***Original Research Article***

Derivation of z-component nonlinear force under wave-particle interaction process in Plasma medium

**Abstract**

On the basis of wave-particle interaction process, instabilities in open and closed plasma medium can be explained. In our work we investigate some features of plasma instabilities in ionosphere on the basis of nonlinear wave-particle interaction process. The accelerated plasma particles may transfer their energy to high frequency plasma waves through modulated waves. It can be seen that a plasma wave which is not in phase relation with ionospheric plasma particles may be amplified at the expense resonant mode energy in plasma medium through modulated field. In this paper we present expressions of nonlinear force due to wave-particle interaction process in plasma under specific conditions.

*Keywords: Plasma medium, Wave-particle interaction, Nonlinear force*

**Introduction**

A partially or fully ionised system is composed of several separate systems- for electrons, ions and neutral particles. To describe phase of such multi-species system in a space, knowledge of position and velocity of each of the particle is necessary. In this regard, a distribution function gives statistical information about the particles in each differential position and velocity range at a given time. To determine some properties of a system, nonlinear differential governing equations are necessary which specifies the phase-space distribution function for a particular species. But nature of inter-particles interaction determines types of governing equations to select in a specific problem. In kinetic theory approach, depends on collision interaction and without long-range interactions, either Boltzmann equation or Fokker-Planck equation are used in an ionised medium. But the Vlasov equation deals with charged particles with long-range interactions but neglecting collision interactions.

Plasma is an ionised, inhomogeneous and anisotropic medium in which waves and perturbations can propagate. These propagating plasma waves and perturbations can absorb or amplify by the plasma medium and generate instabilities. For the nonlinear wave energy exchange among plasma particle species and plasma waves may lead to exhibit different types of enhanced radiation emission phenomena in both space and laboratory plasma environments. Enhancement of nonresonant plasma wave phenomena are observed in space plasma by ground based and satellite based observatories. Plasma maser instability may be one of the possible mode to generate amplified high frequency electrostatic and electromagnetic plasma wave through wave-particle interaction process. At present theoretical investigations on probable wave energy upconversion of nonresonant wave through plasma maser effect [1] in ionospheric plasma are carried on . Though solar wind is the main source of free energy and momenta to this medium, magnetic field are also possible sources of energy of this open plasma system. The research works presented in different literatures [2,3,4,5,6,7,8] on nonlinear wave energy exchange through kinetic theory approach in different ionospheric altitudes after considering ionosphere as homogeneous and inhomogeneous medium in different problems. Gogoi and Deka [9,10,11] had derived probable growth rate expression of electrostatic and electromagnetic plasma waves incorporating different plasma conditions like density and magnetic field gradients and estimate its value by using observational data to testify validity and effectiveness of the plasma maser instability for wave energy upconversion phenomenon in the planetary ionosphere region.

In this paper we present expression of components of nonlinear force due to interaction of Langmuir wave with ion acoustic wave in presence of electrons in Plasma medium.

**Formulation of the Problem**

Consider a semi infinite bounded inhomogeneous plasma which is confined by a magnetic field. Let the plasma density decreases in the x-direction is balanced by a magnetic field that increases with x.

For this system, the electron distribution function is considered as  ****

 **** (1)

 **=** Density gradient

 **=** Electron cyclotron frequency

In this problem neglecting temperature variation and their anisotropy, the density and magnetic field with gradients are taken as

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The interaction of Langmuir wave with ion acoustic wave is generated by the Vlasov-Poison system of equations which are

 **** (2)

 **** (3)

The unperturbed distribution function for electrons, the unperturbed electric field and the unperturbed magnetic fields are taken as

 **** (4)

 ****

 

Where

 =Space and time averaged part of the distribution function

 and =Fluctuating parts due to low frequency ion acoustic turbulence

 = Ordering of the low-frequency ion acoustic wave turbulence field with propagation vector 

Now, equation (6) can be written as

  (5)

To the order of , it has been found

  (6)

Since the system is inhomogeneous in the x-direction, here we consider wave propagation only in the y-z plan.

 

 

  and  (7)

The unperturbed particle orbits [12,13] are found by iterating equations (7) in the small parameter  and these are





  (8)

Where



By integration equations (8),the particle orbits , under the boundary conditions  are

 

 (9)

Using Fourier transform and the method of characteristics [12], it has been found from equation (6) as



 

  (10)

The quasisteady state of the system is now perturb by the test longitudinal Langmuir wave field with wave vector .

The total perturbed electric field, magnetic field and the electron distribution function due to this perturbation are

 

 

  (11)

Where

 and  are modulating fields

 is the fluctuating part due to high frequency Langmuir wave

 and are particle distribution function corresponds to modulating fields

Let, the operator

 

Using equation (11) in Vlasov equation (2) for the perturbed state, it has been found



 (12)

To the order of , it has been found

  (13)

To the order of , it has been found

  (14)

To the order of and applying random phase approximation to omit second order quantities, it has been found

  (15)

Applying Fourier transform and integrating along the unperturbed orbit, we can evaluate the fluctuating part of the distribution function due to high frequency longitudinal Langmuir waveover the particle trajectories. Here

 

 

Using

 

And for weak gradient

 

After lengthy calculations it has been found

 

 (16)

Using Fourier transform and the method of characteristics to equation (14) it has been found



  (17)

Here, after lengthy calculations it has been found

 



 (18)



 

 

 (19)





 

 (20)

For the nonlinear interaction of low frequency resonant ion acoustic wave and high frequency nonresonant Langmuir wave present in the system a nonlinear modulation field is developed and from it a nonlinear force is generated which accelerate electrons .

From equation (15)



 (Say) (21)

Using Fourier transform and integrating along unperturbed orbits it has been found



 



 (22)

The nonlinear force acting on unit volume of particles can be written as









 (23)

Where  represents ensemble average.

The Z-component of this nonlinear force is

 (24)

Where

 (25)

And



 (26)

**Conclusion**

In presence of nonlinear force terms, nonlinear dispersion relation can have by using the methods of Chen [14].With the help of continuity equation, momentum equations and using quasi neutrality condition [14] it has been found dispersion relation from which the approximate growth rate of high frequency wave can be estimated from coupling terms. By using observational data from ground and space based observatories the approximate growth rate of high frequency wave can be evaluated in both homogeneous and inhomogeneous open plasma medium through kinetic theory approach and it may justify the effectiveness of plasma maser effect for amplification of high frequency plasma wave in open plasma system.

**Foot Note:**

The integral form of Bessel’s function is



For n=0



 

 

 

 

 

Using the property



And using reduction formula



Here,



  (neglecting higher order terms)



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