***Original Research Article***

**Innovative Statistical Modelling of Exchange rates and Agricultural Sector Performance** **in Nigeria: The Markov-Switching Vector Autoregressive Modelling.**

**Abstract**

Nigeria has increasingly been interconnected in a global economy, consequently, understanding the dynamic interdependence between exchange rates fluctuations and the real sector performance is necessary for researchers, economic managers and policymakers. This study on innovative statistical modelling employed the Markov Switching Vector Autoregression (MS-VAR) to explore the dynamic relationship between exchange rates and agricultural performance, capture the nonlinear interactions, regime shift, duration of stay in a regime, and the probability of transition from a regime. Quarterly data on exchange rates and agricultural sector gross domestic product spanning through 2000 to 2024, where sourced from the statistical bulletin of the Central Bank of Nigeria. The result of the statistical analysis showed that exchange rate and agricultural performance where stationary at first difference, there was no cointegration, the result identified 2 regimes, and lag selection criteria showed 3 lags while the model selection based on information criteria selected MSI(2)-VAR(3). The result showed varying significant but negative effects of exchange rate on agricultural performance in regime 1 and 2. The result also showed 55.8% probability of staying in regime 1 for a duration of 7 months, and 88.2% probability of staying in regime 2 for a duration of 17 months. Both exchange rates and agricultural sector were self-explained, however the forecast variance indicated that agricultural performance was more exogenous. The study recommended policies that will reduce exchange rates as it showed inverse effects on agricultural performance. Also, regime possibility test should part of pre-diagnostic tests

**Keywords: Markov-Switching Vector Autoregressive Model, MS-VAR Model, MSI-VAR Model, MSM-VAR Model, Exchange Rate, Agricultural Performance.**

**1.1 Introduction**

Agriculture is a crucial sector in Nigeria’s economy, as it contributes considerably to food security, employment, and foreign exchange earnings. Notwithstanding its prominence, the industry has faced challenging production levels impacted by macroeconomic instability. Agricultural production in Nigeria has been undoubtably impacted by macroeconomic factors such as exchange rates, interest rates, inflation, Gross Domestic Product (GDP), government spending, etc. The understanding of the relationship existing between Agriculture and these macro-economic variables is vital for creating strategies to boost agricultural performance and consequently encourage sustainable economic development. Several studies have been caried out to address this relationship but non have considered the unobservable state of regime-dependent nature of these interactions.

Tuaneh and Essi (2021), reported that macroeconomic variables are commonly modelled linearly without recourse to the existence of regimes therefore are inappropriate with incomplete structural inference. According to Tuaneh, Essi and Etuk (2021), linear models ignore the possible unobservable state, regime switches and duration of stay in a regime. Addressing this challenge demands an innovative statisticalmodel that can detect and analyse the regime-dependent relationships, regime switches and duration of stay in a regime.

StatisticalModelling involves the use of mathematical and statistical methods to describe and understand real-world data. Innovative StatisticalModelling on the other hand is the application of modern and advance computational (mathematical and statistical) methods in analysing data, obtaining meaningful insights for valid conclusions and logical decision. Specifically, innovative statistical modelling involves any of the following approaches: (a) Adopting New Models and Approaches, (this include developing innovative statistical models that fit high-dimensional datasets, complex data, dynamic relationships. (b) Carrying out Time Series Analysis, (this includes analysis which innovate the way time-based data is modelled, capturing the dynamics including seasons, trends etc). (c) Utilising the Bayesian Methods by incorporating prior knowledge, and uncertainty quantification. (d) Application of Machine Learning, (this involves the combination of traditional statistical methods and machine learning algorithms for prediction). (e) Using Big Data Techniques, (this includes handling large data, and data visualization). Others include the techniques for analysing nonlinear relationships and interactions.

This study employed the Markov-Switching Vector Autoregression (MS VAR) model as an innovative statistical model that enables the detection of regime transitions, reflecting the non-linear and dynamic interactions between the macroeconomic factor and agricultural performance (Hamilton, 1989), The Markov-Switching Vector Autoregressive (MS-VAR) model will not only fill the literature gab but also give policymakers deeper insights and practical suggestions for enhancing agricultural performance by deepening the knowledge of the complex interactions between exchange rate and agricultural performance in Nigeria. Several authors: Krolzig (2001), Tillman (2003), Clin and shen (2007), Kikanga, Obange, and Adhiambo (2017), Tuaneh, and Essi (2021), Tuaneh, Essi and Etuk (2021), Tuaneh, and Wiri (2022) applied the Markov Switching Vector Autoregressive Model to macroeconomic variables

**1.2 Statement of the Problem.**

The Nigerian agricultural industry has been marked by poor production, which provides a substantial barrier to economic prosperity. Despite multiple governmental interventions and expenditures targeted at rejuvenating the industry, the influence of macroeconomic volatility on agricultural production remains imperfectly understood. Traditional statistical models typically fail to represent the complex, non-linear linkages and regime transitions inherent in the macroeconomic environment. This impedes the creation of relevant and effective policies geared to varied economic situations. Addressing this challenge demands an innovative statistical model that can detect and analyse this regime-dependent relationships. The Markov-Switching Vector Autoregressive (MS VAR) model gives a strong framework for this study, allowing the evaluation of how changing macroeconomic regimes affect agricultural performance.

**1.3 Objectives**

The objectives of the study are to: (i) explore the dynamic relationship between exchange rates and agricultural performance by capturing the nonlinear interactions and determining the effects of exchange rate distortions on agricultural performance, (ii) examine the regime shift, (iii) ascertain the duration of stay in a regime, (iv) find the probability of transition from a regime and (v) determine the short and long run forecast variance

**2.0 Literature Review**

Markov-Switching Vector Autoregressive Models have been applied to very complex multivariate system. The link between exchange rate and agricultural performance has been widely examined, with specific focus on the exchange rate as significant driver of agricultural output with varying directions. This literature review under theoretical, conceptual and empirical literature which emphasise past studies on exchange rate, and it’s influence on agricultural performance giving a lead way to understanding the dynamic interconnections.

**2.1 Theoretical Framework**

The Macroeconomic theory of agricultural development is the theoretical basis for this research study. The theory maintains that stable macroeconomic circumstances are needed for sustained agricultural expansion (Schultz, 1964). This theory highlights the necessity of macroeconomic policies that limit volatility and provide a suitable climate for agriculture investment and production. According to this theory, important macroeconomic factors such as the Naira exchange rate play a vital role in determining agricultural performance by affecting the cost of inputs and market stability (Johnston and Mellor, 1961).

**2.2 Conceptual Framework**

Exchange rate Exchange rate is the price of one country’s currency expressed in terms of some other currencies. Agricultural performance is viewed in terms of its contribution gross Domestic Product. The conceptual foundation for this research focuses on the connection between exchange rate and agricultural performance. It offers that high exchange rate though promotes the competitiveness of exports, hence boost investment in agriculture, it increases the cost of imported agricultural inputs thus reducing investment in Agriculture. A moderate and stable exchange rate is consequently necessary for improved agricultural production,

**2.3 Empirical Literature**

**Markov-Switching Models**

Tuaneh, and Essi (2021) carried out a study on Markov-Switching Vector Autoregressive Modelling (Intercept Adjusted); Application to International Trade and Macroeconomic Stability in Nigeria (2000M1-2019M6) The study modelled and estimated the interdependence existing among Nigeria's International Trade and Macroeconomic Stability, Time series data from January 2000 to June 2019 were sourced from the Statistical Bulletin of the Central Bank of Nigeria. The Markov Switching Intercept Vector Autoregressive (MSI VAR) Model whos used for the analysis, The result showedall variables were I(1). VAR lag length selection criteria choose lag 2. The MS VAR analysis identified two regimes (expansion and contraction), the information criteria selected the Markov-Switching Intercept Autoregressive Heteroschedastic 2 Variance Auto-regression 2 [MSIARH (2)-VAR (2)]. The MS-VAR results showed significant interaction among the macroeconomic variable. However, the variables were also self-explanatory.

Tuaneh, Essi and Etuk (2021) studied Markov-Switching Mean Vector Autoregressive (MSM-VAR) Modelling: Application to Macroeconomic data. The study modelled and estimated the interdependence between macroeconomic variables particularly international trade and exchange rate within the context of the Nigerian economy. The study apart from also investigating the interdependence existing among the study variables, also determined the probabilities of transition from one regime to another and the duration of stay in the regimes. Time series data for 246 months spanning through January 2000 -June 2020 was sourced from the Statistical Bulletin of the Central Bank of Nigeria. The study used the MSM VAR in the Analysis. The results showed that all variables were stationary at first difference. The study chose 2 regimes, and the model selection criteria selected [MS (2)-MVAR (2)]. The result showed relative significance of each random innovation, the variables were largely self-explanatory and very strongly exogenous,

**Exchange Rate and Agricultural Production**

The exchange rate is an important macroeconomic variable that effects agricultural output by changing the cost of imported inputs and the competitiveness of agricultural exports. Fluctuations in the currency rate may contribute to uncertainty and volatility in agricultural markets, influencing farmers' decision-making and investment (Aghion, Bacchetta, Rancière, & Rogoff, 2009). Studies have indicated that a stable exchange rate environment encourages agricultural expansion by decreasing the cost of imported equipment, fertilizers, and other important inputs (Dreger and Zhang, 2014).

Muftaudeen and Abdullahi (2014) Studied the impact of macroeconomic policies on agricultural output between 1978-2011. The study used the Multivariate Vector Error Correction Model to examine both short run and long run relationships. The findings showed a cointegrating relationship among exchange rates and agricultural output. The findings also showed a long run response of agricultural output to changes exchange

In the Nigerian context, the currency rate has been very volatile, partly owing to swings in global oil prices and different macroeconomic policies. This volatility has had conflicting consequences on agricultural productivity. For instance, periods of currency depreciation have made agricultural exports more competitive but have also raised the cost of imported inputs, so reducing productivity (Udoh andAkpan, 2007). Conversely, periods of relative currency rate stability have been linked with better agricultural production owing to reduced input prices and greater market predictability (Olomola and Akinnagbe, 2014).

Interest rates, another crucial macroeconomic variable, greatly effect agricultural productivity by influencing the cost of borrowing and investment in the agricultural industry. Higher interest rates raise the cost of borrowing, deterring farmers from seeking loans to fund their operations and invest in productivity-enhancing technology (Bolarinwa and Fakoya, 2011). Conversely, lower interest rates decrease the cost of borrowing, promoting investment in agricultural operations and allowing the adoption of new farming practices.

Research has shown the susceptibility of agricultural productivity to fluctuations in interest rates, especially in emerging countries like Nigeria. Bolarinwa and Fakoya (2011) showed that high-interest rates have been a key hindrance to agricultural development in Nigeria, restricting farmers' access to cheap finance and diminishing their ability to invest in inputs and technology. Similarly, Ajetomobi and Binuomote (2006) proved that cutting interest rates might dramatically enhance agricultural production by expanding farmers' access to financing and allowing them to engage in yield-improving inputs and techniques.

**3.0 Methods**

## 3.1 Data

The research uses time series data. Specifically, quarterly time series data on exchange rate, and Agricultural Sector Gross Domestic Product spanning from first quarter, 2000 to fourth quarter 2022. The data was obtained from the Central Bank of Nigeria (CBN) Statistical Bulletin.

## 3.2 Variables

The variables of the study are:

The foreign exchange rates (Naira/Dollar) and the Agricultural Performance (Agricultural Sector Gross Domestic Product)

## 3.3 Methods of Data Analysis

The used the Markov-switching Vector Autoregressive (MSVAR) Model. The MSVAR model is a non-linear modelling technique. However, pre-diagnosis and post diagnostic test were conducted.

#### 3.3.1 Pre-estimation Tests

Pre-diagnostic tests were necessary to inquire about certain features of the data.by subjecting the data on the variables to scientific investigation. These include

1. **Unit Root Test**

When the statistical properties (mean, Standard deviation, variance, autocorrelation, etc.) of a time series variable does not change over time, the time series variable is said to be stationary. A test of significance requires all time series variables in the model to be stationary. One common property of time series data is non-stationarity (have unit root). According to Etuk (2012) “the assumption of non-stationarity is essential for the application of the Ordinary Least Squares Regression” This is because unit root can intensely distort the behaviour of the variable and consequently result to a spurious regression. The test for stationarity is therefore necessary to avoid spurious, biased and inconsistent estimates which often lead to forecasting error. According to Granger (1987), a non-stationary time series is integrated of order d [I(d)] if it is stationarity after being differenced d times. (Gujarati, 2013) identified; Augmented Dicky Fuller, the Philips-Peron test, and the graphical method (the correlogram) as methods of unit root test. The study, however, adopted the; the Augmented Dickey-Fuller (ADF) tests and the Philips-Peron (PP) unit root test.

**(a) Augmented Dickey and Fuller ADF Unit Root Test**

ADF test is a conventional unit root test method proposed by Dickey and Fuller (1981). It utilizes t-statistic to test the null hypothesis of a unit against the alternative hypothesis of no unit root.

$Z\_{t}=δ\_{1}Z\_{t-1 }+e\_{t }$ Pure random walk (no intercept, no trend) (3.1)

$Z\_{t}=δ\_{0}+ δ\_{1}Z\_{t-1 }+e\_{t }$ Random walk with drift (intercept, no trend) (3.2)

$Z\_{t}=δ\_{0}+ δ\_{1}Z\_{t-1 }+δ\_{2}t+e\_{t }$Random walk with drift (intercept and trend) (3.3)

The null hypothesis is bi = 1. However, if Zt-1 is subtracted from each side of (3.1) – (3.3)

$Z\_{t}- Z\_{t-1 }=δ\_{1}Z\_{t-1 }- Z\_{t-1 }+e\_{t }$ $\rightarrow $ $∆Z\_{t}=ωZ\_{t-1 }+e\_{t }$ (3.4)

$Z\_{t}- Z\_{t-1 }=δ\_{0}+δ\_{1}Z\_{t-1 }- Z\_{t-1 }+e\_{t }$ $\rightarrow ∆Z\_{t}=δ\_{0}+ωZ\_{t-1 }+e\_{t }$ (3.5)

$Z\_{t}- Z\_{t-1 }=δ\_{0}+δ\_{1}Z\_{t-1 }- Z\_{t-1 }+δ\_{2}T+e\_{t }$ $\rightarrow ∆Z\_{t}=δ\_{0}+ωZ\_{t-1 }+δ\_{2}T+e\_{t}$ (3.6)

Where; $ω= δ\_{1}-1$

The test is conducted with the conventional t-test statistics, the null hypothesis (H0: $ ω=0$)

From the above equation, Granger noted that the presence of serial correlation would render the equation meaningless. He suggested that the lags of ∆Pt's should be used to remove the serial correlation. This augmented test is referred as the Augmented Dickey-Fuller test (ADF).

$∆Z\_{t}=ωZ\_{t-1 }+\sum\_{i-1}^{q}α\_{i}∆Z\_{t-i }+e\_{t }$Pure random walk (3.7)

H0: $ ω=0$

$∆P\_{t}=δ\_{0}+ωZ\_{t-1 }+\sum\_{i-1}^{q}α\_{i}∆Z\_{t-i }+e$ Radom walk with drift (3.8)

H0: $ ω= 0$

$∆P\_{t}=δ\_{0}+ωZ\_{t-1 }+\sum\_{i-1}^{q}α\_{i}∆Z\_{t-i }+b\_{2}t+u\_{t}$ Random walk with drift (3.9)

H0: $ ω=0$

Where;

∆ = First difference operator

Zt = Time series variable under investigation

$ω , δ\_{0}, α\_{i}$ and $b\_{2}$ = parameters estimate of the variables

q = optimal lag length

et = Stochastic term

The presence of non-stationarity is an indication that the variable concerned has a unit root. This shall prompt the testing for difference stationarity process.

1. **The Phillips-Perron Test of Unit Root**

The Phillips-Perron unit root tests named after Peter C. B. Phillips and Pierre Perron differs from the Augmented Dickey-Fuller tests mostly because it corrects serial correlation and heteroscedasticity in the residual of the test regression by modifying the test statistics.

1. **Cointegration Test**

The presence of long-run equilibrium relationship among the study variables is ascertained using the cointegration test. The Johansen and Juselius (1990) procedures were adopted, The null hypothesis is that there is no cointegration and the alternative hypothesis is that the series is cointegrated series at a 5 per cent level. The general specification of the Johansen-Juselius cointegration model is presented below:

$γ\_{trace}\left(r\right)=-T\sum\_{i=r+1}^{n}ln\left(1-\hat{γ}\_{i}\right)$ (3.10)

$γ\_{trace}\left(r,r+1\right)=-Tln\left(1-\hat{γ}\_{i}+1\right)$ (3.11)

Where;

r = Number of cointegrating vectors (under the null hypothesis)

$γ\_{i}$ = The ith ordered Eigen value estimated from the Π matrix or the estimated

value of the characteristic roots

T = Number of observations.

λmax = The Maximum Eigen Statistics Joint test

HO: Number of co-integrating vectors ≤ r,

HA: Number of co-integrating vectors > r.

$γ$trace = This is a separate tests on each Eigen value. The trace statistic is applied to test the null hypothesis that the number of distinct co-integrating vectors is equal to or less than r.

Johansen and Juselius (1990) made available table of critical values for the Maximum Eigen statistics and the Trace Statistics. Test statistics > the critical region from the table means rejecting Ho. Evidence of at least one cointegrating vector at 5 per cent indicating that the underlying time series have a long-run relationship.

**Lag Length Selection Criteria**

It is necessary to scientifically select the lag length understanding that too many lags may result in over fitting and consequently, the increase means square forecast error (MSFE) of the VAR model while too few lags may result in auto-correlated errors whereas. The lag length selection used the information criteria to minimize these errors.

AIC = $Ln|Σ\_{i}|+\frac{2}{T}MK^{2}$ (3.12)

HQ = $Ln|Σ\_{i}|+\frac{2lnT}{T}MK^{2}$ (3.13)

SC = $Ln|Σ\_{i}|+\frac{lnT}{T}MK^{2}$ (3.14) Where;

AIC = Akaike Information Criteria

HQ = Hannan Quinn Information Criteria

SC = Swatch Information Criteria

M = number of parameters including the constant

T = Number of observations

$Ln|Σ\_{i}|$ = Natural log of the determinant of the covariance of the matrix of the

residual of the restricted system

$MK^{2}$ = Number of the parameter to be estimated in the VAR model with order M

**3.4 Model specification**

The general form of a standard VAR model is;

$Yt=ψ\_{i}+ \sum\_{i=1}^{p}δiYt-i+ εt$ (3.15)

Let us consider a vector of n variables Yt that follow a VAR model of order 1, let us also consider that the parameters are subject to unobservable regime shift. The shift results from a latent Markov chain St likely to be in one out of k possible regimes

$Yt=ψ\_{i}\left(s\_{t}\right)+ \sum\_{i=1}^{p}δi\left(s\_{t}\right)Yt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑\left(s\_{t}\right)$) (3.16)

Where; St = 1,2,…, k possible regime and in period T

$Y\_{t}=(Y\_{1t}, Y\_{2t}, ... Y\_{kt}, )$ is KX1 vector of endogenous variable,

$ψ\_{i}$ = KX1 vector of intercept

$ψi$ = KXK matrix of lagged coefficient

To allow for regime change so that Yt follows a VAR procedure which is dependent on unobservable (st) regime, Krolzig 1997, modified the VAR model to; Markov Switching Intercept Vector Autoregressive (MSI-VAR) Models and Markov Switching mean Vector Autoregressive (MSM-VAR). The shift in intercept according to Krolzig 1997 results to smooth adjustment of the time series.

The eight (8) basic classes of Markov Switching Intercept Vector Autoregressive (MSI-VAR) models noted earlier are presented in equation **3.17-3.24** and tabulated in Table 1 below.

MSI(k)-VAR(p):$Yt=ψ\_{0}\left(s\_{t}\right)+ \sum\_{i=1}^{p}δiYt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑$) (3.17)

MSIAR(k)-VAR(p): $Yt=ψ\_{0}\left(s\_{t}\right)+ \sum\_{i=1}^{p}δi\left(s\_{t}\right)Yt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑$) (3.19)

MSIH(k)-VAR(p): $Yt=ψ\_{0}\left(s\_{t}\right)+ \sum\_{i=1}^{p}δiYt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑\left(s\_{t}\right)$) (3.18)

MSIARH(k)-VAR(p): $Yt=ψ\_{0}\left(s\_{t}\right)+ \sum\_{i=1}^{p}δi\left(s\_{t}\right)Yt-i+ εt$ where: ~N(0,$ ∑\left(s\_{t}\right)$) (3.20)

MSAR(k)-VAR(p): $Yt=ψ\_{0}+ \sum\_{i=1}^{p}δi\left(s\_{t}\right)Yt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑$) (3.22)

MSH(k)-VAR(p): $Yt=ψ\_{0}+ \sum\_{i=1}^{p}δiYt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑\left(s\_{t}\right)$) (3.21)

MSARH(k)-VAR(p): $Yt=ψ\_{0}\left(s\_{t}\right)+ \sum\_{i=1}^{p}\left(s\_{t}\right)δiYt-i+ εt$ where: ~N(0,$ ∑\left(s\_{t}\right)$) (3.23)

Linear-(k)-VAR $Yt=ψ+ \sum\_{i=1}^{p}δiYt-i+ εt$ where: $εt$ ~iid, N(0,$ ∑$) (3.24)

Where (k) = Number of regimes, (p) = Number of lags, AR = Autoregressive parameters

H ($∑)$=Heteroschedastic parameters, Nhence;

MSI(k) –VAR(p) = Markov Switching Intercept Vector Autoregressive Model

MSIAR(k) –VAR(p) = Markov Switching Intercept Autoregressive Vector Autoregressive Model

MSIH(k) –VAR(p) = Markov Switching Intercept Heteroschedastic Vector Autoregressive Model

MSIARH(k) –VAR(p) = Markov Switching Intercept Autoregressive Heteroschedastic Model Vector Autoregressive

MS(k) –VAR(p) = Markov Switching Vector Autoregressive Model

MSAR(k) –VAR(p) = Markov Switching Autoregressive Vector Autoregressive Model

MSH(k) –VAR(p) = Markov Switching Heteroschedastic Vector Autoregressive Model

MSARH(k) –VAR(p) = Markov Switching Autoregressive Heteroschedastic Vector Autoregressive Model

**Table 1. Special Cases of Markov Switching Vector Autoregressive Models**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **S/n** | **Markov Switching Intercept (MSI-VAR) models**  | **I** | **AR** | **∑** |
| 1 | MSI(k)-VAR(p) | * $√$
 | * $X$
 | * $X$
 |
| 2 | MSIAR(k)-VAR(p) | * $√$
 | * $√$
 | * $X$
 |
| 3 | MSIH(k)-VAR(p) | * $√$
 | $$X$$ | * $√$
 |
| 4 | MSIARH(k)-VAR(p) | * $√$
 | * $√$
 | * $√$
 |
| 5 | MSAR(k)-VAR(p) | * $X$
 | * $√$
 | * $X$
 |
| 6 | MSH(k)-VAR(p) | * $X$
 | * $X$
 | * $√$
 |
| 7 | MSARH(k)-VAR(p) | * $X$
 | * $√$
 | * $√$
 |
| 8 | LINEAR-(k)-VAR(p) | * $X$
 | * $X$
 | * $X$
 |
| **S/n** | **Markov Switching Mean (MSM-VAR) Models**  | $$μ$$ | **AR** | **∑** |
| 9 | MSM(k)-VAR(p) | * $√$
 | * $X$
 | * $X$
 |
| 10 | MSMAR(k)-VAR(p) | * $√$
 | * $√$
 | * $X$
 |
| 11 | MSMH(k)-VAR(p) | * $√$
 | $$X$$ | * $√$
 |
| 12 | MSMARH(k)-VAR(p) | * $√$
 | * $√$
 | * $√$
 |
| 13 | MSAR(k)-MVAR(p) | * $X$
 | * $√$
 | * $X$
 |
| 14 | MSH(k)-MVAR(p) | * $X$
 | * $X$
 | * $√$
 |
| 15 | MSARH(k)-MVAR(p) | * $X$
 | * $√$
 | * $√$
 |
| 16 | LINEARM(k)-VAR(p) | * $X$
 | * $X$
 | * $X$
 |

*Source: Krulzig, (1998), Guidoli (2012)*

*Where: (r) = Number of regime, (p) = number of lags, AR = Autoregressive Parameter, H=(∑) = Variance (Heteroschedastic parameter), I = Intercept,* $μ$ *= Mean,* $√$ *= Varying,*$X$ *= Invariant*

#### The Markov Switching Mean Vector Autoregressive (MSMVAR) Model:

MSM: $Yt-μ\_{t}\left(st\right)=ψ1\left(s\_{t}\right)(Yt-1-μ\_{t}(s\_{t-1})+…+ψp\left(s\_{t}\right)(Yt-p-μ\_{t}(s\_{t-p}) εt$ **(3.25)**

Where;

$εt \~iid \left(0, ∑\left(s\_{t}\right)\right) and μ\_{t}\left(st\right)$, $ψ1\left(s\_{t}\right),…ψp\left(s\_{t}\right)$, $∑\left(s\_{t}\right)$ are regime dependent parameters to be estimated.

 $μ\_{1}\left(st\right)$ if st = 1

**.**$μ\_{t}\left(st\right)= $

 $μ\_{m}\left(st\right)$ if st = m

**The Markov-Switching Intercept Vector Autoregressive Model of Exchange Rate, and Agricultural Performance with 2 Regimes and 3 Lags (MSI(2) VAR(3)**

The models stated in this section will be used to evaluate the interdependence between the study variables

$st\left(s\_{t-1}\right)\left[\begin{matrix}EXR\\AGP\end{matrix}\right]=\left[\begin{matrix}δ\_{1111}&δ\_{1121}\\δ\_{1211}&δ\_{1221}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-1}\\AGP\_{t-1}\end{matrix}\right]+\left[\begin{matrix}δ\_{1112}&δ\_{1122}\\δ\_{1212}&δ\_{1222}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-2}\\AGP\_{t-2}\end{matrix}\right]$

$$ +\left[\begin{matrix}δ\_{1113}&δ\_{1123}\\δ\_{1213}&δ\_{1223}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-3}\\AGP\_{t-3}\end{matrix}\right]+\left(s\_{t}\right)+\left[\begin{matrix}ϵ\_{11}\\ϵ\_{21}\end{matrix}\right]+st(εtεt^{I})$$

$Y\_{t}$,

 $t\left(s\_{t-1}\right)\left[\begin{matrix}EXR\\AGP\end{matrix}\right]=\left[\begin{matrix}δ\_{2111}&δ\_{2121}\\δ\_{2211}&δ\_{2221}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-1}\\AGP\_{t-1}\end{matrix}\right]+\left[\begin{matrix}δ\_{2112}&δ\_{2122}\\δ\_{2212}&δ\_{2222}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-2}\\AGP\_{t-2}\end{matrix}\right]$

 $+\left[\begin{matrix}δ\_{2113}&δ\_{2123}\\δ\_{2213}&δ\_{2223}\end{matrix}\right]\left[\begin{matrix}EXR\_{t-3}\\AGP\_{t-3}\end{matrix}\right]+\left(s\_{t}\right)+\left[\begin{matrix}ϵ\_{11}\\ϵ\_{21}\end{matrix}\right]+st(εtεt^{I})$ (3.25)

The order of numbering for the autoregressive coefficient is: Regime, Model, Variable, and Lag. The order of numbering for the intercept is; Model, Regime. Such that $δ\_{1,212}$ is autoregressive coefficient for regime 1, model 2, variable 1. And lag 2. Also, $δ\_{2121}$ is autoregressive coefficient for regime 2, model 1, variable 2, lag 1. A MSIARH (2)-VAR (3) model requires the estimation of a 2x2 matrix for each lag and each regime (24 autoregressive parameters), a column of 2 intercept terms for each regime (4 parameters), a matrix of 2 variances and 1 co-variance for each regime (6 parameters), and 2 independent transition probabilities, given a total of 36 parameters.

Note that the regime is unobservable, therefore, the necessity to form the probability inferences of its values, and the equivalent inferences concerning parameter values in equation ….

The assumption is that the state variable is governed by the Markov chain:

$$P\left(s\_{t}=1|s\_{t-1}=1= P^{11}\right)$$

$$P\left(s\_{t}=2|s\_{t-1}=1= P^{12}\right)$$

$$P\left(s\_{t}=1|s\_{t-1}=2= P^{21}\right)$$

$$P\left(s\_{t}=2|s\_{t-1}=2= P^{22}\right)$$

The matrix form is often presented as:

Pij = $\left[\begin{matrix}P\_{11}&P\_{12}\\P\_{21}&P\_{22}\end{matrix}\right]$ The transition probabilities is restricted to ensure that P11 + P12 = 1 while P21 + P22 = 1 and the expected duration is derived by $\left(\frac{1}{1-P\_{ij}}\right)$ Therefore P11 is the probability of stay in Regime 1 and the duration of stay is $\left(\frac{1}{1-P\_{11}}\right)$. Similarly, P22 is the probability of stay in Regime 2 and the duration of stay is $\left(\frac{1}{1-P\_{22}}\right)$

**4.0 Results and Discussions**

The descriptive statistics showed that the mean exchange rate within the period of the study was Two Hundred and Thirty Naira per Dollar, (N240) with a standard deviation of Two Hundred and Four Naira per Dollar (N209). Agricultural Performance on the other hand showed an average of Three Billion Three Hundred and Thirty-One Million Naira with a standard deviation of One Billion Two Hundred and Twenty-Four Million Naira within the period of the study

**4.1 Pre-Diagnostic Tests**

**4.11 Unit Root and Cointegration Test**

The unit root test was conducted using the Augmented Dickey Fuller Statistics and the result showed stationarity at first difference I(1). This was confirmed using the Philips-Perron Unit Root test. The Johannsen Con-integrstion test was consequently conducted and no coingration was found between the two variables hence the use of VAR model

## 4.12: Number of Regime and Lag length selection

**Table 2: Testing for the Number of Unobservable State/Regimes**

Since $S\_{t}$= 1, 2, 3, … , N. $N\geq 2$

|  |  |  |
| --- | --- | --- |
| **Regimes** | **AIC** | **SC** |
| 2 | 18.212 | 18.995 |
| 3 | 20.900 | 22.150 |
| 4 | 24.202 | 21.445 |

The information criteria in Table 2 is used to select the number of regimes. The study used the Akaike information criterion (AIC) and Schwarz information criterion (SIC). The result indicated that 2 Regime has the minimum information criteria with an AIC of 18.212 and SIC of 18.995 therefore 2 regimes where chosen based on the least information criteria. The regimes are identified as expansion (regime 1) and contraction (regime 2). Tuaneh and Essi (2021) in their study on Markov-Switching Vector Autoregressive Modelling (Intercept Adjusted); Application to International Trade and Macroeconomic Stability in Nigeria (2000–2019) choose 2 regimes. **Also, the Lag length selection chose lag 3 based on lag selection criteria.**

## 4.2: Model Selection

Sixteen (16) models, 8 of the Markov Switching Intercept and 8 of Markov Switching Means were estimated. The information criteria results are summarized in Table 3 below.

Table 3: Markov Switching Vector Autoregressive Models Selection

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **S/n** | **Estimated Models**  | **Log-likelihood** | **Akaike info criterion** | **Schwarz criterion** | **Number of coefficients** |
|  | **Markov Switching Intercept (MSI-VAR) Models** |  |  |  |  |
| 1 | MSI(2)-VAR(3) | -1017.71 | 22.10 | 22.10 | 21 |
| 2 | MSIAR(2)-VAR(3) | -921.51 | 20.30 | 21.20 | 33 |
| 3 | MSIH(2)-VAR(3) | -890.22 | 19.45 | 20.10 | 24 |
| **4** | **MSIARH(2)-VAR(3)** | **-854.65** | **18.95** | **19.92** | **36** |
| 5 | MSAR(2)-VAR(3) | -937.52 | 20.61 | 21.45 | 31 |
| 6 | MSH(2)-VAR(3) | -895.07 | 19.51 | 20.10 | 22 |
| 7 | MSARH(2)-VAR(3) | -863.98 | 19.11 | 20.02 | 34 |
| 8 | LINEAR-(2)-VAR(3) | -1024.20 | 22.22 | 22.75 | 20 |
|  | **Markov Switching Mean (MSM-VAR) Models**  |  |  |  |  |
| 9 | MSM(2)-VAR(3) | -1025.22 | 22.26 | 22.82 | 21 |
| 10 | MSMAR(2)-VAR(3) | -941.51 | 21.10 | 22.20 | 33 |
| 11 | MSMH(2)-VAR(3) | -891.38 | 19.48 | 20.12 | 24 |
| 12 | MSMARH(2)-VAR(3) | -871.32 |  20.14 | 21.12 | 36 |
| 13 | MSAR(2)-MVAR(3) | -937.52 | 20.61 | 21.52 | 31 |
| 14 | MSH(2)-MVAR(3) | -885.71 | 19.84 | 20.93 | 22 |
| 15 | MSARH(2)-MVAR(3) | -860.66 | 19.03 | 19.95 | 34 |
| 16 | LINEARM(2)-VAR(3) | -1025.22 | 22.23 | 22.78 | 20 |

Source: Researchers' Computation with E-views 11.0

### The Markov-Switching Models (Markov-Switching intercept and the Markov Switching Mean Vector Autoregressive Models) permit a variety of specifications including intercept switching, means switching, autoregressive parameter switching and heteroscedastic parameter switching leading to the 16 classes of the Markov-Switching Vector Autoregression.

### These 16 types of Markov Switching Vector Autoregressive Model were consequently estimated and summarized in Table 3. The Markov Switching Intercept Autoregressive Heteroskedastic (2) Vector Auto regressive model (3) model had the highest log likelihood (-860.02) and the least information criteria (Akaike Information Criteria = 19.01, Schwarz Information Criteria = 19.07), accordingly, the MSARH(2)-MVAR(3) with 36 parameters was selected.

### 4.3: Estimation of Model Parameters

### Table 4: Estimation of Model Parameters of the selected Model. Markov-Switching Intercept Vector Autoregressive Model Regimes 2 Lags 3 (MS-ARH(2)-VAR(3))

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variables | Regime 1 |   |   | Regime 2 |
| DEXR | DAP |   |   | DEXR | DAP |
| DEXR(-1) |  0.001 | -0.887 |  | DEXR(-1) | -0.019 | -0.311 |
|  | [ 1.150] | [-16.466] |  |  | [-0.823] | [-1.919 |
|  |  |  |  |  |  |  |
| DEXR(-2) |  0.0001 | -1.002 |  | DEXR(-2) | -0.015 | -1.010 |
|  | [ 0.675] | [-81.333] |  |  | [-2.312] | [-20.365 |
|  |  |  |  |  |  |  |
| DEXR(-3) |  0.001 | -0.916 |  | DEXR(-3) | -0.009 | -0.384 |
|  | [ 1.204] | [-16.67] |  |  | [-0.359] | [-2.027] |
|  |  |  |  |  |  |  |
| DAP(-1) | -0.005 |  0.035 |  | DAP(-1) |  2.265 | -1.364 |
|  | [-0.492] | [ 0.055] |  |  | [ 14.175] | [-1.136] |
|  |  |  |  |  |  |  |
| DAP(-2) |  0.003 |  0.112 |  | DAP(-2) | -1.750 |  1.413 |
|  | [ 0.318] | [ 0.180] |  |  | [-2.213] | [ 0.239] |
|  |  |  |  |  |  |  |
| DAP(-3) | -0.092 |  118.109 |  | DAP(-3) |  14.257 |  170.346 |
|  | [-0.565] | [ 11.91] |  |  | [ 2.234] | [ 3.543] |
|  |  |  |  |  |  |  |
| C |  0.135 | -0.033 |  | C |  0.225 | -0.615 |
|   | [ 23.837] | [-0.093] |   |   | [ 1.241] | [-0.451] |
| **Heteroschedastic parameters** |  |  |  |
| SIGMA-DEXR |  1.11 | -7.48 |  | SIGMA-DEXR |  725.51 | -911.72 |
|  | [ 4.997] | [-0.798] |  |  | [ 4.004 | [-0.938] |
|  |  |  |  |  |  |  |
| SIGMA-DAP | -7.48 |  4307.27 |  | SIGMA-DAP | -911.72 |  41097.10 |
|   | [-0.798] | [ 3.907] |   |   | [-0.938] | [ 3.965] |

**Source: Researcher’s computation with Eviews 13.0**

**Regime 1 Variance Covariance Matrix Regime 1 Variance Covariance Matrix**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **SIGMA** | **DEXR** | **DAP** |  | **SIGMA** | **DEXR** | **DAP** |
| **DEXR** | 1.11 | -7.48 |  | **DEXR** | 725.54 | -0.91 |
| **DAP** | -7.48 | 4307.27 |  | **DAP** | -0.91 | 41097.10 |

**Regime 1:** The regime 1 of the Markov Switching Intercept Autoregressive Heteroschedastic Vector Autoregressive Model presented in Table 4 showed that; the lags of agricultural performance and the lags of exchange rate had no significant effect on exchange rate. However, lags 1, 2, and 3 of exchange rate had significant negative effects on agricultural performance while the lags of agricultural performance had significant effects on agricultural performance. This agrees with the findings of Tuaneh and Essi 2021.

**Regime 2:** The regime 2 of the Markov Switching Intercept Autoregressive Heteroschedastic Vector Autoregressive Model shown in Table 4 indicated that; exchange rate lag 2 and agricultural performance lags 1, 2, and 3 had significant effects on exchange rate. On the other hand, all lags of exchange rate had significant negative effects on agricultural performance while only lag 3 of agricultural performance had significant positive effect on agricultural performance. This finding agrees with the study of Rano (2008) found that a negative relationship exist between real exchange rate and agricultural export in Nigeria. The findings also agree with the Yanikkaya et al (2013) who studied the effect of real exchange rates and their volatility on the selected agricultural commodity exports and concluded that an indirect relation ship exist between the.

**Transition probabilities**

The Markov transition probabilities is presented below. Note that P(i, k) = P(s(t)) = k | s (t-1 = i). Where row = i and column = k

$P\_{ij}$= $\left[\begin{matrix}P\_{11}&P\_{12}\\P\_{21}&P\_{22}\end{matrix}\right]$ = $\left[\begin{matrix}0.558&0.442\\0.178&0.822\end{matrix}\right]$

Hence P11 = 0.0.558, and P22 = 2.178. Where; $P\_{11}$+$P\_{12}$=1, $P\_{21}$+$P\_{22}$=1

The result implied that; the probability of transitioning from expansion in a current state to expansion in the next period is 0.558, and the probability of transitioning from expansion in the current state to contraction in the next period is 0.442. Also, the probability of transitioning from contraction in a current state to expansion in the next period is 0.178 and the probability of transitioning from contraction to contraction in the next period is 0.822.

**Expected Duration of Stay**

The expected duration derived by $\left(\frac{1}{1-P\_{ij}}\right)$ is 2.26 in regime 1 and 5.62 in regime 2. This result implied that there is 55.8% probability of staying in regime 1 for a duration of 7 months month. Also, there is 88.2% probability of staying in regime 2 for a duration of 17 months.

**Table 5: Variance Decomposition Table (MSARH(2)-MVAR(2)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Period** | **Regime 2** |  | **Regime 2** |
| **S.E.** | **DERX** | **DAP** |  | **S.E.** | **DERX** | **DAP** |
| **Variance Decomposition of DEXR:** |  |  |  |  |
|  1 |  1.055 | 100.00 | 0.000 |  | 26.93 | 100.00 | 0.000 |
|  2 | 1.055 | 91.29 | 8.70 |  | 460.06 | 3.15 | 96.84 |
|  3 | 1.122 | 88.50 | 11.4 |  | 1088.96 | 2.84 | 97.64 |
| . | . | . |  |  | . | . | . |
| . | . | . |  |  | . | . | . |
| **8** | **4680.48** | **1.21** | **98.78** |  | **10161** | **2.78** | **97.15** |
| **Variance Decomposition of DAP:** |  |  |  |  |
|  1 | 65.62 | 1.16 | 98.83 |  | 202.72 | 2.78 | 97.21 |
|  2 | 65.68 | 1.19 | 98.80 |  | 341.83 | 2.20 | 97.79 |
|  3 | 66.16 | 2.26 | 98.73 |  | 625.36 | 3.35 | 96.64 |
| ‘ | . | . |  |  | . | . | . |
| ‘ | . | . |  |  | . | . | . |
| **8** | **92.14** | **1.22** | **98.77** |  | **23210** | **2.46** | **97.53** |
| Cholesky Ordering: DEXR DAP |

Source: Researcher’s computation with Eviews 13.0.

### Variance Decomposition of the Selected Markov Switching VAR Model Result

### Variance Decomposition of Exchange Rate [MSI (2) –VAR (3)]

**Regime 1**: The percentage of the forecast error variance in regime 1 as tabulated in Table 5 showed that in the short run, 100% forecast variance in exchange rate was self-explained and Agricultural performance was strongly exogenous. Subsequently, exchange rate decreases while Agricultural performance increased and the percentage forecast variance of exchange rate was 1.22% in the long run while agricultural performance, in the long run, was 98.78%.

**Regime 2**: The percentage of the forecast error variance in regime 2 in the short run as shown in Table 5 was 100%. The forecast variance in exchange rate was consequently self-explained. Agricultural performance was strongly exogenous. Afterward, the forecast variance of exchange rate decreases while Agricultural performance increased. The percentage forecast variance of exchange rate was 2.78% in the long run while agricultural performance, in the long run, was 97.15%.

### Variance Decomposition of Agricultural Performance [MSI (2) –VAR (3)]

**Regime 1**: The percentage of the forecast error variance in regime 1 as also tabulated in Table 5 showed that in the short run, 98.83% forecast variance in agricultural performance was self-explained. Exchange rate was strongly exogenous. Moving forward, there was no substantial variation in forecast variance of agricultural performance and exchange rate. The percentage forecast variance of exchange rate was 1.22% in the long run while agricultural performance, in the long run, was 98.77%.

**Regime 2**: The percentage of the forecast error variance in regime 1 in the short run as shown in Table 5 for Agricultural performance was 97.21%. The forecast variance in Agricultural performance was therefore self-explained. exchange rate was strongly exogenous. Afterward, there was no substantial change in the forecast variance of Agricultural performance and exchange rate. The percentage forecast variance of exchange rate was 2.46% in the long run while agricultural performance, in the long run, was 97.53%.

## 5.0 Conclusion and Recommendations

This empirical analysis has shown varying significant and negative effects of exchange rate fluctuation on agricultural performance across different regimes, suggesting that periods of economic expansion and contraction exhibit distinct behavioural patterns. Particularly, the effects of exchange rate on agricultural performance were more in regime one than in regime 2. More so, both exchange rate on agricultural performance were self-explained, however, agricultural performance was exogenous and forecast variance depended more on exchange rate.

The findings of the study underscore the necessity of policies which respond to different economic conditions, predominantly considering Nigeria's unstable exchange rates. The result has shown the need for innovative statistical methodologies that do not only capture the complexities of economic relationships but also capture unobservable states, its transmission and duration of stay. Additionally, this study has provided valuable insights for policymakers who seek to enhance agricultural productivity and resilience in the face of exchange rate volatility. It is thus recommended that endogeneity, unobservable states and number of the regimes should be ascertained as pre-diagnostic tests for multivariate analysis.

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