

## Short Research Article

### Mathematical Modeling of Host-Parasitoid Dynamics for Sustainable Pest Management

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#### ABSTRACT

**Aims:** The management of agricultural pest populations represents a critical challenge in sustainable crop production, particularly for challenging species like the tomato fruit borer, *Helicoverpa armigera*. The research work aimed in developing and analysing a novel mathematical model describing the interaction between the tomato fruit borer and its egg parasitoid, *Trichogramma chilonis*. The main objective of this study to analyze the stability of the proposed system of differential equations.

**Model Formulation:** The model incorporates stage-structured population dynamics and impulsive control measures, providing insights into biological pest control strategies. In this model the tomato borer is represented by the egg and larval stages, and the parasitoid is considered in terms of the parasitized eggs. This model assumes uniform spatial distribution of both host and parasitoid populations, constant environmental conditions affecting parameters like reproduction and mortality rates, and idealized parasitoid release conditions where the released amount  $p$  remains consistent across interventions.

**Methodology:** We used existing secondary data from research articles' instead of conducting primary field experiments for parameter identification for this mathematical model. The use of secondary data from peer-reviewed literature not only provides validated parameter values that have been rigorously tested and verified by multiple researchers, but also allows for the incorporation of findings from diverse geographical locations and climatic conditions.

**Results:** Stability analysis identified three equilibrium points: the trivial, host-only  $E_0$ , coexistence equilibria  $E_1$ , and  $E_2$ . The coexistence equilibrium  $E_2$  found to be locally stable under optimal parasitoid release intervals and quantities. The equilibrium point  $E_2 = (9.14, 0.02, 5.71)$  represents a coexistence state in the host-parasitoid dynamics, where the tomato fruit borer's egg density stabilizes at 9.14, the density of parasitized eggs at 0.02, and the larval density stabilizes at 5.71. This equilibrium is analyzed through its associated eigenvalues. This stability provides a reliable basis for implementing successful biological pest control strategies in tomato cultivation, ensuring a resilient and sustainable ecosystem

*Keywords: biological control, impulsive system, tomato fruit borer, stability*

#### 1. INTRODUCTION

Tomato (*Lycopersicon esculentum*) is one of the most commonly and extensively grown vegetables all over the country. This vegetable being a rich source of vitamins and minerals, occupy an important place in the food basket of Indian consumers. One of the major constraints identified in the production is the increasing incidence of insect pests, diseases and nematodes. a most commonly used vegetable by all classes of people is highly susceptible to the fruit borer attack. Tomato fruit borer *Helicoverpa armigera*, a major pest on developing fruits is responsible for major yield loss. Eggs are laid singly, generally on leaves and flowers in the upper canopy of the plant. The young larvae feeds on the tender foliage and the fourth larval instar bores inside the fruits and eats away the internal content of the fruits. Such fruits are not

preferred by consumers. The holes made on the fruits are circular and the larva feeds on keeping the head portion only inside the hole. A female lays approximately 500 eggs.

Biological control is defined as the reduction of pest population by introducing their natural enemies namely predators, parasitoids and pathogens. The parasitoids are species which develop within or on the host and ultimately kill it. Thus parasitoids are commonly reared in the laboratories and periodically released in high density populations as biological control agents. The biological pest control, which is not only eco-friendly but also cheaper and more effective than other insecticides. The main goal of pest control is to maintain the density of the pest population at an equilibrium level below the economic threshold level. In order to increase the profits and to maintain the environmental quality, economic decisions related regarding pests should be appropriately understood and adopted. Hence the mathematical modeling of these agro ecosystems can help to study their dynamics and choose the biological control strategy. Thus a good strategy of biological pest control based on mathematical model to study the interactions between the crop borer and its parasitoids is highly significant in the present world of agriculture.

In this research work, the mathematical model of interaction between the tomato borer, *Helicoverpa armigae* and its egg parasitoid, *Trichogramma chilonis* represented by system of impulsive differential equations [2] is considered. The model is developed by the process of biological control of the fruit borer that is carried over by periodically releasing the parasitoids to keep the larvae population at an acceptable low level [3]. We are interested to develop a new mathematical model adequate to represent interactions between the pest and its biological control and to determine the optimum level of release of parasitoids. The dependent variables used in the model are the egg population density of the tomato borer, parasitized egg and the larval population. Also fraction of the eggs the larvae emerged at time, fraction of the parasitized eggs from which the adult parasitoids emerge and the fraction of the larval population which moults into pupal stage at time and the number of parasitoids emerged at time.

The aim of the study is to analyse the stability of the impulsive control system, i.e., to maintain the pest population in an equilibrium level below the economic injury level. Relevant computational tools for undergoing numerical simulations for different possible scenarios of biological pest control in Tomato borer based on the mathematical models will be elaborated.

## 1.1 Brief review of work done

The sustainable control of agricultural pests such as *Helicoverpa armigera*, the tomato fruit borer, is still a major problem for contemporary farming. In order to parasitize *Helicoverpa armigera* eggs and inhibit population growth, the application of biological control agents such as egg parasitoids, specifically *Trichogramma chilonis*, has gained significant attention. This review summarizes key studies and advances in modeling pest-parasitoid interactions, with a particular focus on *Helicoverpa armigera* and *Trichogramma chilonis*. The ecological, biological, and operational factors affecting the effectiveness of pest control are also highlighted in this review, which looks at earlier studies on pest-parasitoid dynamics with an emphasis on the creation of mathematical models.

The interaction between *Helicoverpa armigera* and *Trichogramma chilonis* has been used extensively in integrated pest management (IPM) strategies. Abbas et al., S. (2024) investigated the environmentally friendly use of *Trichogramma chilonis* against *Helicoverpa armigera* in tomato crops, showing that targeted releases significantly reduced pest populations. In the same way, Hussain et. (2015) examined parasitoid's ability to parasitize *Helicoverpa armigera*, finding that, it was highly effective in controlled settings. The mass rearing and release techniques have been refined across a range of agricultural systems for *Trichogramma* species.

Mass rearing and release strategies for *Trichogramma* species have been optimized in various agricultural systems. Wang et al. (2014) detailed a comprehensive protocol for large-scale production and application of *Trichogramma* in cornfields, which can be adapted for tomato systems. Their findings underscored the importance of synchronized parasitoid releases to coincide with pest egg-laying periods, ensuring maximum parasitism rates.

Landscape diversity is a critical factor in enhancing the biological control potential of *Trichogramma chilonis*. Liu et al. (2016) examined the influence of landscape heterogeneity on the parasitism of *Helicoverpa armigera* eggs in cotton fields, concluding that diversified landscapes significantly improve parasitoid efficacy. These findings emphasize the need for habitat management strategies that promote biodiversity to support natural enemy populations.

In apple orchards, Liu et al. (2024) demonstrated how the incorporation of functional plants could conserve predator populations, repel pests, and enhance biological control. While the study focused on *Aphis spiraeicola*, its principles are applicable to *Helicoverpa armigera* management in tomato ecosystems.

Mathematical modeling plays a pivotal role in understanding pest-parasitoid interactions and optimizing control strategies. Akman et al. (2018) developed an impulsive stage-structured IPM model incorporating refuge effects. Their analysis highlighted the complex interplay between pest population dynamics, parasitism rates, and refuge availability, providing insights into the timing and intensity of biological control interventions.

Building on this, Liu et al. (2023) proposed pest control switching models that integrate instantaneous and non-instantaneous impulsive effects. These models enable the evaluation of various release schedules and intensities, helping to identify optimal strategies for *Trichogramma chilonis* deployment. Zhang et al. (2023) further explored predator-prey systems with state-dependent impulsive controls, showcasing the rich dynamics that emerge when pest control measures are adaptively adjusted based on real-time population data.

The use of mathematical models to analyze predator-parasitoid interactions extends to other systems as well. Paparella et al. (2016) developed a model to study the biological control of chestnut gall wasps by *Torymus sinensis*. Although focused on a different pest, their work illustrates the utility of modeling in optimizing parasitoid release strategies and predicting long-term ecological outcomes.

The foraging behavior of parasitoids and the environmental context significantly influence biological control outcomes. Liu et al. (2018) examined how tomato yellow leaf curl virus infection alters the behavior and parasitism efficiency of the parasitoid *Encarsia formosa* on *Bemisia tabaci*. While their study focused on a different pest-parasitoid system, the findings are relevant for understanding similar behavioral modifications in *Trichogramma chilonis*.

Thermal requirements and biological parameters also play a crucial role in determining parasitoid efficacy. Oliveira et al. (2017) investigated the thermal tolerance of *T. pretiosum*, a species closely related to *Trichogramma chilonis*. Their results highlighted the importance of selecting suitable release timings and locations to match environmental conditions.

Effective pest management often requires the integration of biological and chemical control methods. Abbas et al. (2015) evaluated the effects of insecticides on *Helicoverpa armigera* and their compatibility with *Trichogramma chilonis* releases. Their findings underscore the importance of selecting insecticides with minimal non-target effects to ensure the success of combined control strategies.

Gulati et al. (2021) formulated a mathematical model of Eggplant fruit and shoot borer controlled through biological control. The dynamical analysis of the system has been discussed and the global stability analysis was studied using Lyapunov technique. The research work analyzed a biological pest control model incorporating chemical treatments for brinjal fruit and shoot borer and the study revealed the potential for synergistic effects when biological and chemical controls are strategically combined. These principles can be extended to *Helicoverpa armigera* management in tomatoes.

Recent advances in pest dynamics modeling provide valuable insights into the complex interactions between pests and their natural enemies and in control optimization. Rubio et al. (2022) developed a predator-parasitoid model for the tomato leafminer (*Tuta absoluta*), demonstrating how mathematical approaches can guide the design of effective control measures. Similarly, Molter et al. (2023) analyzed the dynamics of sugarcane borers and their parasitoids, providing a framework for multi-parasitoid systems that could inform *Helicoverpa armigera* management.

Zevika et al. (2024) investigated predator-prey dynamics in oil palm plantations, emphasizing the importance of optimal control strategies in achieving pest eradication. Their findings highlight the potential for adaptive management approaches that dynamically respond to changing pest and parasitoid populations.

The development of novel mathematical models to describe *Helicoverpa armigera* and *Trichogramma chilonis* interactions has the potential to revolutionize pest management strategies. Singh et al. (2021) explored fractional models incorporating memory effects, providing a new dimension to the analysis of pest dynamics. These approaches could be applied to capture the temporal complexity of *Helicoverpa armigera* populations and the lagged effects of *Trichogramma chilonis* releases.

Wang et al. (2024) analyzed pest control models with multiple delays and impulsive effects, offering a sophisticated framework for studying time-dependent interactions. Their work underscores the importance of incorporating temporal dynamics into IPM models to enhance the precision and reliability of control strategies.

The integration of biological and mathematical research has significantly advanced our understanding of pest-parasitoid interactions. Studies on *Trichogramma chilonis* and *Helicoverpa armigera* have demonstrated the potential for eco-friendly pest management through targeted parasitoid releases. Mathematical models provide invaluable tools for optimizing these strategies, enabling researchers and practitioners to predict outcomes, identify optimal release schedules, and adapt to changing ecological conditions.

The existing literature underscores the complexity of pest-parasitoid interactions and the critical role of mathematical modeling in understanding and managing these biological systems. The proposed research offers new insights into the dynamics of interaction between *Helicoverpa armigera* and *Trichogramma Chilonis* represented through the mathematical model by impulsive dynamical systems. Future research should focus on integrating behavioral, ecological, and operational factors into comprehensive models, ensuring the sustainable management of *Helicoverpa armigera* in tomato crops.

## **2. MATHEMATICAL MODEL FORMULATION AND METHODOLOGY**

The research work aimed in developing and analysing a novel mathematical model describing the interaction between the tomato fruit borer, *Helicoverpa armigera* and its egg parasitoid, *Trichogramma chilonis*. The model incorporates stage-structured population dynamics and impulsive control measures, providing insights into biological pest control strategies. We propose a simple mathematical model of interaction between the tomato borer and its egg parasitoid. In this model the tomato fruit borer is represented by the egg and larval stages, and the parasitoid is considered in terms of the parasitized eggs. Consider two main stages of development of the tomato fruit borer *Helicoverpa armigera*, the egg and larval stages and there exists only an egg parasitoid *Trichogramma chilonis* in a common environment. Let  $x_1(t)$ , the egg density of *Helicoverpa armigera*,  $x_2(t)$ , the density of parasitized eggs and  $x_3(t)$ , the larval density of *Helicoverpa armigera* be the state variables representing different population components. Then, by assuming the logistic growth for the egg population of tomato borer, the system dynamics are described by the following set of differential equations:

$$\frac{dx_1}{dt} = \alpha \left(1 - \frac{x_1}{k}\right) x_1 - m_1 x_1 - \eta_1 x_1 - \beta x_1 x_2$$

$$\frac{dx_2}{dt} = \beta x_1 x_2 - m_2 x_2 - \eta_2 x_2$$

$$\frac{dx_3}{dt} = \eta_1 x_1 - m_2 x_3 - \eta_3 x_3, \quad t \neq n\tau$$

with impulsive control conditions

$$\Delta x_1(t) = 0$$

$$\Delta x_2(t) = p$$

$$\Delta x_3(t) = 0, \quad t = n\tau, \quad n \in \mathbb{Z}$$

Where the model parameters are described as :

$\alpha$ : net reproduction rate

$\beta$ : parasitism rate

$k$ : environmental carrying capacity

$m_1, m_2, m_3$ : natural mortality rates of the egg, parasitized egg and larvae populations

$\eta_1$ : the fraction of the eggs from which the larvae emerge at time  $t$

$\eta_2$ : the fraction of the parasitized eggs from which the adult parasitoids emerge at time  $t$

$\eta_3$ : the fraction of the larvae population which moults into pupal stage at time  $t$

$p$ : Release amount of parasitoids

$\tau$ : Time interval between releases

## 2.1 Salient Features of the Model Description

This mathematical model presents a stage-structured representation of host-parasitoid dynamics, specifically focusing on the *Helicoverpa armigera* and *Trichogramma chilonis* system. The significant characteristics of this model lies in the integration of density-dependent host population growth through the logistic term  $\alpha(1-x_1/k)x_1$ , along with explicit tracking of both unparasitized and parasitized egg stages, and subsequent larval development. We included an impulsive control conditions ( $\Delta x_2(t) = p$  at  $t = n\tau$ ) that which is useful for the simulation and optimization of periodic parasitoid release strategies. making it particularly valuable for practical Integrated Pest Management (IPM) implementation. The model also includes key biological processes including parasitism ( $\beta x_1 x_2$ ), natural mortality rates ( $m_1, m_2, m_3$ ), and stage transitions ( $\eta_1, \eta_2, \eta_3$ ), providing a realistic framework for understanding population dynamics and intervention effects.

## 2.2 Assumptions made in the formulation of the Model

This model operates under several simplifying assumptions that should be considered when interpreting its predictions. It assumes uniform spatial distribution of both host and parasitoid populations, constant environmental conditions affecting parameters like reproduction and mortality rates, and idealized parasitoid release conditions where the released amount  $p$  remains consistent across interventions. These assumptions limit the ability of the model to capture spatial heterogeneity, seasonal variations, and real-world release efficiency variations.

Despite these limitations, the model serves as a valuable tool for determining optimal release intervals ( $\tau$ ) and quantities ( $p$ ) of parasitoids, predicting population trajectories under different management scenarios, establishing economic control thresholds, and evaluating the cost-effectiveness of biological control strategies. The stage-structured approach particularly enhances its utility in practical applications by allowing managers to track and respond to changes in specific life stages of the pest population.

### 2.3 Identification of Parameter Values

In this research work, it was decided to use existing secondary data from research articles' instead of conducting primary field experiments for parameter identification for this mathematical model. Field experiments for *Helicoverpa armigera* and *Trichogramma chilonis* interactions would require extensive time spanning multiple pest generations, substantial resources, and carefully controlled conditions to generate reliable data across different weather patterns and seasonal variations. Also it was understood that parameter estimation through field trials faces significant challenges due to the complex nature of host-parasitoid dynamics, environmental stochasticity, and the difficulty in maintaining consistent observation protocols across multiple life stages. The use of secondary data from peer-reviewed literature not only provides validated parameter values that have been rigorously tested and verified by multiple researchers, but also allows for the incorporation of findings from diverse geographical locations and climatic conditions. This approach ensures model robustness while acknowledging the practical limitations of conducting comprehensive field trials, particularly given the time-sensitive nature of developing effective pest management strategies. The values in Tables 1, 2, and 3 are justified based on their alignment with data reported in previous studies focusing on pest and parasitoid dynamics. The values used for *Helicoverpa armigera* and *Trichogramma chilonis* are justified as they align with biological and ecological parameters reported in relevant studies (S Abbas et al., 2024, Hussain et al., 2015 and Abbas et al., 2015). The net reproduction rate, parasitism rate, and mortality rates are reflective of data specific to pest-parasitoid systems in tomato ecosystems (Liu et al., 2016; Hussain et al., 2015). Stage transition rates, including egg to larval and parasitoid emergence rates, are validated by prior research on developmental timings under similar agroecological conditions (Oliveira et al., 2017). These parameters ensure the model's applicability to *H. armigera* and *T. chilonis* interactions in tomato crop systems.

**Table 1: Population Parameters**

Parameter	Value	Unit	Description
$\alpha$	0.42	day <sup>-1</sup>	Net reproduction rate
$\beta$	0.035	day <sup>-1</sup>	Parasitism rate
k	200	eggs/plant	Environmental carrying capacity

**Table 2: Mortality Rates**

Parameter	Value	Unit	Description
$m_1$	0.15	day <sup>-1</sup>	Natural mortality rate of eggs
$m_2$	0.12	day <sup>-1</sup>	Natural mortality rate of parasitized eggs
$m_3$	0.18	day <sup>-1</sup>	Natural mortality rate of larvae

**Table 3: Stage Transition Rates**

Parameter	Value	Unit	Description
$\eta_1$	0.25	day <sup>-1</sup>	Egg to larval transition rate
$\eta_2$	0.20	day <sup>-1</sup>	Parasitoid emergence rate
$\eta_3$	0.22	day <sup>-1</sup>	Larval to pupal transition rate

**Table 4: Control Parameters**

Parameter	Value	Unit	Description
$p$	50	parasitoids/release	Release amount
$\tau$	7	days	Time interval between releases

The above values represent averages under standard field conditions of temperature and rainfall range (25±2°C, 65±5% RH). The values may vary significantly based on local environmental conditions and pest-parasitoid strain characteristics.

### 3. RESULTS AND DISCUSSION

#### 3.1 Stability Analysis of the Host-Parasitoid System

The system of differential equations incorporated with the parameter values is given by

$$\frac{dx_1}{dt} = 0.42 \left(1 - \frac{x_1}{200}\right) x_1 - 0.15x_1 - 0.25x_1 - 0.035x_1x_2 \text{ -----(1)}$$

$$dx_2/dt = 0.035x_1x_2 - 0.12x_2 - 0.20x_2 \text{ -----(2)}$$

$$dx_3/dt = 0.25x_1 - 0.18x_3 - 0.22x_3 \text{ -----(3)}$$

where  $\Delta x_1(t) = 0$ ,  $\Delta x_2(t) = 50$  and  $\Delta x_3(t) = 0$ ,  $t = 7n$ ,  $n \in Z$

The stability of the system dynamics is studied by finding the equilibrium points. Equate the above equations to zero to find the equilibrium points of the dynamical biological system.

$$0.42 \left(1 - \frac{x_1}{200}\right) x_1 - 0.15x_1 - 0.25x_1 - 0.035x_1x_2 = 0 \text{ -----(4)}$$

$$0.035x_1x_2 - 0.12x_2 - 0.20x_2 = 0 \text{ -----(5)}$$

$$0.25x_1 - 0.18x_3 - 0.22x_3 = 0 \text{ -----(6)}$$

Solving the system of equations we get the equilibrium points  $(x_{1*}, x_{2*}, x_{3*})$

$$0.42 \left(1 - \frac{x_{1*}}{200}\right) x_{1*} - 0.15x_{1*} - 0.25x_{1*} - 0.035x_{1*} x_{2*} = 0$$

$$0.035 x_{1*} x_{2*} - 0.12x_{2*} - 0.20x_{2*} = 0$$

$$0.25x_{1*} - 0.18x_{3*} - 0.22x_{3*} = 0$$

We used Python code to solve the above complex systems to get the equilibrium values.

Hence we have three equilibrium values for  $(x_{1*}, x_{2*}, x_{3*})$  given by

$E_0 = (0.0, 0.0, 0.0)$ , the trivial points

$E_1 = (9.52, 0.0, 5.95)$ , the host only state equilibrium and

$E_2 = (9.14, 0.02, 5.71)$ , the co-existence equilibrium

The Jacobian matrix at any point  $(x_1, x_2, x_3)$  is:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}$$

$$J = \begin{bmatrix} 0.42(1 - 2x_1/200) - 0.40x_2 + 0.035x_2 & -0.035x_1 & 0 \\ 0.035x_2 & 0.035x_1 - 0.32 & 0 \\ 0.25 & 0 & -0.40 \end{bmatrix}$$

The stability of the dynamic system is fundamentally characterized by the nature of its eigenvalues. Substitute each equilibrium point into the Jacobian matrix to evaluate J at that point and then compute the eigenvalues of J at each equilibrium. When all eigenvalues possess negative real parts, the system achieves local stability, indicating that small perturbations from the equilibrium point will result in a return to the original state. Conversely, if any eigenvalue exhibits a positive real part, the system becomes unstable, meaning that minor deviations from the equilibrium will lead to progressively larger oscillations or divergence, potentially causing significant system disruption. This eigenvalue analysis provides a critical mathematical framework for understanding the qualitative behavior and long-term dynamics of complex systems across various scientific and engineering domains. The eigen values of the Jacobian matrix calculated around the three equilibrium points are given by

**Table 5: Eigen values around the equilibrium points**

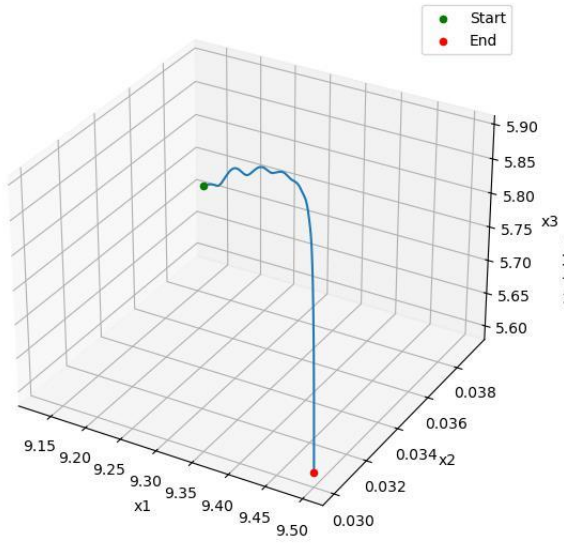
Equilibrium points	Eigen values	Result
$E_0 = (0.0, 0.0, 0.0)$	$(-0.4, 0.02, -0.32)$	Unstable
$E_1 = (9.52, 0.0, 5.95)$	$(-0.4, -0.019984, 0.0132)$	Unstable
$E_2 = (9.14, 0.02, 5.71)$	$(-0.4, -0.009594, -0.011567)$	Stable

### 3.2 Simulation of the system dynamics

In this research study, the dynamics of the pest-parasitoid interaction is visualised by 3D phase space and time series graph. A 3D plot shows trajectories in the  $x_1, x_2, x_3$  space, giving insight into the system's attractors or behavior and the time-series graphs show how these variables evolve over time.

The graphs given below show how  $x_1(t)$ , the egg density of *helicoverpa armigera*,  $x_2(t)$ , the density of parasitized eggs and  $x_3(t)$ , the larval density of *helicoverpa armigera* evolve over time, allowing us to analyze the system's behavior from different starting points.

3D Phase Space for Initial Condition: [9.5, 0.03, 5.6]



Time Series Dynamics

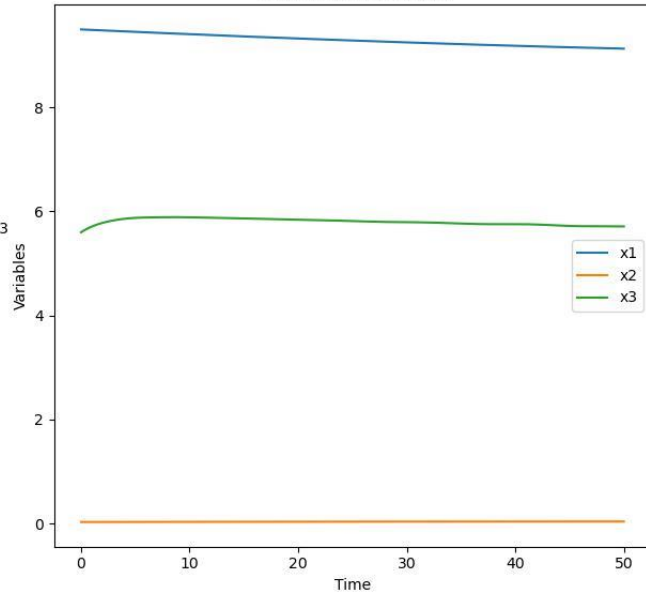
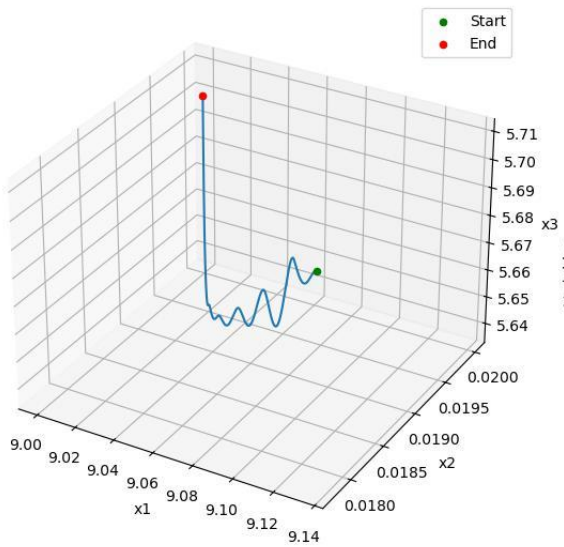


Fig. 1. 3D Phase space and time series dynamics for the initial condition (9.5, .03,5.6)

3D Phase Space for Initial Condition: [9.0, 0.02, 5.7]



Time Series Dynamics

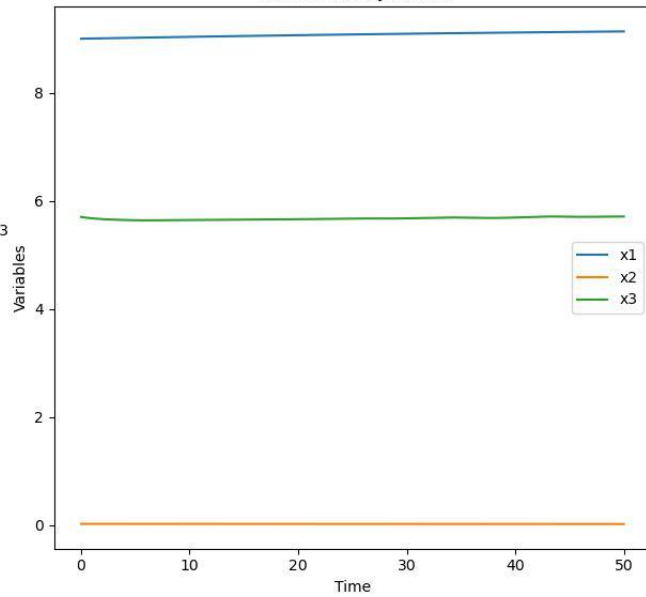
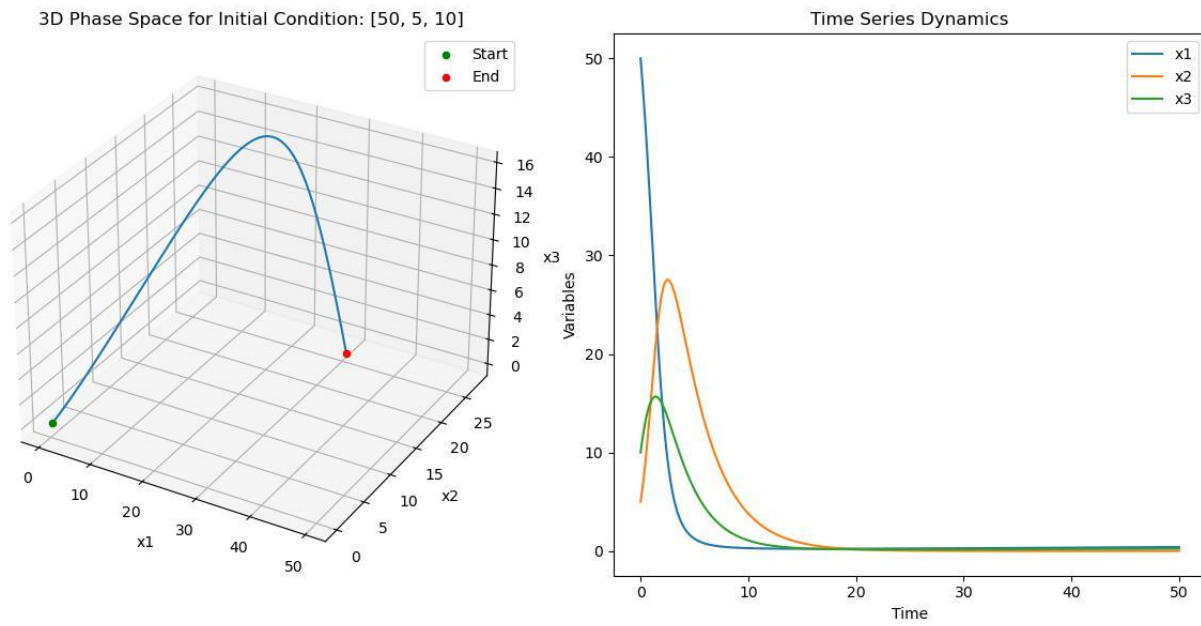
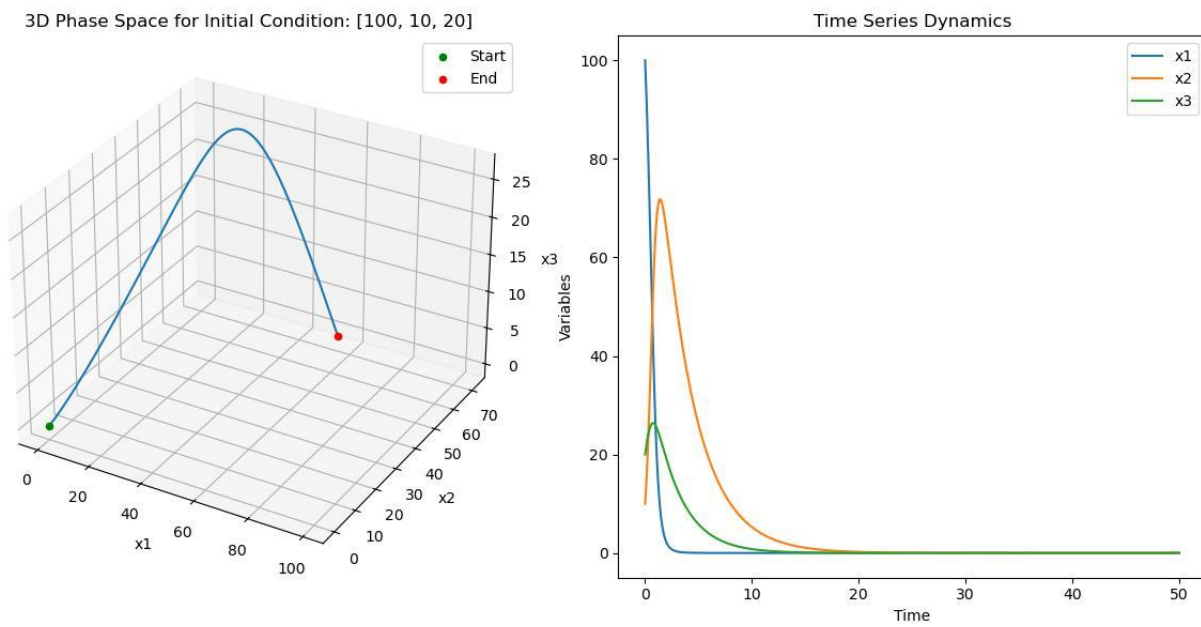


Fig. 2. 3D Phase space and time series dynamics for the initial condition (9.0, .02,5.7)





**Fig. 3. 3D Phase space and time series dynamics for the initial condition (50, 5,10)**



**Fig. 4. 3D Phase space and time series dynamics for the initial condition (100,10,20)**

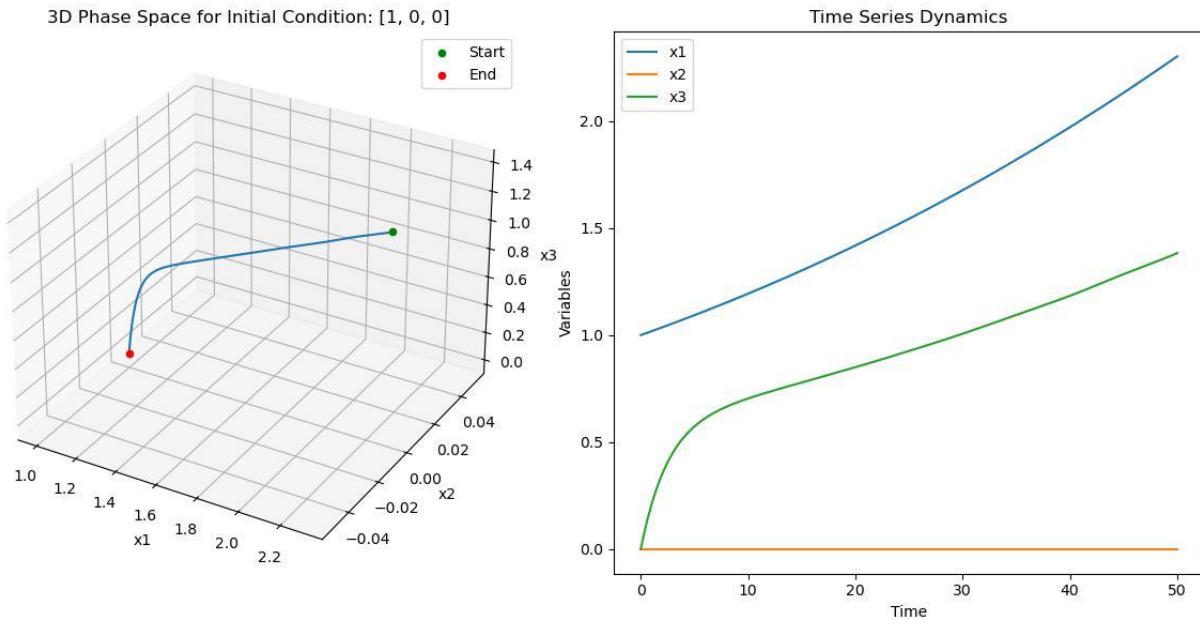


Fig. 5. 3D Phase space and time series dynamics for the initial condition (1, 0,0)

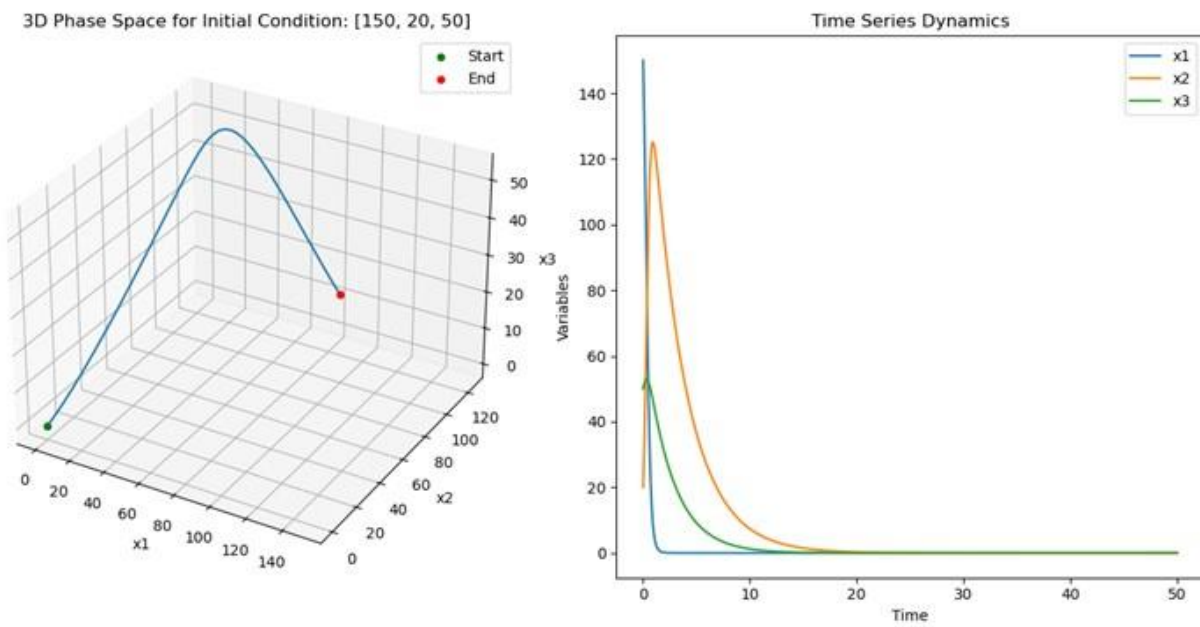


Fig. 6. 3D Phase space and time series dynamics for the initial condition (150,20,50)

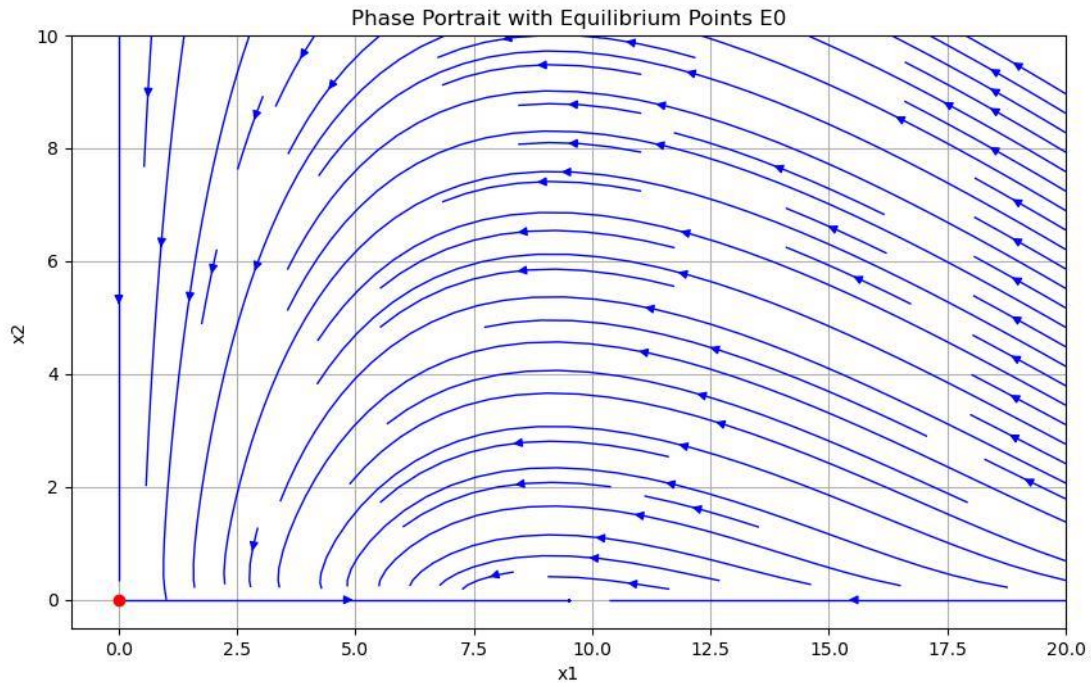


Fig 7. Phase Portraits at the Equilibrium point  $E_0$

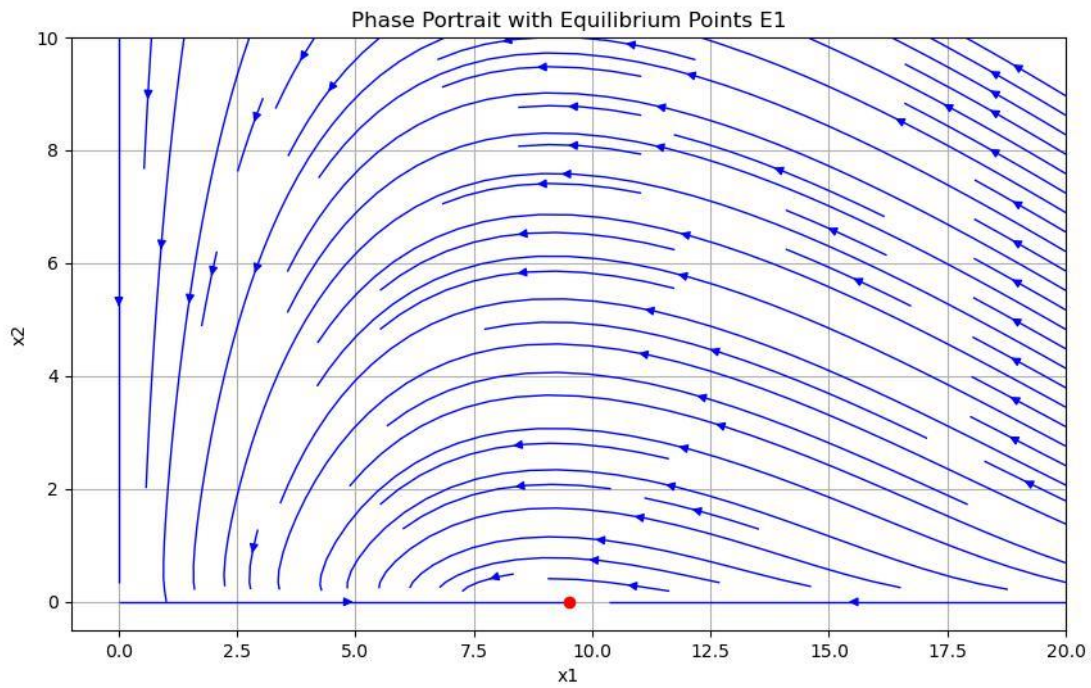
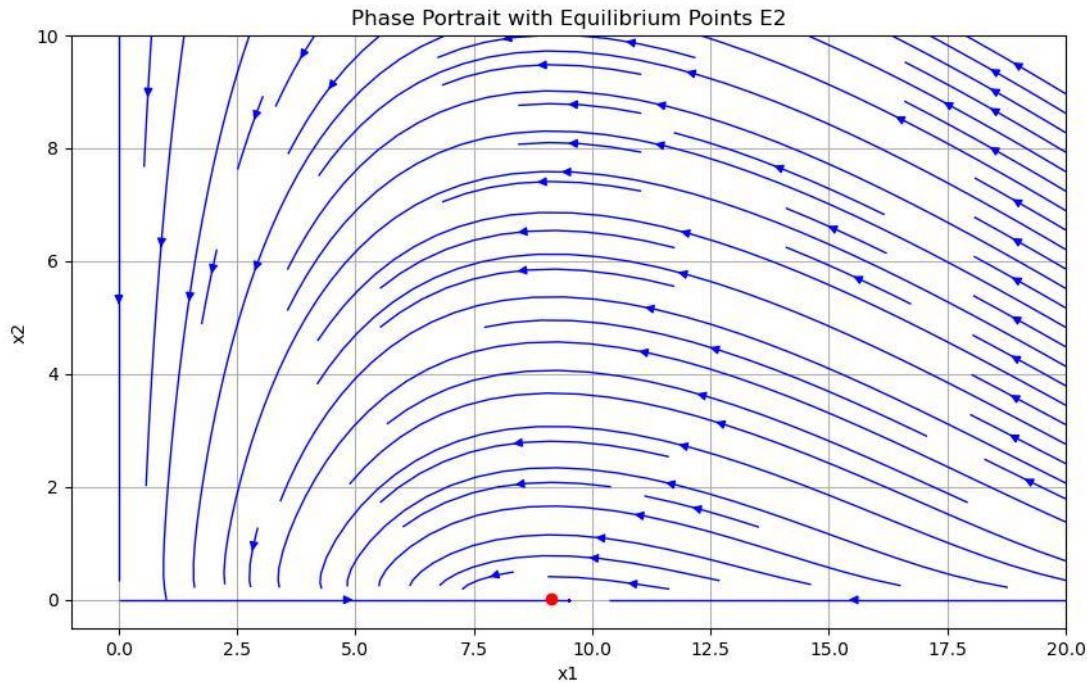


Fig 8. Phase Portraits at the Equilibrium point  $E_1$



**Fig 9. Phase Portraits at the Equilibrium point  $E_2$**

#### 4. CONCLUSION

The mathematical model offers a robust framework for understanding host-parasitoid dynamics. The model underscores the importance of regular monitoring and the stability of the system is strongly influenced by natural mortality rates, emphasizing the importance of precise parameter adjustments.

#### 5. RECOMMENDATIONS

Effective implementation of the model requires regular monitoring of field populations to validate its predictions, while adjusting parasitoid release parameters based on optimized model outcomes. To enhance its impact, these strategies should be integrated with existing Integrated Pest Management (IPM) approaches. Future model enhancements should prioritize incorporating environmental variability, spatial components to account for heterogeneity, and validation using field data. Additionally, integrating economic thresholds will boost the model's practical applicability and decision-making relevance, ultimately leading to more effective and sustainable pest management practices.

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