
The coefficient of correlation is 0.99.

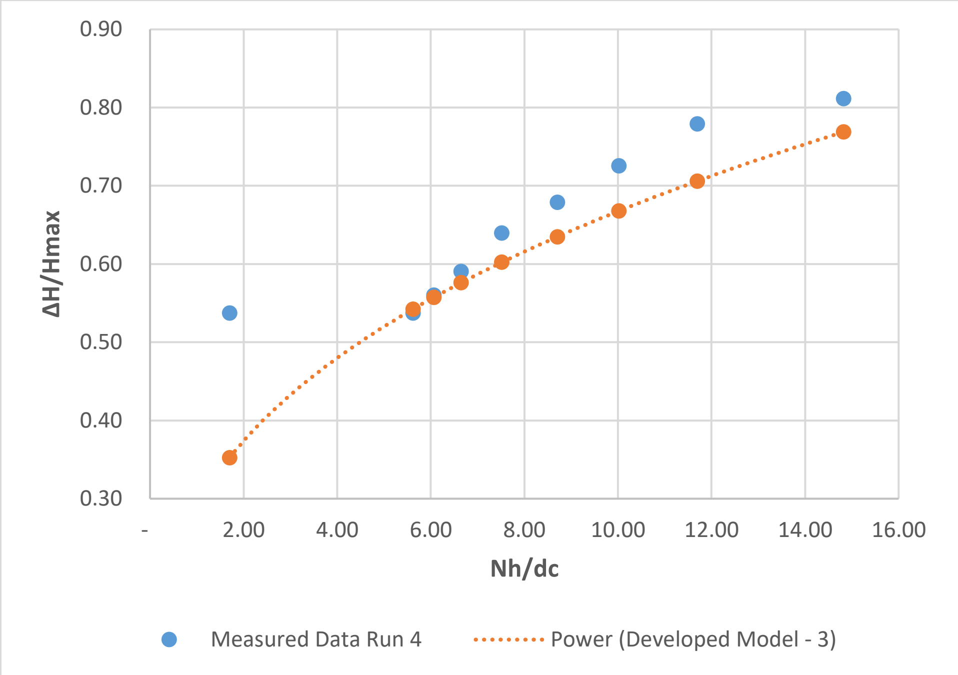

Figure 16: $\Delta H/H_{max}$ as a function of Nh/d_c between 2.00 and 20.00,
The coefficient of correlation is 0.88.

Figures 6 to 16 depict the energy loss rates as a function of the expression of a dam height divided by the critical depth for the measured data, the developed analytical formulation (9) to (11)).
The results from the developed models, Eq (9) to Eq (11), compare well with the measured data sets (Run 1 to Run 4) in terms of energy dissipation, with the coefficients of correlation that range between 0.95 and 1.0. The models are simple to use.
From Figure 6 to Figure 16, energy losses for a given discharge rise progressively with an increasing dam height, which is consistent with [27].

4. CONCLUSION

The figures depict the energy loss rates as a function of the expression of a dam height divided by the critical depth for the measured data as well as the developed analytical formulation. Both the measured dstasets and the estimated datasets distribution also show same traditional concave shape, which is consistent with [27]. For all the measured data sets, the results from the three number developed models, in terms of the dimensionless energy losses rates distribution compare well with the measured datasets for all the flow regimes with the coefficients correlations that range from 0.87 to 1.00. All the measured datasets and estimated datasets are in good agreement. The models are simple and straightforward to use, and they produce accurate results.

DISCLAIMER (ARTIFICIAL
Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

REFERENCE

Beitz E, Lawless M. (1992). Hydraulic Model Study for darn on GHFL 3791 lsaac River al Burton Gorge. Water Resources Commission Report, Ref. No. REP/24.1, Sept., Brisbane, Australia.
Bindo M, Gautier J, Lacrotx F, (1993). The Stepped Spillway of :Vl"Bah Darn. lntl Water Power and Dam Construction. 1-5(1):35-36.
Boes R, Hager W (2003a). Two-phase flow characteristics of stepped spillways. J. Hydraul. Eng. ASCE. 129(9): 661-670.
Boes RM. Scale effects in modeling two-phase stepped spillway flow. In Proc. Intl. Workshop on Hydraulics of Stepped Spillways, Minor HE and Hager WH, eds. Steenwijk, The Netherlands: A. A. Balkema. 2000:53-60.
Bung D, Schlenkhoff A. Prediction of oxygen transfer in self-aerated skimming flow on embankment stepped spillways, 33rd IAHR World Congress. Vancouver, Canada 123 Author's personal copy Environ Fluid Mech; 2009.
Carosi G, Chanson H.Air-water time and length scales in skimming flow on a stepped spillway. Application to the spray characterisation. Report No. CH59/06, Division of Civil Engineering, the University of Queensland, Brisbane, Australia, July; 2006.
Chamani R, Rajaratnam N. Jet Flow on Stepped Spillways. J. Hydraul. Engg. ASCE 1994;120(2):254–259.
Chanson H, Carosi G. Advanced post-processing and correlation analyses in high-velocity air–water flows. Environ Fluid Mech. 2007;7(6):495–508.
Chanson H, Toombes L. Air–water flows down stepped chutes: turbulence and flow structure observations. Int J Multiph Flow. 2002a;28(11):1737–1761.
Chanson H, Toombes L. Energy dissipation and air entrainment in stepped storm waterway: experimental study. J Irrig Drain Eng ASCE. 2002b; 128(5):305–315.
Chanson H, Toombes L. Supercritical Flow at an Abrupt Drop: Flow Patterns and Aeration". Can. J. Civil Eng. 1998;25(5):956–966.
Chanson H. The hydraulics of stepped chutes and spillways. Balkema, Lisse. 2001:418
Chanson H. Air bubble entrainment in open channels. Flow structure and bubble size distributions. Int J Multiph Flow. 1997a; 23(1):193–203
Chanson H. Air–water flow measurements with intrusive phase-detection probes. Can we improve their interpretation? J Hydraul Eng ASCE. 2002;128(3):252–255,
Chanson H. Measuring air–water interface area in supercritical open channel flow. Water Res. 1997b;31(6):1414–1420
Chanson, H. 1994a. Hydraulics of skimming flows over stepped channels and spillways. Journal of Hydraulic Research, 32(3): 445-460.
Chanson, H. 1994b. Comparison of energy dissipation between nappe and skimming flow regimes on stepped chutes. Journal of Hydraulic Research, 32(2): 213-218
Degouite G, Peyras L, Royet P. Skimming Flow in Stepped Spillways - Discussion. Jl of Hyd. Engrg., ASCE. 1992;118(1): 111-114.
Essery ITS, Horner MW. The hydraulic design of stepped spillways, Rep. 33, Construction Industry Research and Information Assoc., London, U.K; 1971.
Felder S, Chanson H. Air–water flow properties in step cavity down a stepped chute. Int J Multiph Flow. 2011;37(7): 732–745
Felder S, Chanson H. Energy dissipation and flow resistance on flat slope stepped spillways. 5th IAHR International Symposium on Hydraulic Structures, Brisbane, Australia, 25-27 June 2014. Brisbane, Australia: The University of Queensland; 2014
Felder S, Chanson H. Energy dissipation, flow resistance and gas-liquid interfacial area in skimming flows on moderate-slope stepped spillways. Environ Fluid Mech. 2009a;9(4):427–441.

Felder S, Chanson H. Turbulence, dynamic similarity and scale effects in high-velocity free surface flows above a stepped chute. Exp Fluids. 2009b;47(1): 1–18

Felder S. Air–water flow properties on stepped spillways for embankment dams: aeration, energy dissipation and turbulence on uniform, non-uniform and pooled stepped chutes. PhD Thesis, The University of Queensland, Australia; 2013.

Gonzalez CA. An experimental study of free-surface aeration on embankment stepped chutes. PhD Thesis, Department of Civil Engineering, The University of Queensland, Brisbane, Australia; 2005.

Guenther P, Felder S, Chanson H. Flow aeration, cavity processes and energy dissipation on flat and pooled stepped spillways for embankments. Environ Fluid Mech. 2013;13(5):503–525.

Ohtsu I, Yasuda Y, Takahashi M (2004) Flow characteristics of skimming flows in stepped channels. J Hydraulic Eng ASCE 130(9):860–869

Ozueigbo, O, Agunwamba J, New Equations for Energy Dissipation down a Stepped Spillway. Journal of Engineering Research and Reports 23(4): 1-14, 2022; Article no.JERR.90654 ISSN: 2582-2926, DOI: 10.9734/JERR/2022/v23i417602.

Ozueigbo, O. (2021). Analysis of flow and the energy dissipation down a stepped spillway. Ph.D. thesis, University of Nigeria, Nsukka.

Ozueigbo O and Agunwamba J. (2023). New Equations for Rate of Energy Dissipation of a Stepped Spillway with Slope less than Critical and Specific Step Height September 2023.Journal of Engineering Research and Technology VOLUME 10(2):49 – 57 DOI: 10.33976/JERT.10.2/2023/2

Ozueigbo, O. and Agunwamba J. (2023). New Equations for Rate of Energy Dissipation of a Stepped Spillway w https://journals.iugaza.edu.ps/index.php/JERT/issue/view/352ith Slope less than Critical and Specific Step Height https://doi.org/10.33976/JERT.10.2/2023/2

Peyras L, Royet P, Degoutte G. Flow and Energy Dissipation over Stepped Gabion Weirs." Jl of Hyd. Engrg. ASCE. 1992;118(5):707-717.

Rajaratnam N. Skimming flow in stepped spillways. J. Hydr. Engrg., ASCE. 1990;116(4):587-591

Sorensen, RM. 1985. Stepped spillway hydraulic model investigation. Journal of Hydraulic Engineering, 111(12): 1461-1472.

Stefan Felder, Hubert Chanson (2014). Aeration and air–water mass transfer on stepped chutes with embankment dam slopes, Environ Fluid Mech; 2014. DOI 10.1007/s10652-014-9376-x

Stephenson, D. (1991). "Energy Dissipation down Stepped Spillways." International Water Power & Dam Construction, Sept., pp. 27-30.

Takahashi, M., Yasuda, Y., and Ohtsu, I. (2008). Flow Patterns and Energy Dissipation over Various Stepped Chutes. Discussion. Jl of Irrigation and Drainage Engineering, ASCE, Vol. 134, No. 1, pp. 114-116.