Original Research Article

ON THE EFFICIENCY OF GENERALIZED VERSION EXPONENTIAL RATIO CUM TO DUAL-RATIO CUM TYPE ESTIMATOR VIA MEASURE OF LOCATION

Abstract

Ratio-cum-type to Dual-ratio of population mean is addressees under known influence of auxiliary variable. The expression for the bias, mse and mmse of the proposed estimator are study and optimality is tested. Theoretical efficiency comparison of the proposed estimator over some existing of the same characteristics reviews are established. Numerical illustration in support of theoretical results is however consider on two artificial data sets and simulated data sets using R package. Results indicate the superiority of proposed estimator over exiting estimators accept some few cases.

Keywords: Ratio, Product, Dual-ratio, Dual-Product, Ratio Cum, Product cum, Ratio to Dual-Ratio Cum, Product to Dual-Product Cum, Linear Combination, Optimum Values.

1 Introduction

The simple method of population mean is the sample mean obtained by using simple random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample surveys, along with the study variable Y , information on auxiliary variable X , correlated with Y , is also collected. This information on auxiliary variable X, may be utilized to obtain a more efficient estimator of the population mean. Ratio method of estimation is an attempt

in this direction. This method of estimation may be used when (i) X represents the same character as Y, but measured at some previous date when a complete count of the population was made and (ii) the character X is cheaply, quickly and easily available. Numerous study in field of sampling survey as regard to proposed an estimator for estimating population parameters such as population mean, population variance, population median, etc under the assumption that complete information about sampling units is available as stated before. some authors (Cochran[1]), (Robson[2]), (Srivenkataramana[3]), (Bandyopadhyay[4]), (Sisodia and Dwivedi[5]), (Pandey and Dubey[6]), (Bahl and Tuteja[7]), (Upadhyaya and Singh[8]), (Koyuncu and Kadilar[9]), (Sharma and Tailor[10]), (Abd-Elfattah et al.,[11]), (Yan and Tian[12]), (Yadav[13]), (Tailor et al.,[14]), (Singh et al.,[15]), (Lone and Tailor[16]), (Ahmed et al.,[17]), (Lone et al.,[18]), (Ahmed et al.,[19]), (Ikughur et al.,[20]), (Audu et al.,[21]) this study, therefore embraced concept of unknown waited in estimator formulation utilized linear combination.
But so many authors are also work extensively when the population is homogeneous nature that the study population can be taken into different accounts of strata and estimate the same as mention above such population mean, variance, proportion, median, correlation coefficient of variation (Vishwakarma et al.,[22]), (Tailor et al.,[23]), (Tailor et al.,[24]), (Singh and Singh[25]), (Tailor and Chouhan[26]), (Tailor et al.,[27]), (Shahzad et al.,[28]), (Shahzad et al.,[29]), (Audu et al.,[30]), (Javed and Irfan[31]), (Audu et al.,[32]), (Ahmad et al.,[33]), (Rather and Kadilar[34]), (Serdar et al.,[35]), (Siraj et al.,[36]), (Suleiman et al.,[37]) are some of current estimators proposed methods under stratified random sampling however, all the estimators proposed by aforementioned authors are functions of population mean of auxiliary variable $\bar{X}$ and since $\bar{X}$ is known the proposed estimator can be applied to real life situations.

Consider a finite population $U_i = (X_i, Y_i), i = 1, 2, ..., N$ has a pair of values. N and n: Population and sample size of study and auxiliary variables, while $y = (y_n, x_n)$ are the n sample values. $\bar{Y}$ and $\bar{X}$ are the population mean study and auxiliary variables. respectively. Let S_Y^2 and S_X^2 be the population variances of Y and X respectively. $\bar{y}$ and $\bar{x}$ are the sample mean of the study and auxiliary variable, s_{y^2} and s_x^2 be respective sample variance base on the random sample of size n drawn without replacement.

2 Estimators in Review

Usual ratio and product estimators proposed by (Cochran[1]) and (Robson[2]) are respectively given in equations (2.1) and (2.2) below:

$$\hat{\bar{Y}}_{11} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{2.1}$$

$$\hat{\bar{Y}}_{12} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \tag{2.2}$$

Biases and Mean squared error of the estimators (2.1) and (2.2) are given up first order as (2.3), (2.4),(2.5) and (2.6) are accordingly defined

$$Bias\left(\hat{\bar{Y}}_{11}\right) = \frac{(1-f)}{n}\bar{Y}C_x^2(1-\rho_{yx}C_y/C_x) \tag{2.3}$$

$$Bias\left(\hat{\bar{Y}}_{12}\right) = \frac{(1-f)}{n}\bar{Y}\rho_{yx}C_xC_y \tag{2.4}$$

$$MSE(\hat{\bar{Y}}_{11}) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_x^2(1-2\rho_{yx}C_y/C_x)\right) \tag{2.5}$$

$$MSE(\hat{\bar{Y}}_{12}) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_x^2(1+2\rho_{yx}C_y/C_x)\right) \tag{2.6}$$

Srivenkataramana[3] and Bandyopadhyay[4] envisaged dual-ratio and dual-product estimators each, the Bias and mean square error reported as

$$\hat{\bar{Y}}_{13} = \bar{y}\left(\frac{\bar{x}'}{\bar{X}}\right) \tag{2.7}$$

$$\hat{\bar{Y}}_{14} = \bar{y}\left(\frac{\bar{X}}{\bar{x}'}\right) \tag{2.8}$$

The Biases of the dual to ratio and product estimators in (2.7) and (2.8) together with their MSEs under simple random sampling are given by

$$Bias\left(\hat{\hat{\Upsilon}}_{13}\right) = -\frac{(1-f)}{n}\bar{Y}dC_x^2(d-\rho_{yx}C_x/C_y) \tag{2.9}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{14}\right) = -\frac{(1-f)}{n}\bar{Y}d\rho_{yx}C_xC_y \tag{2.10}$$

$$MSE(\hat{\hat{\Upsilon}}_{13}) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + dC_x^2(d-2\rho_{yx}C_y/C_x)\right) \tag{2.11}$$

$$MSE(\hat{\hat{\Upsilon}}_{14}) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + dC_x^2(d+2\rho_{yx}C_y/C_x)\right) \tag{2.12}$$

Exponential ratio cum, product cum and dual-ratio cum, dual-product cum estimators suggested by (Bahl and Tuteja[7]) and (Sharma and Tailor[10]) are define respectively as.

$$\hat{\hat{\Upsilon}}_{15} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{2.13}$$

$$\hat{\hat{\Upsilon}}_{16} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right) \tag{2.14}$$

$$\hat{\hat{\Upsilon}}_{17} = \bar{y}\exp\left(\frac{\bar{x}^t-\bar{X}}{\bar{x}^t+\bar{X}}\right) \tag{2.15}$$

$$\hat{\hat{\Upsilon}}_{18} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}^t}{\bar{X}+\bar{x}^t}\right) \tag{2.16}$$

The biases of the ratio cum, product cum exponential estimators of the (2.13) and (2.14) proposed by (Bahl and Tuteja[7]), while the dual-ratio cum, dual-product cum estimators of (2.15) and (2.16) proposed by (Sharma and Tailor[10]) in simple random sampling are given as

$$Bias\left(\hat{\hat{\Upsilon}}_{15}\right) = \frac{(1-f)}{8n}\bar{Y}C_x^2(3-4\rho_{yx}C_y/C_x) \tag{2.17}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{16}\right) = -\frac{(1-f)}{8n}\bar{Y}C_x^2(1-4\rho_{yx}C_y/C_x) \tag{2.18}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{17}\right) = \frac{(1-f)}{8n}\bar{Y}C_x^2(3d-4\rho_{yx}C_y/C_x) \tag{2.19}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{18}\right) = \frac{(1-f)}{8n}\bar{Y}C_x^2(d-4\rho_{yx}C_y/C_x) \tag{2.20}$$

The MSE of the estimators $\hat{\hat{\Upsilon}}_{15}$, $\hat{\hat{\Upsilon}}_{16}$, $\hat{\hat{\Upsilon}}_{17}$, and $\hat{\hat{\Upsilon}}_{18}$ under simple random sampling are given by

$$MSE(\hat{\hat{\Upsilon}}_{15}) = \frac{(1-f)}{n}\left(C_y^2 + C_x^2\left(\frac{1}{4} - \rho_{yx}C_y/C_x\right)\right) \tag{2.21}$$

$$MSE(\hat{\hat{\Upsilon}}_{16}) = \frac{(1-f)}{n}\left(C_y^2 + C_x^2\left(\frac{1}{4} + \rho_{yx}C_y/C_x\right)\right) \tag{2.22}$$

$$MSE(\hat{\hat{\Upsilon}}_{17}) = \frac{(1-f)}{n}\left(C_y^2 + dC_x^2\left(\frac{d}{4} - \rho_{yx}C_y/C_x\right)\right) \tag{2.23}$$

$$MSE(\hat{\hat{\Upsilon}}_{18}) = \frac{(1-f)}{n}\left(C_y^2 + dC_x^2\left(\frac{d}{4} + \rho_{yx}C_y/C_x\right)\right) \tag{2.24}$$

Hassen et al.,[38] suggested new Exponential ratio type later again (Hassen et al.,[39]) proposed Exponential product estimators to (Bahl and Tuteja[7]) classical ratio, product cum by introduced unknown scale parameter λ_a, λ_b, δ_1 and δ_2 to sample and population mean of auxiliary variable X such that the proposed estimator $\hat{\hat{\Upsilon}}_{15}$ and $\hat{\hat{\Upsilon}}_{16}$ estimate population mean precisely.

$$\hat{\hat{t}}_{19} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\lambda_a \bar{x}}\right) \tag{2.25}$$

$$\hat{\hat{t}}_{20} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\lambda_b \bar{X}}\right) \tag{2.26}$$

$$\hat{\hat{t}}_{21} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\delta_1 \bar{x}}\right) \tag{2.27}$$

$$\hat{\hat{t}}_{22} = \bar{y}\exp\left(\frac{\bar{x}-\bar{X}}{\delta_2 \bar{X}}\right) \tag{2.28}$$

The Bias and MSE of the proposed estimator (2.13) and (2.14) are obtained as equations below

$$Bias\left(\hat{\hat{\Upsilon}}_{19}\right) = \frac{(1-f)}{n\lambda_a} S_x^2 \left(R\left(1+\frac{1}{2\lambda_a}\right) - \rho_{yx}S_y/S_x\right)\bar{X}^{-1} \tag{2.29}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{20}\right) = -\frac{(1-f)}{n\lambda_b} S_x^2 (1/2\lambda_b - \rho_{yx}S_y/S_x) \tag{2.30}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{21}\right) = \frac{(1-f)}{n} S_x^2 (1/2\delta_1 - 1 + \rho_{yx}S_y/S_x) \tag{2.31}$$

$$Bias\left(\hat{\hat{\Upsilon}}_{22}\right) = -\frac{(1-f)}{n} S_x^2 (1/2\delta_1 + \rho_{yx}S_y/S_x) \tag{2.32}$$

$$MSE(\hat{\hat{\Upsilon}}_{19}) = \frac{(1-f)}{n}\left(S_y^2 + R\frac{1}{\lambda_a}S_x^2\left(R\frac{1}{\lambda_a} - 2\rho_{yx}S_y/S_x\right)\right) \quad \lambda_a = \lambda_b = RS_x/\rho_{yx}S_y \tag{2.33}$$

$$MSE(\hat{\hat{\Upsilon}}_{20}) = \frac{(1-f)}{n}\left(S_y^2 + R\frac{1}{\lambda_b}S_x^2\left(R\frac{1}{\lambda_b} - 2\rho_{yx}S_y/S_x\right)\right) \tag{2.34}$$

$$MSE(\hat{\hat{\Upsilon}}_{21}) = \frac{(1-f)}{n}\left(S_y^2 + R\frac{1}{\delta_1}S_x^2\left(R\frac{1}{\delta_1} + 2\rho_{yx}S_y/S_x\right)\right) \quad \delta_1 = \delta_2 = -RS_x/\rho_{yx}S_y \tag{2.35}$$

$$MSE(\hat{\hat{\Upsilon}}_{22}) = \frac{(1-f)}{n}\left(S_y^2 + R\frac{1}{\delta_2}S_x^2\left(R\frac{1}{\delta_2} + 2\rho_{yx}S_y/S_x\right)\right) \tag{2.36}$$

Sharma and Tailor[10] Introduced ratio to dual-ratio estimators of population mean motivated the work of (Cochran[1]) and (Srivenkataramana[3]) the Bias and mean square error of the proposed estimator is defined be as

$$\hat{\hat{\Upsilon}}_{23} = \bar{y}\left(\vartheta\frac{\bar{X}}{\bar{x}} + (1-\vartheta)\frac{\bar{x}^t}{\bar{X}}\right) \tag{2.37}$$

Where ϑ are constant parameter to minimized MSE of estimator, the equation of bias, mean square error, and minimum square error MMSE, up to first term approximation are given as below.

$$B(\hat{\hat{\Upsilon}}_{23}) = \frac{(1-f)}{n}\bar{Y}C_x^2\left(\vartheta_1 - \vartheta_1\rho_{yx}C_y/C_x\right) \tag{2.38}$$

$$MSE(\hat{\hat{\Upsilon}}_{23}) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \vartheta_1 C_x^2\left(\vartheta_1 - 2\rho_{yx}C_y/C_x\right)\right) \quad \Rightarrow \quad \vartheta_{min} = -\frac{\left(\rho_{yx}C_y/C_x + d\right)}{(d-1)} \tag{2.39}$$

Yadav[13] introduced ratio cum to dual-ratio cum estimator of finite population mean by taking linear combination of estimators (2.13) and (2.15) of (Bahl and Tuteja[7]) and (Sharma and Tailor[10]) respectively in simple random sampling

$$\hat{\hat{\Upsilon}}_{24} = \bar{y}\left(\varDelta\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] + (1-\varDelta)\,exp\left[\frac{\bar{x}^t-\bar{X}}{\bar{x}^t+\bar{X}}\right]\right) \tag{2.40}$$

The expressions of Bias, mean square errors (MSE) and MMSE up to the first term of approximation are as follow

$$Bias\left(\hat{\bar{Y}}_{24}\right) = \frac{(1-f)}{n}\bar{Y}\left(\Delta_2\frac{C_x^2}{8} + \frac{\Delta_1}{2}\rho_{yx}C_zC_y\right) \tag{2.41}$$

$$MSE\left(\hat{\bar{Y}}_{24}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \Delta_1C_x^2\left(\frac{\Delta_1}{4} - \rho_{yx}C_y/C_x\right)\right) \tag{2.42}$$

$$MSE_{min}\left(\hat{\bar{Y}}_{24}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(1-\rho_{yx}^2\right)C_y^2 \quad \Rightarrow \quad \Delta_{opt} = \frac{\left(2\rho_{yx}C_x/C_y + d\right)}{(1+d)} \tag{2.43}$$

Singh et al.,[15] Followed the study of (Bahl and Tuteja[7]) equation (2.14) and (Sharma and Tailor[10]) equation (2.16) exponential product cum to dual-product cum estimators of finite population mean is proposed in simple random sampling

$$\hat{\bar{Y}}_{25} = \bar{y}\left(\Gamma\exp\left[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right] + (1-\Gamma)\exp\left[\frac{\bar{X}-\bar{x}^t}{\bar{X}+\bar{x}^t}\right]\right) \tag{2.44}$$

The expressions of Bias and mean square errors (MSE) and minimum mean square error (MMSE) up to the first term of approximation are respectively as follow

$$Bias\left(\hat{\bar{Y}}_{25}\right) = \frac{1}{8}\frac{(1-f)}{n}\bar{Y}C_x^2\left(\Gamma_2 + 4\Gamma_1\rho_{yx}C_y/C_x\right) \tag{2.45}$$

$$MSE\left(\hat{\bar{Y}}_{25}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \frac{\Gamma_1}{4}C_x^2\left(\Gamma_1 + 4\rho_{yx}C_y/C_x\right)\right) \tag{2.46}$$

$$MSE_{min}\left(\hat{\bar{Y}}_{25}\right) = \frac{(1-f)}{n}\bar{Y}^2\left(1-\rho_{yx}^2\right)C_y^2 \quad \Rightarrow \quad \Gamma_{opt} = -\frac{\left(2\rho_{yx}C_y/C_x + d\right)num}{(1-d)} \tag{2.47}$$

Sisodia and Dwivedi[5] suggested a ratio estimator of population mean using the coefficient of variation of auxiliary variable, the estimator and weighted constant parameter are

$$\hat{\bar{Y}}_{26} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right), \; where \;\; \theta_i = \bar{X}/(\bar{X}+C_x) \tag{2.48}$$

Pandey and Dubey[6] proposed a ratio and product estimators of population mean using the coefficient of variation of auxiliary variable, the estimators and weighted constant parameters are defined as

$$\hat{\bar{Y}}_{27} = \bar{y}\left(\frac{\bar{X}\beta_1+\beta_1}{\bar{x}\beta_1+\beta_2}\right), \; where \;\; \theta_i = \beta_2\bar{X}/(\bar{X}\beta_1+\beta_2) \tag{2.49}$$

$$\hat{\bar{Y}}_{28} = \bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right), \; where \;\; \theta_i = \bar{X}/(\bar{X}+C_x) \tag{2.50}$$

Singh et al.,[42] envisaged ratio estimator using unknown waited constants of coefficient variation and skewness such as C_x and β_1

$$\hat{\bar{Y}}_{29} = \bar{y}\left(\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right) \; where \;\; \theta_i = \beta_2\bar{X}/(\bar{X}C_x+\beta_1) \tag{2.51}$$

Upadhyaya and Singh[8] Introduced coefficient of variation and kurtosis as the function of auxiliary variable to ratio and product estimators

$$\hat{\bar{Y}}_{30} = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right) \; where \;\; \theta_i = C_x\bar{X}/(\bar{X}C_x+\beta_2) \tag{2.52}$$

$$\hat{\bar{Y}}_{31} = \bar{y}\left(\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right) \; where \;\; \theta_i = \beta_2\bar{X}/(\bar{X}\beta_2+C_x) \tag{2.53}$$

$$\hat{\bar{Y}}_{32} = \bar{y}\left(\frac{\bar{x}C_x+\beta_2}{\bar{X}C_x+\beta_2}\right) \; where \;\; \theta_i = C_x\bar{X}/(\bar{X}C_x+\beta_2) \tag{2.54}$$

$$\hat{\bar{Y}}_{33} = \bar{y}\left(\frac{\bar{x}\beta_2 + C_x}{\bar{X}\beta_2 + C_x}\right) \;\; where \;\; \theta_i = \beta_2\bar{X}/\left(\bar{X}\beta_2 + C_x\right) \tag{2.55}$$

Yan and Tian[12] suggested a ratio estimator of population mean using the coefficient of variation, skeweness and kurtposis as the effect of auxiliary variable, the estimator and weighted constant parameter is

$$\hat{\bar{Y}}_{34} = \bar{y}\left(\frac{\bar{X}\beta_1 + \beta_2}{\bar{x}\beta_1 + \beta_2}\right) \;\; where \;\; \theta_i = \beta_1\bar{X}/\left(\bar{X}\beta_1 + \beta_2\right) \tag{2.56}$$

$$\hat{\bar{Y}}_{35} = \bar{y}\left(\frac{\bar{X}C_x + \beta_1}{\bar{x}C_x + \beta_1}\right) \;\; where \;\; \theta_i = C_x\bar{X}/\left(\bar{X}C_x + \beta_1\right) \tag{2.57}$$

Jeelani and Maqbool[41] suggested a ratio estimator of population mean using the skeweness and quantile deviation as the effect of auxiliary variable, the estimator and weighted constant parameter is

$$\hat{\bar{Y}}_{36} = \bar{y}\left(\frac{\bar{X}\beta_1 + QD}{\bar{x}\beta_1 + QD}\right) \;\; where \;\; \theta_i = \beta_1\bar{X}/\left(\bar{X}\beta_1 + \beta_2\right) \tag{2.58}$$

Yadav et al.,[43] suggested new ratio type estimators to population mean when sample size and correlation coefficient is effect of auxiliary variable the estimators define as

$$\hat{\bar{Y}}_{37} = \bar{y}\left(\frac{n\bar{X} + \rho}{n\bar{x} + \rho}\right) \;\; where \;\; \theta_i = n\bar{X}/\left(n\bar{X} + \rho\right) \tag{2.59}$$

$$\hat{\bar{Y}}_{38} = \bar{y}\left(\frac{n\bar{X} + C_x}{n\bar{x} + C_x}\right) \;\; where \;\; \theta_i = n\bar{X}/\left(n\bar{X} + C_x\right) \tag{2.60}$$

Jerajuddin and Kishun[40] suggested new ratio type estimator to population mean when sample size is effect of auxiliary variable the estimator define as

$$\hat{\bar{Y}}_{39} = \bar{y}\exp\left(\frac{\bar{X} + n}{n + \bar{x}}\right) \;\; where \;\; \theta_i = \bar{X}/\left(\bar{X} + n\right) \tag{2.61}$$

Zakari et al.,[45] suggested improve new ratio type estimator of population mean by introducing unknown parameter k to the estimator of [40] to be determined such that mse of the proposed estimator is minimized, the proposed estimator is define as

$$\hat{\bar{Y}}_{40} = \bar{y}k\exp\left(\frac{\bar{X} + n}{n + \bar{x}}\right) \;\; where \;\; \theta_i = \bar{X}/\left(\bar{X} + n\right) \tag{2.62}$$

The general term θ_i of the Bias and mean square error of ratios and products estimator listed above in equations (2.48), (2.49),..., (2.62) when the auxiliary variable is a function of Traditional Measure of dispersion as mention in this section 2 below up to fist order of approximation with i^{th} of θ are respectively given as

$$Bias\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\theta_i\bar{Y}\left(C_x^2 - \rho_{yx}C_yC_{xh}\right), \; \forall \; \theta_i \;\; where \;\; i = \begin{Bmatrix} (26), & (27), & (29), & (30), \\ (31), & (34), & (35), & (36), \\ (37), & (38), & (39), & (40), \end{Bmatrix} \tag{2.63}$$

$$Bias\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\theta_i\bar{Y}\left(C_x^2 + \rho_{yx}C_yC_{xh}\right), \quad \forall \quad \theta_i \quad where \quad i = \{(28), (32), (33)\} \tag{2.64}$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_iC_x^2\left(\theta_i - 2\rho_{yx}C_y/C_x\right)\right) \tag{2.65}$$

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_iC_x^2\left(\theta_i + 2\rho_{yx}C_y/C_x\right)\right) \tag{2.66}$$

The are several modifications of ratio product dual estimators of population mean but this study focus the estimator that satisfied the criteria for building the proposed estimator this criteria are one auxiliary variable, duality, measure of locations such as Gini Difference (G), Downtown (D), Probability Weighted Mass (PWM), coefficient of variation (C_x), Tri-mean (TM), Kurtosis (β_2), Skeweness (β_1), Decile Mean (DM), Mid-range (MR), Hodges-Lehmann (HL), Quantile Deviation QD, Mean Deviation (MD).

3 Proposed Estimator Under Study

Adopting the same formation as outlined in (Sharma and Tailor[10]) and recently from (Yadav[13]) the following exponential ratio-cum-type to ratio-dual estimator has been proposed to defined as:

$$\hat{\hat{\Upsilon}}^{*}_{pj} = \bar{y}\left(\delta_{\alpha}\exp\left(\frac{\left(A_{pj}\bar{X}+B_{pj}\right)-\left(A_{pj}\bar{x}+B_{pj}\right)}{\left(A_{pj}\bar{x}+B_{pj}\right)+\left(A_{pj}\bar{X}+B_{pj}\right)}\right)+\delta_{\beta}\exp\left(\frac{\left(A_{pj}\bar{x}^{t}+B_{pj}\right)-\left(A_{pj}\bar{X}+B_{pj}\right)}{\left(A_{pj}\bar{x}^{t}+B_{pj}\right)+\left(A_{pj}\bar{X}+B_{pj}\right)}\right)\right) \tag{3.1}$$

Where $\delta_{\alpha}+\delta_{\beta}=1$ and δ_{α}, δ_{β} are constant parameters to minimized the mean square error and bias while A_{pj} and B_{pj} represent known function of auxiliary variable such as gini and other as mention earlier in section 2. In order to study the large sample model based properties of the proposed estimators, we define sample means as

$$\left.\begin{array}{ll} \bar{x}^{t}=(1+d)\bar{X}-d\bar{x} & d=n(N-n)^{-1} \\ \bar{x}=\bar{X}(1+\eta_{1}) & \bar{y}=\bar{Y}(1+\eta_{0}) \\ E(\eta_{0})=E(\eta_{1})=0 & E(\eta_{0}^{2})=(1-f)n^{-1}C_{y}^{2} \\ E(\eta_{1}^{2})=(1-f)n^{-1}C_{x}^{2} & E(\eta_{0}\eta_{1})=(1-f)n^{-1}\rho_{yx}C_{y}C_{x} \end{array}\right\} \tag{3.2}$$

3.1 Properties(Bias and MSE) of Proposed Estimators

To obtain mean square error and bias. Using the error terms equation in (3.2) we write the proposed estimator as:

$$\hat{\hat{\Upsilon}}^{*}_{pj} = \bar{y}\left(\delta_{\alpha}\exp\left(\frac{-A_{pj}\bar{X}\eta_{1}}{2\left(A_{pj}\bar{x}+B_{pj}\right)+A_{pj}\bar{X}e_{1}}\right)+\delta_{\beta}\exp\left(\frac{-A_{pj}d\bar{X}\eta_{1}}{2\left(A_{pj}\bar{X}+B_{pj}\right)-A_{pj}d\bar{X}\eta_{1}}\right)\right) \tag{3.3}$$

From eq. 3.3 Let $\lambda=A_{pj}\bar{X}(A_{pj}\bar{X}+B_{pj})^{-1}$ and $\beta=A_{pj}d\bar{X}(A_{pj}\bar{X}+B_{pj})^{-1}$ $\forall$ $p=1,2$ and $j=0,1,2,3,4,5,6,7,8,9$ where
$A_{11}=A_{16}=A_{18}=A_{22}=A_{23}=A_{24}=1, A_{12}=A_{14}=G, A_{13}=A_{15}=A_{21}=PWM, B_{11}=B_{20}=B_{25}=C_{x}, A_{18}=TM, A_{17}=A_{20}=DM, A_{25}=B_{14}=B_{17}=B_{18}=B_{19}=B_{23}=\beta_{2}, B_{12}=B_{13}=B_{16}=D, B_{15}=B_{22}=B_{24}=\beta_{1}$.

$$\hat{\hat{\Upsilon}}^{*}_{pj} = \bar{y}\left(\delta_{\alpha}\exp\left(\frac{-\lambda\eta_{1}}{2+\lambda\eta_{1}}\right)+\delta_{\beta}\exp\left(\frac{-\beta\eta_{1}}{2-\beta\eta_{1}}\right)\right) \tag{3.4}$$

$$\hat{\hat{\Upsilon}}^{*}_{pj} = \bar{Y}\left(1+\eta_{0}\right)\left(1-\left(\beta+\left(\lambda-\beta\right)\delta_{\alpha}\right)\frac{\eta_{1}}{2}-\left(\beta^{2}-\left(3\lambda^{2}+\beta^{2}\right)\delta_{\beta}\right)\frac{\eta_{1}^{2}}{8}\right)$$

Denote again $\delta_{\alpha_{1}}=\left(\beta+\left(\lambda-\beta\right)\delta_{\alpha}\right)$ and $\delta_{\beta_{1}}=\left(\beta^{2}-\left(3\lambda^{2}+\beta^{2}\right)\delta_{\beta}\right)$ expand (3.5) and taking expectation of both side, after simplification the BIAS is obtain as,

$$\hat{\hat{\Upsilon}}^{*}_{pj}-\bar{Y}=\bar{Y}\left(\eta_{0}-\delta_{\alpha_{1}}\frac{\eta_{1}}{2}-\delta_{\beta_{1}}\frac{\eta_{1}^{2}}{8}-\delta_{\alpha_{1}}\frac{\eta_{0}\eta_{1}}{2}\right) \tag{3.5}$$

Taking expectation of above and substituting the values defined at (3.2), the bias of $\hat{\hat{\Upsilon}}^{*}_{pj}$ is obtain as

$$B(\hat{\hat{\Upsilon}}^{*}_{pj})=-\frac{(1-f)}{n}\bar{Y}\left(\delta_{\alpha_{1}}\frac{C_{x}^{2}}{8}+\delta_{\beta_{1}}\frac{\rho_{yx}C_{y}C_{x}}{2}\right) \tag{3.6}$$

In other to subject the mean square error of proposed estimator of 3.1. Take leading terms of equation (3.5) squaring both sides and taking expectation the MSE of $\hat{\hat{\Upsilon}}^{*}_{pj}$ is obtain:

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})=\frac{1-f}{n}\bar{Y}^{2}\left(C_{y}^{2}+\delta_{\alpha_{1}}C_{x}^{2}\frac{1}{4}\left(\delta_{\alpha_{1}}-4\rho_{yx}C_{y}C_{x}^{-1}\right)\right) \tag{3.7}$$

To obtain minimum mean square error of proposed we differentiate (3.7) w.r.t.δ_{α_1} and equate the result to zero therefore arrived as

$$f'(\delta_{\alpha_1}) = \frac{\delta_{\alpha_1} C_x^2}{2} - \rho_{yx} C_y C_x \qquad \Longrightarrow \qquad \delta_{\alpha_1} = 2\rho_{yx} C_y / C_x \tag{3.8}$$

After solving the above equating in terms of δ_α therefore attend the optimal value to minimize the mean square error

$$\delta_\alpha = \left(2\rho_{yx} C_y / C_x - \beta\right)(\lambda - \beta)^{-1} \tag{3.9}$$

Substitute the R.H.S. of (3.9) into equation (3.7) place of L.H.S. simplify we obtain the minimum mean square error as follow

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} = \frac{1-f}{n} \bar{Y} C_y^2 \left(1 - \rho_{yx}^2\right) \tag{3.10}$$

Substituting the value of α from equation 3.1 in equation yields the 'asymptotically optimum estimator' (AOE) as

$$\hat{\hat{\Upsilon}}^*_{pj} = \bar{y}\left(\left(\frac{2\rho_{yx} C_y / C_x - \beta}{\lambda - \beta}\right) exp\left(\frac{(A_{pj}\bar{X} + B_{pj}) - (A_{pj}\bar{x} + B_{pj})}{(A_{pj}\bar{x} + B_{pj}) + (A_{pj}\bar{X} + B_{pj})}\right) + \left(1 - \left(\frac{2\rho_{yx} C_y / C_x - \beta}{\lambda - \beta}\right)\right) exp\left(\frac{(A_{pj}\bar{x}^t + B_{pj}) - (A_{pj}\bar{X} + B_{pj})}{(A_{pj}\bar{x}^t + B_{pj}) + (A_{pj}\bar{X} + B_{pj})}\right)\right) \tag{3.11}$$

3.2 Theoretical Efficiency Compassion

The investigation of theoretical efficiency conditions of the proposed estimator $\hat{\hat{\Upsilon}}^*_{pj}$ against existing estimators $\bar{\Upsilon}_i$ review on the study were established in this section.
To test if **Proposed Estimator 3.1** is to be superior than **(Cochran[1])** and **(Robson[2])** ratio, product estimators, respectively require

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{11}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + 2)/4C_y C_x^{-1} \tag{3.12}$$

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{12}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - 2)/C_y C_x^{-1} \tag{3.13}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Srivenkataramana[3])** and **(Bandyopadhyay[4])** dual-ratio, dual-product estimator, respectively require

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{13}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + 2d)/4C_y C_x^{-1} \tag{3.14}$$

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{14}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - 2d)/4C_y C_x^{-1} \tag{3.15}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Bahl and Tuteja[7])** ratio cum and product cum estimators, respectively require

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{15}), \qquad \Longrightarrow \qquad \rho_{yx} > \left(\delta_{\alpha_1}^2 + 1\right)/4C_y C_x^{-1} \tag{3.16}$$

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{16}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - 1)/4C_y C_x^{-1} \tag{3.17}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Sharma and Tailor[10])** dual-ratio cum, dual-product cum estimators, respectively require

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{17}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + d)/4C_y C_x^{-1} \tag{3.18}$$

$$M(\hat{\hat{\Upsilon}}^*_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{18}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - d)/4C_y C_x^{-1} \tag{3.19}$$

To test if **Proposed Estimator 3.1** is to be superior than (**Hassen et al.,[?]**) ratio estimator, require. Where $\delta_1 = \delta_2$ and $\lambda_a = \lambda_b$

$$M(\hat{\hat{\Upsilon}}^*_{ij})_{min} > m(\hat{\hat{\Upsilon}}_{19}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - 2/\delta_1)/4C_y C_x^{-1} \tag{3.20}$$

$$M(\hat{\hat{\Upsilon}}^*_{ij})_{min} > m(\hat{\hat{\Upsilon}}_{19}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + 2\lambda_a)/4C_y C_x^{-1} \tag{3.21}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Sharma and Tailor[10])** ratio to dual-ratio estimator, require

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{21}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + 2\vartheta_1)\,/4C_yC_x^{-1} \tag{3.22}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Yadav[13])** ratio cum to dual-ratio cum estimator, require

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{22}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + \Delta_1)\,/4C_yC_x^{-1} \tag{3.23}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Singh et al.,[15])** product cum to dual-product cum estimator, require

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{23}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - \Gamma_1)\,/4C_yC_x^{-1} \tag{3.24}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Sisodia and Dwivedi[5])**, **(Pandey and Dubey[6])**, **(Singh et al.,[42])**, **(Upadhyaya and Singh[8])**, **(Yan and Tian[12])**, **(Jeelani and Maqbool[41])**, **(Yadav et al.,[43])**, **(Jerajuddin and Kishun[40])**, **(Zakari et al.,[45])** ratio proposed estimators of θ_i for i refer to equation (2.63)

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{i}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} + 2\theta_i)\,/4C_yC_x^{-1} \tag{3.25}$$

To test if **Proposed Estimator 3.1** is to be superior than **(Pandey and Dubey[6])** and **(Upadhyaya and Singh[8])** product proposed estimators of θ_j for i go to equation (2.64)

$$M(\hat{\hat{\Upsilon}}^{*}_{pj})_{min} > m(\hat{\hat{\Upsilon}}_{i}), \qquad \Longrightarrow \qquad \rho_{yx} > (\delta_{\alpha_1} - 2\theta_i)\,/4C_yC_x^{-1} \tag{3.26}$$

Established from the equations list as follow; (3.12), (3.13), (3.14), (3.15), (3.16), (3.17), (3.18), (3.19), (3.20), (3.21), (3.22), (3.23), (3.24), (3.25), (3.25) is an indication that for proposed estimator to be more those quantity most provided. To test the efficiency conditions, of the above, the sample statistics from sample observations can be used.

4 Numerical Efficiency Comparison

In this section, we study the performance of the proposed estimator over review estimator consider section (2) using two approaches, real and simulation, the detail and summary description are given in subsections (4.1) and (4.2).

4.1 Empirical Study under Populations Data

Population 1: Mir Subzar et al.,[46] source :([Page 228]Murthy [47]) $N = 80$, $n = 20$, $\bar{Y} = 5182.637$, $S_y = 1835.659$, $C_y = 0.354193$, $\bar{X} = 1126.463$, $S_x = 845.610$, $C_x = 0.7506772$, $\beta_1 = 1.050002$, $\beta_2 = -0.063386$, $\rho_{yx} = 0.941$, $MD = 757.5$, $TM = 931.562$, $MR = 1795.5$, $QD = 80.25$, $HL = 1040.5$, $DM = 588.325$, $G = 901.081$, $D = 801.38$, $pwm = 791.364$, $MD = 1150.7$:
Population 2: Yadav and Zaman[44] $N = 150$, $n = 40$, $\bar{Y} = 79.58$, $S_y = 62.1785$, $C_y = 0.781333$, $\bar{X} = 6.5833$, $S_x = 4.3564$, $C_x = 0.661726$, $\beta_1 = 1.4984$, $\beta_2 = 5.408$, $\rho_{yx} = 0.9363$, $MD = 6.22$, $TM = 6$, $MR = 11$, $QD = 3$, $HL = 7$, $DM = 3$, $G = 8.2298$, $D = 9.2542$, $pwm = 9.3707$:
Population 3: Mir Subzar et al.,[46] source :([Page 177]Singh and Chaudhary[48]) $N = 34$, $n = 20$, $\bar{Y} = 856.4117$, $\bar{X} = 199.4412$, $\rho_{yx} = 0.4455$, $S_y = 733.1407$, $S_x = 150.2150$, $C_y = 0.8561$, $C_x = 0.7531$, $\beta_2 = 1.0445$, $\beta_1 = 1.1823$, $MD = 142.5$, $TM = 165.562$, $MR = 320$, $QD = 89.375$, $HL = 184$, $DM = 89.375$, $G = 162.996$, $D = 144.481$, $pwm = 206.944$, $DM = 206.944$:

4.2 Empirical Study Using Simulation Data

In this section, simulation study is conducted to examine the superiority of the proposed estimator over other related estimators consider in the section two. For this purpose Data size of 1000 units were generated using function defined in table (3) study population were generated using simple linear regression with a_1 slope of 30 and a_2 intercept of 60. Sample size of 60 is selected 10,000 time by method (SRSWOR) the Bias, MSE and PRE of the proposed and considered estimator are computed using eqn (4.1), (4.2) and (4.3) the results is indicated in table (2).

$$Bias(\hat{\theta}_s) = \frac{1}{10000}\sum_{s=1}^{10000}\left(\bar{y} - \bar{\Upsilon}\right), \hat{\hat{\Upsilon}}_i, \forall \imath = \{11, 12, ..., 38\}, \hat{\hat{\Upsilon}}^*_{pj}, \forall\ p, j = \{(1,2), (0,1,2,3,4,5,6,7,8,9)\} \tag{4.1}$$

$$MSE(\hat{\theta}_s) = \frac{1}{10000}\sum_{s=1}^{10000}\left(\bar{y} - \bar{\Upsilon}\right)^2, \hat{\hat{\Upsilon}}_i, \forall \imath = \{11, 12, ..., 38\}, \hat{\hat{\Upsilon}}^*_{pj}, \forall\ p, j = \{(1,2), (0,1,2,3,4,5,6,7,8,9)\} \tag{4.2}$$

$$PRE(\hat{\theta}_s) = \left(\frac{VAR(\bar{y})}{MSE(\hat{\hat{\Upsilon}}^*_{pj})}\right) \times 100, \hat{\hat{\Upsilon}}_i, \forall \imath = \{11, 12, ..., 38\}, \hat{\hat{\Upsilon}}^*_{pj}, \forall\ p, j = \{(1,2), (0,1,2,3,4,5,6,7,8,9)\} \tag{4.3}$$

Table 1: Parameters and Distributions used to Simulate Populations

Populations	Auxiliary variables	Study variables
Distribution 1	$X \sim Gamma(1.2, 1.8)$	$Y = a_1 + a_2X + \varepsilon \ \ \forall\ \varepsilon \sim (0,5)$
Distribution 2	$X \sim Normal(3,5)$	$Y = a_1 + a_2X + \varepsilon \ \ \forall\ \varepsilon \sim (0,4)$
Distribution 3	$X \sim Uniform(0,1)$	$Y = a_1 + a_2X + \varepsilon \ \ \forall\ \varepsilon \sim (0,4)$

Table 2: Statistical Analysis of Three different Data based on Artificial Populations

S/No.	Estimators	Population One			Population Two			Population Three		
		Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
1	$\hat{\bar{\Upsilon}}_{10}$	0	126361	100	0	70.8797	100	0	11067.1	100
2	$\hat{\bar{\Upsilon}}_{11}$	60.8932	189939	66.527	-0.06742	9.30871	761.434	4.93805	10960.8	100.969
3	$\hat{\bar{\Upsilon}}_{12}$	48.6256	1197974	10.5479	0.70628	234.131	30.2736	5.06211	28301.8	39.1038
4	$\hat{\bar{\Upsilon}}_{13}$	-48.6256	21421.3	589.885	-0.70628	36.7256	192.998	-5.06211	16158.7	68.4898
5	$\hat{\bar{\Upsilon}}_{14}$	-0.0055	268698	47.0272	0.35521	125.868	56.3129	-13.9074	42625.7	25.9634
6	$\hat{\bar{\Upsilon}}_{15}$	-56.1817	16251.1	777.551	-1.17298	27.3842	258.834	-6.37416	8872.9	124.729
7	$\hat{\bar{\Upsilon}}_{16}$	83.5613	520269	24.2876	-1.3327	139.795	50.7025	8.8742	17543.4	63.084
8	$\hat{\bar{\Upsilon}}_{17}$	-101.815	58124.6	217.397	1.44159	52.122	135.988	-11.91	9243.39	119.73
9	$\hat{\bar{\Upsilon}}_{18}$	110.941	226130	55.8797	1.49967	92.9987	76.2158	15.4815	21629.8	51.1659
10	$\hat{\bar{\Upsilon}}_{19}$	-27.0362	462032	27.349	0.07454	257.291	27.5484	-2.49966	17650.6	62.7008
11	$\hat{\bar{\Upsilon}}_{20}$	-14.5404	14915.8	847.16	0.091	14.045	504.661	-1.25673	8873.03	124.727
12	$\hat{\bar{\Upsilon}}_{21}$	80.5601	410261	20.2876	-1.3017	129.795	37.7025	8.8701	17520.4	60.084
13	$\hat{\bar{\Upsilon}}_{22}$	-99.805	58112.2	215.337	1.41112	41.122	113.982	-10.91	9211.39	114.73
14	$\hat{\bar{\Upsilon}}_{23}$	22.3562	140920	89.6684	0.7685	46.0197	154.02	14.101	20873.7	53.0192
15	$\hat{\bar{\Upsilon}}_{24}$	60.7797	189519	66.6745	-0.11428	10.7129	661.627	4.882	10929.2	101.262
16	$\hat{\bar{\Upsilon}}_{25}$	60.7797	1196883	10.5575	-0.11428	215.001	32.9672	4.882	28204.9	39.2382
17	$\hat{\bar{\Upsilon}}_{26}$	158.159	189975	66.5146	0.58031	24.4889	289.436	14.932	10917	101.375
18	$\hat{\bar{\Upsilon}}_{27}$	60.9059	189987	66.5104	-0.18794	30.8473	229.776	4.83536	10902.8	101.507
19	$\hat{\bar{\Upsilon}}_{28}$	62.7162	196710	64.2373	-0.07764	9.52145	744.421	4.88437	10930.5	101.25
20	$\hat{\bar{\Upsilon}}_{29}$	158.1644	1198097	10.5468	0.44227	131.151	54.0442	14.8897	28124.1	39.3509
21	$\hat{\bar{\Upsilon}}_{30}$	161.001	1215455	10.3962	1.30914	230.241	30.785	14.9721	28209	39.2324
22	$\hat{\bar{\Upsilon}}_{31}$	60.9023	189973	66.5152	-0.18966	19.4833	363.797	4.87237	10923.7	101.313
23	$\hat{\bar{\Upsilon}}_{32}$	60.682	189157	66.802	-0.17182	15.3851	460.704	4.82199	10895.3	101.577
24	$\hat{\bar{\Upsilon}}_{33}$	12.05357	14470.8	70.553	-0.10934	56.0087	126.551	-0.561560	9960.995	111.104
25	$\hat{\bar{\Upsilon}}_{34}$	60.9059	189987	66.5104	-0.18794	30.8473	229.776	4.83536	10902.8	101.507
26	$\hat{\bar{\Upsilon}}_{35}$	62.7162	196710	64.2373	-0.07764	9.52145	744.421	4.88437	10930.5	101.25
27	$\hat{\bar{\Upsilon}}_{36}$	158.1644	1198097	10.5468	0.44227	131.151	54.0442	14.8897	28124.1	39.3509
28	$\hat{\bar{\Upsilon}}_{37}$	161.001	1215455	10.3962	1.30914	230.241	30.785	14.9721	28209	39.2324
29	$\hat{\bar{\Upsilon}}_{38}$	60.9023	189973	66.5152	-0.18966	19.4833	363.797	4.87237	10923.7	101.313
30	$\hat{\bar{\Upsilon}}_{39}$	60.682	189157	66.802	-0.17182	15.3851	460.704	4.82199	10895.3	101.577
31	$\hat{\bar{\Upsilon}}_{40}$	12.05357	14470.8	70.553	-0.10934	56.0087	126.551	-0.561560	9960.995	111.104
32	$\hat{\bar{\Upsilon}}^{*}_{11}$	1.0601	14470.7	873.218	-0.41764	8.74246	810.752	-5.39844	8872.57	124.734
33	$\hat{\bar{\Upsilon}}^{*}_{12}$	1.05801	14470.7	873.218	-0.43766	8.74246	810.752	-5.39959	8872.57	124.734
34	$\hat{\bar{\Upsilon}}^{*}_{13}$	1.05616	14470.7	873.218	-0.43197	8.74246	810.752	5.39796	8872.57	124.734
35	$\hat{\bar{\Upsilon}}^{*}_{14}$	-6.06e+28	14470.7	873.218	-398538	8.74246	810.752	5.15e+21	8872.57	124.734
36	$\hat{\bar{\Upsilon}}^{*}_{15}$	-0.61791	21426	589.754	-0.27253	15.8528	447.112	-2.92167	9006.35	122.881
37	$\hat{\bar{\Upsilon}}^{*}_{16}$	-2.69e+17	14470.7	873.218	-372.615	8.74246	810.752	2.2E+10	8872.57	124.734
38	$\hat{\bar{\Upsilon}}^{*}_{17}$	-1.07e+14	14470.7	873.218	-0.59458	8.74246	810.752	-5.40389	8872.57	124.734
39	$\hat{\bar{\Upsilon}}^{*}_{18}$	-9.20e+19	14470.7	873.218	-67.3499	8.74246	810.752	7.4E+09	8872.57	124.734
40	$\hat{\bar{\Upsilon}}^{*}_{19}$	838.124	14470.7	873.218	-1.13533	8.74246	810.752	-4.85365	8872.57	124.734
41	$\hat{\bar{\Upsilon}}^{*}_{20}$	1.07139	14470.7	873.218	-0.40214	8.74246	810.752	-5.39177	8872.57	124.734
42	$\hat{\bar{\Upsilon}}^{*}_{21}$	1.07138	14470.7	873.218	-0.41325	8.74246	810.752	-5.39174	8872.57	124.734
43	$\hat{\bar{\Upsilon}}^{*}_{22}$	13.0925	14470.7	873.218	-0.66063	8.74246	810.752	-4.46914	8872.57	124.734
44	$\hat{\bar{\Upsilon}}^{*}_{23}$	1.02959	14470.9	873.208	-0.07905	22.9412	308.962	-5.40092	8872.57	124.734
45	$\hat{\bar{\Upsilon}}^{*}_{24}$	0.68124	14507.5	871.007	-1.43006	86.9975	81.4732	-5.40174	8872.57	124.734
46	$\hat{\bar{\Upsilon}}^{*}_{25}$	1.3232	14470.7	873.218	-0.39916	8.74246	810.752	-5.40171	8872.57	124.734

Table 3: Statistical Analysis of three different populations based on simulated data

S/No.	Estimators	Population One(Gamma)			Population Two(Normal)			Population Three(Uniform)		
		Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
1	$\hat{\Upsilon}_{10}$	3.2151	30.2243	100	3.62089	1186.04	100	0.26375	4.09683	100
2	$\hat{\Upsilon}_{11}$	-1.25312	10.3262	292.695	0.74494	39.013	3040.12	-0.07348	3.82492	98.426
3	$\hat{\Upsilon}_{12}$	-3.54134	47.2015	49.0679	-12.3542	4351.47	26.9252	-0.32232	4.55324	89.9761
4	$\hat{\Upsilon}_{13}$	2.85809	23.5834	128.159	2.96209	1008.58	117.595	0.22761	2.92214	128.835
5	$\hat{\Upsilon}_{14}$	-2.36864	24.437	94.7775	-3.7824	1312.35	89.2781	-0.20351	4.12321	99.3601
6	$\hat{\Upsilon}_{15}$	0.85749	1.46839	2058.34	1.22226	212.557	557.988	0.06674	0.30476	1235.3
7	$\hat{\Upsilon}_{16}$	-2.88217	33.3444	69.4595	-7.07126	2423.92	48.3366	-0.25808	4.3	95.2751
8	$\hat{\Upsilon}_{17}$	3.39566	33.8946	89.1714	3.96227	1281.25	92.5688	0.28217	4.23073	88.985
9	$\hat{\Upsilon}_{18}$	-2.24734	22.5443	102.735	-2.87688	1105.23	106.009	-0.19132	4.08394	100.316
10	$\hat{\Upsilon}_{19}$	-32.3143	1046.73	2.88751	-31.1976	1001.2	118.462	-31.8767	1016.97	0.37019
11	$\hat{\Upsilon}_{20}$	0.78966	1.22335	2470.63	-2.43656	337.054	351.885	0.14127	0.82294	457.475
12	$\hat{\Upsilon}_{21}$	80.5601	410261	20.2876	-1.3017	129.795	37.7025	8.8701	17520.4	60.084
13	$\hat{\Upsilon}_{22}$	-99.805	58112.2	215.337	1.41112	41.122	113.982	-10.91	9211.39	114.73
14	$\hat{\Upsilon}_{23}$	2.90449	21.3223	108.623	5.93455	102.689	1140.97	0.19132	42.5503	9.62819
15	$\hat{\Upsilon}_{24}$	-2.10901	15.1159	199.95	-3.58853	672.603	176.336	-3.79129	3.28249	114.691
16	$\hat{\Upsilon}_{25}$	-0.52028	0.80018	2894.44	-5.95458	450.173	260.264	-0.06132	0.01222	33532.6
17	$\hat{\Upsilon}_{26}$	-0.78376	2.0363	1484.28	-2.95013	1570.14	75.5375	0.62623	10.4711	35.9534
18	$\hat{\Upsilon}_{27}$	-0.72798	1.75531	1721.88	-2.92102	1590.9	74.5518	0.21387	1.62911	231.09
19	$\hat{\Upsilon}_{28}$	-3.46284	42.3618	71.3481	0.77261	11.5933	10230.4	3.26122	436692	0.00086
20	$\hat{\Upsilon}_{29}$	-0.16843	1.85215	1631.85	-9.09901	1051.15	111.462	0.04214	237.019	1.58836
21	$\hat{\Upsilon}_{30}$	0.91633	2.48207	933.123	-0.7307	6.77882	17283.8	-1.90922	34.5922	11.8432
22	$\hat{\Upsilon}_{31}$	-1.22282	4.99201	605.454	-0.81947	2823.18	42.0109	-0.02609	0.03174	11861.6
23	$\hat{\Upsilon}_{32}$	-1.32201	5.99127	501.908	-2.79971	1674.75	70.8189	-0.3112	34.9204	11.1833
24	$\hat{\Upsilon}_{33}$	-1.32606	5.88127	513.908	-2.79971	1674.75	70.8189	-0.3164	36.9604	10.1858
25	$\hat{\Upsilon}_{34}$	-0.72798	1.75531	1721.88	-2.92102	1590.9	74.5518	0.21387	1.62911	231.09
26	$\hat{\Upsilon}_{35}$	-3.46284	42.3618	71.3481	0.77261	11.5933	10230.4	3.26122	436692	0.00086
27	$\hat{\Upsilon}_{36}$	-0.16843	1.85215	1631.85	-9.09901	1051.15	111.462	0.04214	237.019	1.58836
28	$\hat{\Upsilon}_{37}$	0.91633	2.48207	933.123	-0.7307	6.77882	17283.8	-1.90922	34.5922	11.8432
29	$\hat{\Upsilon}_{38}$	-1.22282	4.99201	605.454	-0.81947	2823.18	42.0109	-0.02609	0.03174	11861.6
30	$\hat{\Upsilon}_{39}$	-1.32201	5.99127	501.908	-2.79971	1674.75	70.8189	-0.3112	34.9204	11.1833
31	$\hat{\Upsilon}_{40}$	-1.32606	5.88127	513.908	-2.79971	1674.75	70.8189	-0.3164	36.9604	10.1858
32	$\hat{\Upsilon}^*_{11}$	0.28817	0.36961	8177.34	2.0202	15.3477	7727.83	-0.02243	0.30156	1248.43
33	$\hat{\Upsilon}^*_{12}$	0.05862	0.50244	6015.48	5.21147	87.5933	1354.03	-0.02116	0.30937	1216.89
34	$\hat{\Upsilon}^*_{13}$	0.18846	0.38388	7873.36	-0.41505	1.66589	71195.6	-0.0168	0.30806	1222.08
35	$\hat{\Upsilon}^*_{14}$	0.05804	0.50323	6006.06	-4.4717	64.8619	1828.57	-0.02125	0.3094	1216.77
36	$\hat{\Upsilon}^*_{15}$	0.92099	1.62615	1858.64	-2.9385	521.163	227.576	0.19076	0.99928	376.742
37	$\hat{\Upsilon}^*_{16}$	0.31523	0.37668	8023.89	-1.33608	7.75801	15288	0.01302	0.30225	1245.55
38	$\hat{\Upsilon}^*_{17}$	0.13655	0.41731	7242.59	0.41283	1.65715	71571.1	-0.05334	0.32289	1165.96
39	$\hat{\Upsilon}^*_{18}$	0.28817	0.36961	8177.34	2.0202	15.3477	7727.83	-0.02243	0.30156	1248.43
40	$\hat{\Upsilon}^*_{19}$	-0.24533	1.08855	2776.58	0.77604	12.3453	9607.27	-0.04496	0.40449	930.736
41	$\hat{\Upsilon}^*_{20}$	-8.27773	241.569	12.5117	-0.39063	33.1598	3576.74	0.4923	13.035	28.8817
42	$\hat{\Upsilon}^*_{21}$	-8.12475	233.163	12.9628	-0.39259	31.9233	3715.28	-0.49178	13.0104	28.9363
43	$\hat{\Upsilon}^*_{22}$	0.21214	0.37463	8067.74	-1.46453	8.98638	13198.2	0.02461	0.30147	1248.79
44	$\hat{\Upsilon}^*_{23}$	0.21106	0.37497	8060.37	0.2561	1.11417	106451	0.08648	0.31087	1211.05
45	$\hat{\Upsilon}^*_{24}$	0.14261	0.41247	7327.7	-0.44071	1.76927	67035.8	-0.10415	0.35865	1049.7
46	$\hat{\Upsilon}^*_{25}$	0.37207	0.40618	7441.05	5.12272	172.072	689.271	0.05286	0.30294	1242.74

5 Discussion of Results

Table (2) and (3) shows the criteria for better judgment between proposed and related existing estimators, such as BIASs, MSEs and PREs. As indications from Tables (2) and (3) the proposed estimator eqn (3.1) have less BIAS and mean square error MSE and also have larger PRE than existing estimators eqn (2.1), (2.2), (2.7), (2.8), (2.13), (2.14), (2.15), (2.16), (2.25), (2.26), (2.27),(2.28), (2.37), (2.44), (**??**), (2.48), (2.49), (2.50), (2.51), (2.52), (2.53), (2.54), (2.55), (2.56), (2.57), (2.58), (2.59), (2.60), for all data sets I, II, III and simulated data under Gamma, Normal, Uniform distribution accept some few cases

6 Conclusion

In section (2) the existing estimators of ratio, product, ratio cum, product cum, dual-ratio, dual-product, dual-ratio cum, dual-product cum of all the family of usual ratio and product estimators were reviews and in section (3) linear combination of generalized exponential ratio cum to dual-ratio cum estimators for the population mean of the study variable is developed within the parameters of a simple random sampling plan. The suggested estimator's properties are deduced up to the first order of approximation. Both the theoretical and empirical comparisons of the suggested estimator's efficacy are made with that of other existing estimators. Evaluation of the suggested estimator's performance using data from a known natural population and simulated. Findings are shown in Tables (2) and (3), which demonstrates that the proposed linear combination of generalized exponential estimator outperforms other existing estimators by having less BIASs and MSEs.

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The authors(s) of this manuscript solemnly declare that they have not utilized any forms generative AI tools, including Large Language Models (such as ChatGPT, Perplexity and COPILOT) and text-to-image generators, during writing process. This work is the result of their own original thinking, research, and writing effort, without any assistance from AI-generate content.

Competing Interests

Authors have declared that no competing interests exist.

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