Original Research Article

Aircraft Pitch Control using Transformation Matrix T

ABSTRACT

This research aims to design a controller that is used on aircraft pitches using the Ackerman formula. The Ackerman formula makes it possible to obtain linear state feedback control in an explicit form with the desired eigenvalue.Pitch control is very important in take-off and stabilization when flying down. This critical criterion exists because the pitch can cause the aircraft in turbulence if the aircraft's pitch control is not calibrated correctly.Aircraft are considered rigid objects, and their movements consist of small interference originating from external conditions. There are two equations of an aircraft namely a lateral equation and a longitudinal equation. Matrix K feedback method is used to regulate the pitch angle. Transformation matrix T is one of the methods that can be used to find the feedback gain matrix K.The controller is used to find out the response value by several experiments with fixed setpoints, changed setpoints, and by adding load to the system. based on the results of simulations that have been done with a fixed setpoint, the system response has a value τ = 0.47, and td = 0.39. From several simulation that have been done show the system response can follow the setpoint well without passing the setpoint that has been determined.

Keywords: Aircraft Pitch, Overshooting Control, Ackerman Formula, Transformation matrix T

1. INTRODUCTION

For ensuring the safe operation of modern aviation, aircraft control systems are critical. One of the critical parameters that need accurate control is the aircraft's pitch. The pitch control influences the aircraft's ability and keep a level flight. Conventional pitch control technique basically depends on classical control theories like Proportional-Integral-Derivative (PID) controller. However, this technique may sometimes fall short when applied in complex aerodynamic, especially in modern aviation.

Along with the development of increasingly modern times, technology embedded in aircraft is also increasingly sophisticated [1]. Basically, an aircraft is regulated by elevator, rudder, and aileron. The elevator serves to regulate the pitch motion of the aircraft. [2]. Basic control of the aircraft has 3 basic movements in determining the longitudinal stability of the plane including pitch and to establish lateral stability there roll, and yaw. [3]. Pitch control is very important in take-off and stabilization when flying down. This critical criterion exists because the pitch can cause the aircraft in turbulence if the aircraft's pitch control is not calibrated correctly [4].

Overshooting issues are very important in some practical applications such as in the production process, which can result in damage to the product, another example is in aircraft [5]. The control system on the aircraft must have high precision control, which overshoot can risk the aircraft. In the past few years, there have been many studies on the controller methods to reach an overshoot response to linear systems [6 - 9].

Digital control theory usually assumes the same sampling interval [10]. This research aims to design a controller that is used on aircraft pitches using the Ackerman formula. The Ackerman formula makes it possible to obtain linear state feedback control in an explicit form with the desired eigenvalue [11]. The Ackerman formula used is determine of matrix K using the matrix transformation method, it was chosen because the method used is quite simple and can produce the desired response.

This article consists of 5 parts. Part I contains the background of this research, the description of the problems to be studied, and the purpose of this research. Part II contains a mathematical model of the plant, which is a mathematical model of the aircraft pitch. Part III explains the control method proposed in the study. Part IV contains the implementation of the control method in the plant and its results. Part V contains conclusions from experiments that have been carried out.

2. MODELLING OF AIRCRAFT PITCH CONTROL

This section presents the mathematical model of the aircraft's pitch control system as a basis for simulations for the development of proposed control techniques. Aircraft are considered rigid objects, and their movements consist of small interference originating from external conditions [12]. The are two equations of an aircraft namely a lateral equation and a longitudinal equation. Figure 1 is the control system of the plane's pitch where Xb, Yb, Zb are the aerodynamic forces section and θ, Φ, δe are the pitch, roll and deflection angle of the elevator respectively [12 – 13].

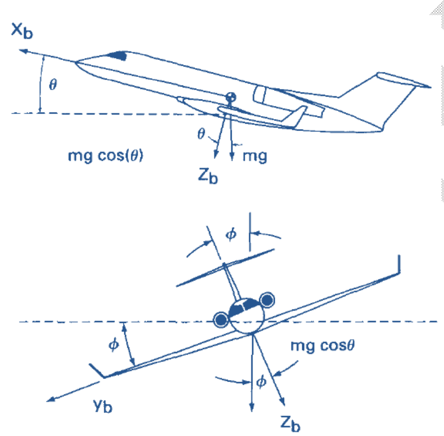

Fig. 1 Aircraft Pitch Control.

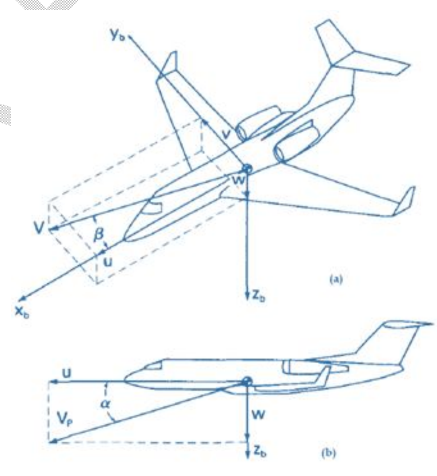

Fig. 2 The components in body coordinate.

.Several conditions must be considered for modelling. First, aircraft conditions must be stable at a constant speed and altitude, so that the thrust and resistance and weight balance cannot be considered. Second, there is no impact on the modification in pitch for aircraft speed. The dynamic equations of an aircraft are presented in (1), (2), and (3).

$$X - mgS_\theta = m(\dot{u} + qv - rv) \quad (1)$$

$$Z + mgC_\theta C_\Phi = m(\dot{W} + pv - qu) \quad (2)$$

$$M = Iy\dot{q} + rq(Ix + Iz) + Ixz(p^2 - r^2) \quad (3)$$

The longitudinal parameters utilized in this paper are presented in Table I [14].

TABLE I. LONGITUDINAL PARAMETERS

Longitudinal Derivatives	Devices $Q = 36.8lb/ft2$; $QS = 6771lb$; $QS\bar{c} = 38596ft.lb;,\bar{c}/2u0 = 0.016s$		
	X (S^{-1})	Z (F^{-1})	Pitching Moment (FT^{-1})
Rolling Velocities	$X_u = -0.045$	$Z_u = -0.369$	$M_u = 0$
Yawing Velocities	$X_w = 0.03$	$Z_w = -2.02$ $Z_{\dot{w}} = 0$	$M_w = 0.05$ $M_{\dot{w}} = 0.051$
Angle of attack	$X_\alpha = 0$ $X_{\dot{\alpha}} = 0$	$Z_\alpha = -355.42$ $Z_{\dot{\alpha}} = 0$	$M_\alpha = -8.8$ $M_{\dot{\alpha}} = -0.8976$
Pitching rate	$X_q = 0$	$Z_q = 0$	$M_q = -2.05$
Elevator Deflection	$X\delta_e = 0$	$X\delta_e = -28.15$	$X\delta_e = -28.15$

Small disturbance can be applied to linearization in equations (1), (2), and (3). The formula are substituted by variables as follows.

$$u = u_0 + \Delta u;\ v = v_0 + \Delta v;\ w = w_0 + \Delta w$$
$$p = p_0 + \Delta p;\ q = q_0 + \Delta q;\ r = r_0 + \Delta r;$$
$$\delta = \delta_0 + \Delta\delta \quad (4)$$

This indicatesthat the expression $v_0 = p_0 = q_0 = r_0 = w_0 = 0$. Afterwards, using linearization the (5), (6) and (7) are as follows

$$\left(\frac{d}{dt} - Xu\right)\Delta u - X_w\Delta w + (gcos\theta_0)\Delta\theta = X\delta_e\Delta\delta_e \quad (5)$$
$$-Z_u\Delta u + \left[(1 - Z_w)\frac{d}{dt} - Z_w\right]\Delta w - \left[(u0 + Z_q)\frac{d}{dt} - gsin\theta_0\right]\Delta\theta = Z_{\delta e}\Delta\delta_e \quad (6)$$
$$-M_u\Delta u - \left(M_w\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta\theta = M_{\delta e}\Delta\delta_e \quad (7)$$

By manipulating and substituting longitudinal stability parameter values derived from table I, the following transfer function for the transformation is presented in (8).

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{-\left(M_{\delta e}+\frac{M_{\dot{\alpha}}Z_{\delta e}}{U_0}\right)s-\left(\frac{M_\alpha Z_{\delta e}}{u_0}-\frac{M_{\delta e}Z_\alpha}{u_0}\right)}{s^2-\left(M_q+M_{\dot{\alpha}}+\frac{Z_\alpha}{u_0}\right)+\left(\frac{Z_\alpha M_q}{u_0}-M_\alpha\right)} \quad (8)$$

The expression of pitch angle deviation to elevator angle deviation is obtained through the pitch rates change to elevator angle change.

$$\Delta q = \Delta\dot{\theta} \quad (9)$$
$$\Delta q(s) = s\Delta\theta(s) \quad (10)$$
$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s} * \frac{\Delta q(s)}{\Delta\theta(s)} \quad (11)$$

Therefore, the expression of the aircraft pitch control system is shown as follows.

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s} * \frac{-\left(M_{\delta e}+\frac{M_{\dot{\alpha}}Z_{\delta e}}{u_0}\right)s-\left(\frac{M_\alpha Z_{\delta e}}{u_0}-\frac{M_{\delta e}Z_\alpha}{u0}\right)}{s^2-\left(M_q+M_{\dot{\alpha}}+\frac{Z_\alpha}{u_0}\right)+\left(\frac{Z_\alpha M_q}{u_0}-M\alpha\right)} \quad (12)$$
$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{11.7304s+22.578}{s^3 +4.9676s2+12.941s} \quad (13)$$

The expression ischanged to the state space form as follows.

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix}-2.02 & 1 & 0\\ -6.9868 & -2.9476 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix}0.16\\ 11.7304\\ 0\end{bmatrix}[\Delta\delta_e] \quad (14)$$

$$y = [0 \quad 0 \quad 1]\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + [0] \quad (15)$$

3. TRANSFORMATION MATRIX T

In this paper, matrix K feedback method is used to regulate the pitch angle. Transformation matrix T is one of the methods that can be used to find the feedback gain matrix K. This method is selected because it can provide good results on a system. In general, finding the feedback matrix K using transformation matrix T is done in five steps. The first step is checking the system controllability. To check controllability, we can find the system rank. The matrix T is needed in case the system is completely uncontrollable. The transformation of matrix T is expressed as follows.

$$\boldsymbol{T} = [\boldsymbol{B} \ \vdots \ \boldsymbol{AB} \ \vdots \cdots \vdots \boldsymbol{A}^{n-1}\boldsymbol{B}]\begin{bmatrix}a_{n-1} & a_{n-2} & \cdots & a_1 & 1\\ a_{n-2} & a_{n-2} & \cdots & 1 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ a_1 & 1 & \cdots & 0 & 0\\ 1 & 0 & \cdots & 0 & 0\end{bmatrix} \quad (16)$$

The second step is determining the values of a(1,) a_2 ,⋯,an by using the characteristic polynomial.

$$|s\boldsymbol{I} - \boldsymbol{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n \quad (17)$$

Then, the third step is utilizing the desired eigenvalue and obtain the value of ∝1,∝_2),⋯,∝_n.

(s - μ1)(s - μ_2)…(s - μ_n)=s^n+ ∝_1 s^(n-1)+⋯+∝_(n-1) s+ ∝_n (18)

The last step is determining feedback gain matrix K using equation that expressed.

$$\boldsymbol{K} = [\propto_n - a_n \vdots \propto_{n-1} - a_{n-1} \vdots \cdots \vdots \propto_2 - a_2 \vdots \propto_1 - a_1] \quad (19)$$

4.RESULT AND DISCUSSION

To get the performance of the proposed method, the MATLAB Simulink is employed. The open loop system of the aircraft pitch is presented in Fig 3.

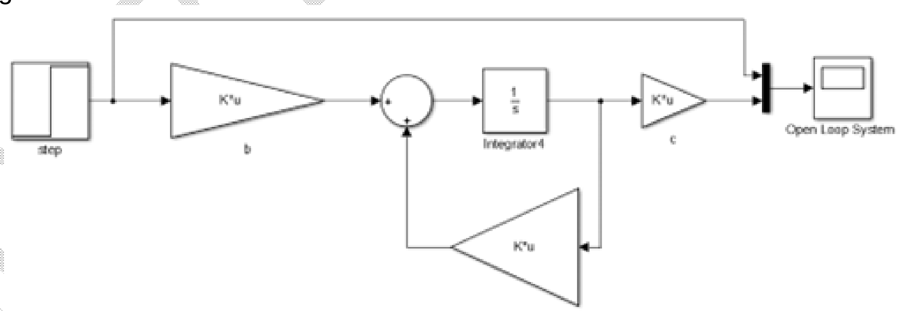

Fig. 3 open-loop block diagram in MATLAB

The response of diagram block in Fig 3 is presented in Fig. 4. The open loop response cannot follow the setpoint. the value of the response will continue to rise with time. with results like this, the controller is needed with the aim of the response of the system to follow the setpoint because the aircraft pitch requires high precision.

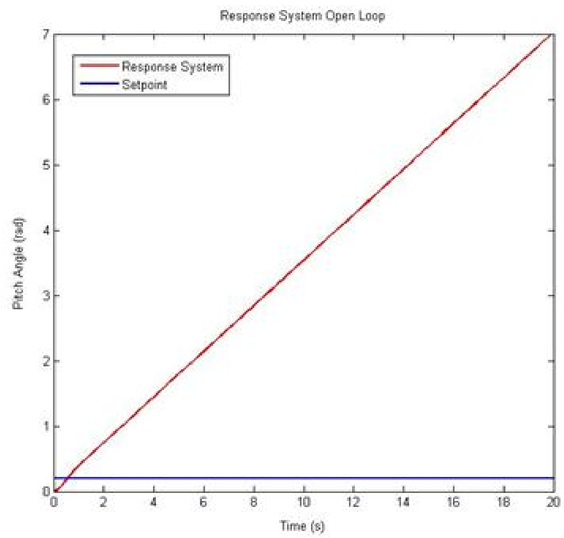

Fig. 4 response system without controller

Controller matrix K is obtained by means of transformation T matrix using equations (16)-(19). The parameters needed to use the matrix T transformation method must have the input, output and pole of the aircraft pitch system. The input and output of the aircraft are shown in equations (14) and (15), while the missing parameter is the pole. In this article the determination of the pole value of the system is obtained from the formula in MATLAB by entering the input and the output of the plant. After all the parameters exist, the next step is to determine the K matrix using equations (16) – (19) with the required parameters. From the calculation results, the K matrix value is shown in equation (20).

$$K = [0 \quad 0 \quad 0.87] \tag{20}$$

After obtaining the controller formula, the proposed controller is applied to control aircraft pitch. The proposed controller is examined using three conditions namely fixed reference, the variation of the reference and a load change. the reference employed in this paper is 0.2 rad. The result of fixed reference is depicted in Fig 5.

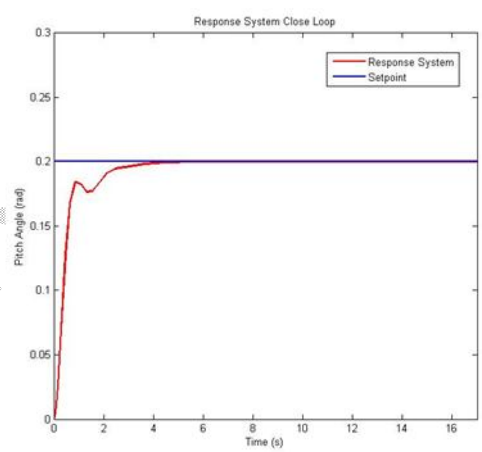

Fig. 5 System waveform of the pitch angle using Fixed reference

The proposed method is applied, and the performance of the system follows the reference. From the figure, the response of the system after using the proposed method, the time constant is 0.47 s. the detail performance of the system is presented in Table 2.

Table 2. Response System with fixed Setpoint

Reference (rad)	t_d	Rise Time		Settling time	
		t_r (5%-95%)	t_r (10%-90%)	t_s 5%	t_s 2%
0.2	0.39	2.003	1.54	2.09	2.93

The next examination is the variation of reference namely 0.2 rad, 0 rad and 0.2 rad. The performance of the proposed controller is depicted in Fig. 6. It can be seen that at t=8 s, the reference is decreased to 0 rad and the pitch of the system can follow the reference. Then, at t=16 s the pitch reference is back to 0.2 rad and again the pitch response can follow the reference.

The last test is the additional load with the aim to determine the durability of the system. Adding load is done by adding a load block to the main block diagram with the aim of disrupting the system response. The response of the pitch after adding the load are presented in Figure 7. The load is added to the system at t=13 s and t=18 s. As presented in the Figure, the proposed method can maintain the pitch although there are the changes in the load. The detail performance of the proposed method is presented in Table 3.

TABLE 3. THE PERFORMANCE OF THE SYSTEM AFTER ADDING THE LOAD

Reference (rad)	the pitch angle deviation (rad)	
	$t = 13\,s$	$t = 18\,s$
0.2	0.042	0.039

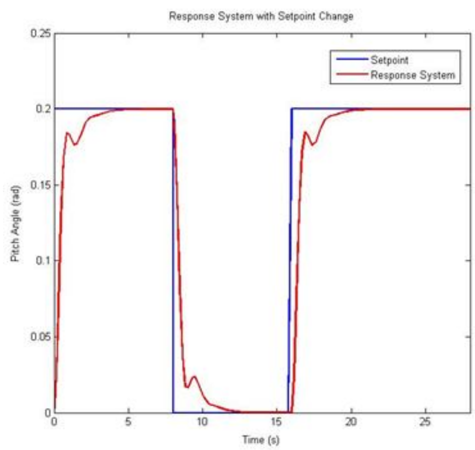

Fig. 6 System waveform of the pitch angle after changing the reference

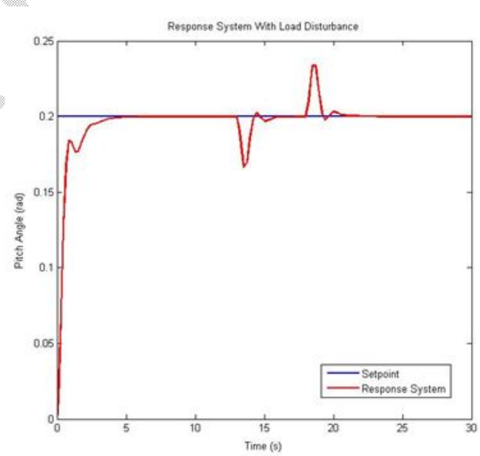

Fig. 7 System waveform of the pitch angle after adding the load disturbance

5. CONCLUSION

In this article we have learned how to search for aircraft pitch controllers using Formula Ackerman namely transformation matrix T, then it will be implemented in MATLAB software to determine the performance of the proposed method. The controller is used to find out the response value by several experiments with fixed setpoints, changed setpoints, and by adding load to the system. based on the results of simulations that have been done with a fixed setpoint, the system response has a value τ = 0.47, and td = 0.39. From several simulation that have been done show the system response can follow the setpoint well without passing the setpoint that has been determined.

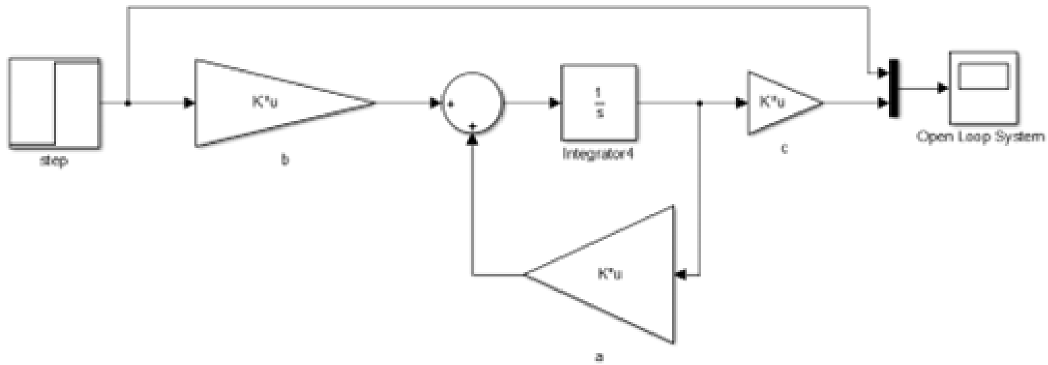

Fig 8-

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