A MODEL STUDY OF THE IMPACT OF ENLIGHTENMENT RATE ON THE DYNAMICS OF TYPHOID FEVER

Abstract

In this study, a mathematical investigation of the effect of enlightenment rate on the spread of typhoid fever is considered, using a system of nonlinear first order ordinary differential equations and a MATLAB ODE45 numerical scheme. The result shows that a decrease in vaccination rate (θ) significantly increases the size of the susceptible (S) class and the response functions, R_0 or R_e thereby heightening the tendency for the disease to be endemic. If $R_0 > 1$ then the typhoid infection will spread throughout the population, but if $R_0 < 1$ then the infection will not be able to take hold and will eventually die out. Furthermore, it was shown that, the basic reproduction number has the tendency to reveal faster if a disease will result in an epidemic than the effective reproduction number. Finally, as enlightenment rates approach zero, it was observed that, over time, the disease will result in an epidemic. It is therefore recommended that enlightenment rate of the exposed, enlightenment rate to go for treatment and enlightenment rate to go for vaccination should be taken seriously by policy makers in order to stem the spread of typhoid.

Keywords: Typhoid fever, basic reproduction number, effective reproduction number, epidemic, mathematical model

Introduction

Mushayabasa et al. (2013) stated that, typhoid fever, also called enteric fever, is an infectiousdisease caused by a bacterium known as *Salmonella enteric serotype Typhi* also called *Salmonella typhi* (S.Typhi). It is an infection of the intestinal tract and bloodstream (Ivanoff et al., 1994). It is life-threatening and highly contagious; it can spread through the urine and faeces of an infected person (Nsutebu, 2003). Typhoid has an incubation period of about 10-14 days.

Symptoms of typhoid include high fever, headache, stomach pain, constipation or diarrhea. Intestinal perforation can result from a more complicated case, causing leakages of intestinal contents into the abdomen. An individual who eats food or drinks water contaminated with small amount of infected faeces or urine is likely to become infected and develop typhoid fever(NHS, 2024; Healthline, 2024).

The implementation of mathematical modelling helps researchers to concentrate on the process by which an infectious disease is transmitted in a given population. To understand different infectious diseases and their dynamical properties, researchers have proposed and developed several mathematical models (Butt et al., 2022; Butt et al., 2023). Musa et al. (2021) specificallystudied the dynamics of the transmission of typhoid fever using a mathematical model. They evaluated how public health education initiatives impact the pathogenesis of typhoid fever, which is likely to result in serious outbreaks, in areas with limited resources. Also, Adeboye et al. (2015)proposed and studied a mathematical model of typhoid and malaria co-infection to tackle the control of the transmission of malaria and typhoid simultaneously.

In this study, a mathematical model is considered to investigate the impact of enlightenment rate of the exposed class and, enlightenment rates to go for treatment and vaccination. The study also considers the basic and effective reproduction numbers to determine which of the two reproduction numbers gives early signal in the event of an epidemic occurring.

Materials and Methods

The following system describing the transmission dynamics of typhoid, as given by Atokolo and Omale (2018), is considered for this study:

$$\frac{dS}{dt} = \tau - \psi(1-x)BS + \alpha R - [\theta(1+z) + \mu]S \tag{1}$$

$$\frac{dE}{dt} = \psi(1-x)BS + \delta(1-\omega)BV - (\lambda + \mu)E \tag{2}$$

$$\frac{dI}{dt} = \lambda E - [\gamma(1+y) + (\xi + \mu)]I \tag{3}$$

$$\frac{I_T}{dt} = \gamma(1+y)I - (\phi + \zeta + \mu)I_T \tag{4}$$

$$\frac{dV}{dt} = \theta(1+z)S - [\delta(1-\omega)B - \mu]V \tag{5}$$

$$\frac{dR}{dt} = \phi I_T - (\alpha + \mu)R \tag{6}$$

$$\frac{dB}{dt} = \kappa I - \mu_1 B \tag{7}$$

In the above system, the total population under consideration at time t represented by $N(t)$ is divided into six classes of individuals: Susceptible$S(t)$, Exposed$E(t)$, Infected$I(t)$, Infected but on treatment$I_T(t)$, Vaccinated$V(t)$ and Recovered$R(t)$. Equation representing the bacteria disease is incorporated

$$N(t) = S(t) + E(t) + I(t) + I_T(t) + V(t) + R(t)$$

So that

$$\frac{dN(t)}{dt} = \tau - N\mu - (\kappa + \xi)I - \zeta I_T - \mu_1 B \tag{8}$$

The variables and parameter values in the model are presented in Tables 1 and 2.

Table 1: Description of model variables

Variables	Description
S	Susceptible class
E	Exposed class
I	Infected class
I_T	Infected but on treatment
V	Vaccinated class
R	Recovered class
B	Bacteria class

Table 2: Description of model parameters

Variables/ Parameters	Description	Estimated Values	Source
τ	Recruitment rate	500	Atokolo and Mbah (2018)
ω	Declining rate of vaccine	$0 < \omega < 1$	Nthiri (2016)
ψ	Interaction rate	0.0011	Nthiri (2016)
α	Rate at which the recovered are susceptible again	0.9	Atokolo and Mbah (2018)
γ	Treatment rate	0.9	Nthiri (2016)
μ	Natural death rate of human	0.016	Nthiri (2016)
μ_1	Death rate of bacteria	0.0345 per day	Jones (2015)
ξ	Death rate for the infected	0.005	Nthiri (2016)
ζ	Death rate for infected but on treatment	0.001	Atokolo and Mbah (2018)
θ	Vaccination rate	0.5	Atokolo and Mbah (2018)
ϕ	Recovery rate	0.0357 per day	Nthiri (2016)
δ	Exposure rate of vaccinated class	$0 < \delta < 1$	Atokolo and Mbah (2018)
κ	Shedding rate of bacteria	0.014 per day	Jones (2015)
B	Contact rate of bacteria	0.0002	Nthiri (2016)
λ	Infection rate	0.2	Muhammad (2015)
x	Enlightenment rate of the exposed	$0 < x < 1$	Atokolo and Mbah (2018)
y	Enlightenment rate to go for treatment	$0 < y < 1$	Atokolo and Mbah (2018)
z	Enlightenment rate to go for vaccination	$0 < z < 1$	Atokolo and Mbah (2018)

Mathematical Preliminaries:

Existence and Uniqueness of Solution

To determine the conditions for the existence and uniqueness of solution for the model equations (1) – (7), let

$$h_1(t,m) = \tau - \psi(1-x)BS + \alpha R - \theta(1+z)S - \mu S, \tag{8}$$

$$h_2(t,m) = \psi(1-x)BS + \delta(1-\omega)BV - (\lambda+\mu)E, \tag{9}$$

$$h_3(t,x) = \lambda E - \gamma(1+y)I - (\xi+\mu)I, \tag{10}$$

$$h_4(t,m) = \gamma(1+y)I - (\phi+\zeta+\mu)I_T, \tag{11}$$

$$h_5(t,m) = \theta(1+z)S - \delta(1-\omega)BV - \mu V. \tag{12}$$

$$h_6(t,m) = \phi I_T - (\alpha+\mu)R \tag{13}$$

$$h_7(t,m) = \kappa I - \mu_1 B \tag{14}$$

Such that

$$\frac{dm}{dt} = h(t,m) = h(m). \tag{15}$$

Theorem 1. Let A represent the region

$$|t-t_0| \le k_1,\ \ \|m-m_0\| \le k_2,\ \text{ and }\ m = (m_1, m_2, \dots, m_n) = (m_{10}, m_{20,} \dots, m_{n0}) \tag{16}$$

with $h(t,m)$ satisfying the Lipschitz condition

$$\|h(t,m_1) - h(t_1,m_2)\| \le k\|m_1 - m_2\| \tag{17}$$

for (t,m_1) and (t_1,m_2) in A and $k > 0$. Then, there exists a constant $\delta > 0$ such that a unique continuous vector solution $\overline{m}(t)$ of equations (8) – (14) exists in $|t-t_0| \le \delta$.

$\frac{\partial h_i}{\partial m_j}$, $i,j = 1,2,\dots,n$ is continuous and bounded in A and fulfilled the condition in equation (17)

Lemma 1. If $h(t,m)$ is continuous and has partial derivative $\frac{\partial h_i}{\partial m_j}$ on a bounded closed convex domain $\mathbb{R}$, then it satisfies a Lipschitz condition in $\mathbb{R}$.

The region of interest is given by

$$1 \leq \epsilon \leq \mathbb{R} \tag{18}$$

and bounded solution of the form below is sought for:

$$0 < \mathbb{R} < \infty \tag{19}$$

Below is the proof of the existence theorem:

Theorem 2: If A represents the region defined in (17) such that (18) and (19) hold, then ∃ a solution of the model equations (8) – (14) bounded in the region A.

Proof. Considering equations $(8) - (14)$, it will be shown that the continuity of $\frac{\partial h_i}{\partial m}, i = j = 1, 2, 3, 4, 5, 6, 7$ exists. Differentiating h_i partially with respect to S, E, I, I_T, V, R and B, give:

$$\left|\frac{\partial h_1}{\partial S}\right| = -|[\psi(1-x)B + \theta(1+z) + \mu]| < \infty \tag{20}$$

$$\left|\frac{\partial h_1}{\partial E}\right| = |0| < \infty \tag{21}$$

$$\left|\frac{\partial h_1}{\partial I}\right| = |0| < \infty \tag{22}$$

$$\left|\frac{\partial h_1}{\partial I_T}\right| = |0| < \infty \tag{23}$$

$$\left|\frac{\partial h_1}{\partial V}\right| = |0| < \infty \tag{24}$$

$$\left|\frac{\partial h_1}{\partial R}\right| = |\alpha| < \infty \tag{25}$$

$$\left|\frac{\partial h_1}{\partial B}\right| = |-\psi(1-x)S| < \infty \tag{26}$$

$$\left|\frac{\partial h_2}{\partial S}\right| = |\psi(1-x)B| < \infty \tag{27}$$

$$\left|\frac{\partial h_2}{\partial E}\right| = |-(\lambda+\mu)| < \infty \tag{28}$$

$$\left|\frac{\partial h_2}{\partial I}\right| = |0| < \infty \tag{29}$$

$$\left|\frac{\partial h_2}{\partial I_T}\right| = |0| < \infty \tag{30}$$

$$\left|\frac{\partial h_2}{\partial V}\right| = |\delta(1-\omega)B| < \infty \tag{31}$$

$$\left|\frac{\partial h_2}{\partial R}\right| = |0| < \infty \quad (32)$$

$$\left|\frac{\partial h_2}{\partial B}\right| = |\psi(1-x)S + \delta(1-\omega)V| < \infty \quad (33)$$

$$\left|\frac{\partial h_3}{\partial S}\right| = |0| < \infty \quad (34)$$

$$\left|\frac{\partial h_3}{\partial E}\right| = |\lambda| < \infty \quad (35)$$

$$\left|\frac{\partial h_3}{\partial I}\right| = |-[\gamma(1+y) + (\xi + \mu)]| < \infty \quad (36)$$

$$\left|\frac{\partial h_3}{\partial I_T}\right| = |0| < \infty \quad (37)$$

$$\left|\frac{\partial h_3}{\partial V}\right| = |0| < \infty \quad (38)$$

$$\left|\frac{\partial h_3}{\partial R}\right| = |0| < \infty \quad (39)$$

$$\left|\frac{\partial h_3}{\partial B}\right| = |0| < \infty \quad (40)$$

$$\left|\frac{\partial h_4}{\partial S}\right| = |0| < \infty \quad (41)$$

$$\left|\frac{\partial h_4}{\partial E}\right| = |0| < \infty \quad (42)$$

$$\left|\frac{\partial h_4}{\partial I}\right| = |\gamma(1+y)| < \infty \quad (43)$$

$$\left|\frac{\partial h_4}{\partial I_T}\right| = |-(\phi + \zeta + \mu)| < \infty \quad (44)$$

$$\left|\frac{\partial h_4}{\partial V}\right| = |0| < \infty \quad (45)$$

$$\left|\frac{\partial h_4}{\partial R}\right| = |0| < \infty \quad (46)$$

$$\left|\frac{\partial h_4}{\partial B}\right| = |0| < \infty \quad (47)$$

$$\left|\frac{\partial h_5}{\partial S}\right| = |\theta(1+z)| < \infty \quad (48)$$

$$\left|\frac{\partial h_5}{\partial E}\right| = |0| < \infty \quad (49)$$

$$\left|\frac{\partial h_5}{\partial I}\right| = |0| < \infty \quad (50)$$

$$\left|\frac{\partial h_5}{\partial I_T}\right| = |0| < \infty \tag{51}$$

$$\left|\frac{\partial h_5}{\partial V}\right| = |-[\delta(1-\omega)B + \mu]| < \infty \tag{52}$$

$$\left|\frac{\partial h_5}{\partial R}\right| = |0| < \infty \tag{53}$$

$$\left|\frac{\partial h_5}{\partial B}\right| = |0| < \infty \tag{54}$$

$$\left|\frac{\partial h_6}{\partial S}\right| = |0| < \infty \tag{55}$$

$$\left|\frac{\partial h_6}{\partial E}\right| = |0| < \infty \tag{56}$$

$$\left|\frac{\partial h_6}{\partial I}\right| = |0| < \infty \tag{57}$$

$$\left|\frac{\partial h_6}{\partial I_T}\right| = |\phi| < \infty \tag{58}$$

$$\left|\frac{\partial h_6}{\partial V}\right| = |0| < \infty \tag{59}$$

$$\left|\frac{\partial h_6}{\partial R}\right| = |-(\alpha + \mu)| < \infty \tag{60}$$

$$\left|\frac{\partial h_6}{\partial B}\right| = |0| < \infty \tag{61}$$

$$\left|\frac{\partial h_7}{\partial S}\right| = |0| < \infty \tag{62}$$

$$\left|\frac{\partial h_7}{\partial E}\right| = |0| < \infty \tag{63}$$

$$\left|\frac{\partial h_7}{\partial I}\right| = |\kappa| < \infty \tag{64}$$

$$\left|\frac{\partial h_7}{\partial I_T}\right| = |0| < \infty \tag{65}$$

$$\left|\frac{\partial h_7}{\partial V}\right| = |0| < \infty \tag{66}$$

$$\left|\frac{\partial h_7}{\partial R}\right| = |0| < \infty \tag{67}$$

$$\left|\frac{\partial h_7}{\partial B}\right| = |-\mu_1| < \infty \tag{68}$$

The partial derivatives (20) – (68) of the right hand side of (1) – (7) with respect to S, E, I, I_T, V, R and B are continuously differentiable and bounded. Hence, by Theorem 2, it is locally Lipschitz, therefore, $(S(t), E(t), I(t), I_T(t), V(t), R(t), B(t))$ is a unique solution to the system of equations (1) – (7) with the initial conditions $S_{10}, E_{10}, I_{10}, I_{T_{10}}, V_{10}, R_{10}, B_{10}$ in the region A.

To show that the solution satisfies the Lipschitz condition, from equation (1), it can be seen that

$$|G(t,S_1) - G(t,S_2)| = |(\tau - \psi(1-x)BS_1 + \alpha R - \theta(1+z)S_1 - \mu S_1) - (\tau - \psi(1-x)BS_2 + \alpha R - \theta(1+z)S_2 - \mu S_2)|$$

$$\leq (\psi(1-x)BS_1 + \alpha R - \theta(1+z)S_1 - \mu S_1)|(S_1 - S_2)|$$

This means that $|G(t,S_1) - G(t,S_2)| \leq M|(S_1 - S_2)|$ where

$$M = (\psi(1-x)BS_1 + \alpha R - \theta(1+z)S_1 - \mu S_1)$$

is a Lipschitz constant.

Similarly, the other variables satisfy the Lipschitz condition, hence $\exists$ a unique solution $E(t), I(t), I_T(t), V(t), R(t), B(t)\ \ \forall\, t \geq 0$.

Invariant Region

Lemma 2. The region $A \subset \mathbb{R}_+^7$ is positively invariant for the equations $(1) - (7)$ with whole number initial condition in $\mathbb{R}_+^7$.

Proof. Following equation (8), it is shown that

$$\frac{dN(t)}{dt} \leq \tau - N\mu$$

$$\Rightarrow N(t) \leq N(0)e^{\mu t} + \frac{\tau}{\mu}(1 - e^{\mu t})$$

$$\Rightarrow N(t) \leq \frac{\tau}{\mu} \text{ if } N(0) \leq 0.$$

Therefore, the feasible region $A \subset \mathbb{R}_+^7$ for the continuous system $(1) - (7)$ becomes

$$A = \left\{(S, E, I, I_T, V, R, B) \epsilon\, \mathbb{R}_+^7 : S + E + I + I_T + V + R + B \leq \frac{\tau}{\mu}\right\}$$

Positivity of Solution

Lemma 3. The zeros of the system of equations $(1) - (7)$, $\{S, E, I, I_T, V, R, B\}$, with initial condition $\left\{S_{10}, E_{10}, I_{10}, I_{T_{10}}, V_{10}, R_{10}, B_{10} \geq 0\right\} \epsilon\, A$ will remain non-negative $\forall$ time $t \geq 0$.

Proof. From equation (1),

$$\frac{dS}{dt} = \tau - \psi(1-x)BS + \alpha R - \theta(1+z)S - \mu S$$

$$\leq -[\theta(1+z) - \mu]S$$

$$\Rightarrow S \geq S_{10}e^{-\int[\theta(1+z)-\mu]dt} \geq 0 \qquad \forall\, t > 0$$

Similarly, equations (2) – (7) show that $\forall\, t > 0$,

$$E \geq E_{10}e^{-\int(\lambda+\mu)dt} \geq 0,\, I \geq I_{10}e^{-\int[\gamma(1+y)+(\xi+\mu)]dt} \geq 0,\, I_T \geq I_{T_{10}}e^{-\int(\phi+\zeta+\mu)dt} \geq 0,$$
$$V \geq V_{10}e^{-\int \mu dt} \geq 0,\, R \geq R_{10}e^{-\int(\alpha+\mu)dt} \geq 0 \text{ and } B \geq B_{10}e^{-\int \mu dt} \geq 0$$

Hence, it can be concluded that whenever $t \geq 0$, the solution of the system (1) – (7) is positive.

Boundedness of Solution

Lemma 4. The zeros $\{S, E, I, I_T, V, R, B\}$ of the system of equations $(1) - (7)$ with initial condition $\left\{S_{10}, E_{10}, I_{10}, I_{T_{10}}, V_{10}, R_{10}, B_{10} \geq 0\right\} \epsilon\, A$ are bounded and remain in the region

$$A = A_H \times A_B \tag{69}$$

Where

$$A_H = \left\{\left(S, E, I, I_{T,} V, R\right) \in\ \mathbb{R}_+^6 : 0 \leq (S(t) + E(t) + I(t) + I_T(t) + V(t) + R(t) \leq \frac{\tau}{\mu}\right\} \tag{70}$$

And

$$A_B = \left\{B \leq \mathbb{R}_+ : 0 \leq B(t) \leq \frac{\kappa}{\mu_1}\right\} \tag{71}$$

Proof:
Splitting the model (1) – (7) into human population $H(t)$ and bacteria population $B(t)$, where

$$H(t) = S(t) + E(t) + I(t) + I_T(t) + V(t) + R(t)$$

Hence,

$$\frac{dH}{dt} = \tau - H\mu - (\kappa + \xi)I - \zeta I_T \leq \tau - H\mu$$

$$\therefore H \leq \frac{\tau}{\mu} + Ce^{-\mu t} \qquad (C \text{ is an arbitrary constant})$$

At $t = 0$, $H \leq \frac{\tau}{\mu} + \left(H(0) - \frac{\tau}{\mu}\right)e^{-\mu t}$

$$\lim_{t\to\infty} H \leq \frac{\tau}{\mu}$$

Similarly,

$$\frac{dB}{dt} = \kappa I - \mu_1 B \leq \kappa - \mu_1 B$$

$$\lim_{t\to\infty} B \leq \frac{\kappa}{\mu_1}$$

It implies that the human and bacteria populations are biologically feasible in the region (70) and (71) respectively. Hence, the solution of (1) – (7) with the given initial condition is bounded in the invariant region (69) $\forall\ t \geq 0$.

Disease-free Equilibrium (DFE) and Endemic Equilibrium (EE)

Following George (2023), at an equilibrium point,

$$\frac{dS(t)}{dt} = \frac{dE(t)}{dt} = \frac{dI(t)}{dt} = \frac{dI_T(t)}{dt} = \frac{dV(t)}{dt} = \frac{dR(t)}{dt} = \frac{dB(t)}{dt} = 0.$$

Hence, the disease-free equilibrium (DFE) of the model (1) – (7) is given by

$$\left(S,^0 E^0, I^0, {I_T}^0, V^0, R^0, B^0\right) = \left(\frac{\tau}{\theta(1+z)+\mu}, 0, 0, 0,\ \frac{\theta(1+z)\tau}{\mu[\theta(1+z)+\mu]}, 0, 0\right), \qquad (72)$$

since at the DFE, $E = I = I_T = 0$.

At the endemic equilibrium (EE), $E \neq 0,\ I \neq 0,\ I_T \neq 0$. Hence,

$$s^* = \frac{\mu}{\psi(1-x)\kappa I^* + \mu[\theta(1+z)+\pi]}\left[\frac{\tau + \alpha\gamma\theta(1+z)I^*}{(\phi+\zeta+\mu)(\alpha+\mu)}\right]$$

$$E^* = \frac{1}{\gamma}[\gamma(1+y) + (\xi+\mu)]I^*$$

$$I^* = \frac{\mu}{\kappa}B^*$$

$$ {I_T}^* = \frac{\gamma(1+z)}{(\phi+\zeta+\mu)}I^*$$

$$V^* = \frac{\mu^2\theta(1+z)}{\{\delta(1-\omega)\kappa+\mu^2\}\{\psi(1-x)\kappa I^* + \mu[\theta(1+z)+\pi]\}}\left[\frac{\tau+\alpha\gamma\theta(1+z)}{(\phi+\zeta+\mu)(\alpha+\mu)}\right]I^*$$

$$R^* = \frac{\gamma\theta(1+z)}{(\phi+\zeta+\mu)(\alpha+\mu)}I^*$$

$$B^* = \frac{\kappa}{\mu}I^*$$

The Effective Reproduction Number (R_e)

The effective reproduction number (R_e) can be calculated by multiplying the basic reproduction number R_0 by the fraction of the population who are susceptible, leading to the equation

$$R_e = R_0 s \qquad (73)$$

where *s* is the fraction of the host population who are susceptible to the disease.

The basic reproduction number (R_0) is a critical threshold value in epidemiology used to measure the transmission potential of a disease. In the calculation of R_0, it is assumed that the entire population is susceptible to the disease. This assumption may not always be the case since some individuals will be immune due to a prior infection creating life-long immunity, or as a result of vaccination. To consider this, the effective reproduction number (R_e) is used.

Following the next generation matrix approach proposed by Driessche and Watmough (2002), the basic reproduction number R_0 is given by

$$R_0 = \frac{\tau\lambda\kappa(1-x)\psi}{(\theta(1+z)+\mu)\{\mu_1(\lambda+\mu)[\gamma(1+y)+(\xi+\mu)]-\lambda\kappa[\psi(1-x)S+\delta(1-\omega)V]\}}$$

Hence,

$$R_e = \frac{\tau\lambda\kappa(1-x)\psi s}{(\theta(1+z)+\mu)\{\mu_1(\lambda+\mu)[\gamma(1+y)+(\xi+\mu)]-\lambda\kappa[\psi(1-x)S+\delta(1-\omega)V]\}}$$

The magnitude of R_0 not only indicates the speed of how a disease will spread, but whether it will spread at all. If it is the case that $R_0 > 1$ then the typhoid infection will spread throughout the population, but if $R_0 < 1$ then the infection will not be able to take hold and will eventually die out. The greater the value of R_0, the harder it is to control an epidemic.

RESULTS AND DISCUSSION

Simulated data are generated using a MATLABODE45 scheme to study the effect of enlightenment rates on R_0, R_e and the state variables, $S, E, I, I_T, V, R,$ and B. The results are presented and discussed hereunder:

Table 1a: Impact of vaccination rate (θ) on R_0 and R_e

θ	R_0	R_e
0.5	1.20×10^{-5}	4.21×10^{-6}
0.45	2.12×10^{-5}	4.66×10^{-6}
0.40	2.38×10^{-5}	5.22×10^{-6}
0.35	2.70×10^{-5}	5.94×10^{-6}
0.30	3.13×10^{-5}	6.88×10^{-6}
0.25	3.72×10^{-5}	8.18×10^{-6}
0.20	4.59×10^{-5}	1.01×10^{-5}
0.15	5.98×10^{-5}	1.31×10^{-5}
0.10	8.57×10^{-5}	1.88×10^{-5}
0.05	1.51×10^{-4}	3.32×10^{-5}

Table 1a shows that a decrease in vaccination rate (θ) increases the response functions, R_0andR_e respectively thereby heightening the tendency for the endemicity of the disease.

Table 1b: Impact of vaccination rate (θ) on S, E, I, I_T, V, R and B

θ	S	E	I	I_T	V	R	B
0.50	500.86	4267.59	864.89	13175.90	4.12	507.34	1329.45
0.45	514.77	4263.38	864.07	13167.80	3.81	507.04	1329.31
0.40	529.47	4258.90	863.19	13159.26	3.48	506.72	1329.16
0.35	545.03	4254.12	862.26	13150.26	3.14	506.39	1329.00
0.30	561.53	4249.01	861.26	13140.77	2.77	506.04	1328.84
0.25	579.04	4243.54	860.19	13130.74	2.38	505.67	1328.67
0.20	597.67	4237.69	859.05	13120.12	1.97	505.27	1328.49
0.15	617.53	4231.37	857.83	13108.86	1.52	504.86	1328.29
0.10	638.74	4224.57	856.51	13096.90	1.05	504.42	1328.09
0.05	661.45	4217.22	855.09	13084.18	0.54	503.95	1327.87

Table 1b reveals that, decreasing the rate of vaccination (θ) significantly increases the size of the susceptible (S) class. It can be inferred from Tables 1a and b that the more the number of susceptible people, the higher the possibility for the disease to spread faster.

Table 2a: Impact of enlightenment rate (x) of the exposed on R_0 and R_e

x	R_0	R_e
0.080	1.92×10^{-5}	4.21×10^{-6}
0.072	1.93×10^{-5}	4.25×10^{-6}
0.064	1.95×10^{-5}	4.28×10^{-6}
0.056	1.97×10^{-5}	4.32×10^{-6}
0.048	1.98×10^{-5}	4.36×10^{-6}
0.040	2.00×10^{-5}	4.39×10^{-6}
0.032	2.02×10^{-5}	4.43×10^{-6}
0.024	2.03×10^{-5}	4.46×10^{-6}
0.016	2.05×10^{-5}	4.50×10^{-6}
0.008	2.07×10^{-5}	4.54×10^{-6}

Table 2a shows that a decrease in enlightenment rate (x) of the exposed increases $R_0$and$R_e$respectively, signaling the tendency for the endemicity of the disease.

Table 2b: Impact of enlightenment rate (x) of the exposed on S, E, I, I_T, V, R and B

x	S	E	I	I_T	V	R	B
0.080	500.86	4267.57	864.89	13175.90	4.11	507.34	1329.45
0.072	497.83	4268.34	865.04	13177.93	4.09	507.42	1329.48
0.064	494.83	4269.08	865.19	13179.94	4.07	507.49	1329.52
0.056	491.87	4269.81	865.34	13181.93	4.04	507.57	1329.55
0.048	488.94	4270.54	865.48	13183.89	4.02	507.64	1329.59
0.040	486.05	4271.26	865.63	13185.82	3.99	507.72	1329.62
0.032	483.20	4271.97	865.77	13187.73	3.97	507.79	1329.66
0.024	480.37	4272.67	865.91	13189.62	3.95	507.86	1329.69
0.016	477.58	4273.36	866.05	13191.49	3.92	507.79	1329.732
0.008	474.82	4274.05	866.19	13193.33	3.90	508.00	1329.76

Table 2b reveals that decreasing the enlightenment rate (x) of the exposed gradually moved individuals from susceptible class to the exposed (E) and infected classes. It can be concluded from Tables 2a and b that reducing the enlightenment rate (x) of the exposed individuals is likely to increase the possibility for the disease to spread faster.

Table 3a: Impact of enlightenment rate to go for treatment (y) on R_0 and R_e

y	R_0	R_e
0.070	1.92×10^{-5}	4.21×10^{-6}
0.063	1.90×10^{-5}	4.18×10^{-6}
0.056	1.89×10^{-5}	4.15×10^{-6}
0.049	1.88×10^{-5}	4.13×10^{-6}
0.042	1.87×10^{-5}	4.10×10^{-6}
0.035	1.86×10^{-5}	4.07×10^{-6}
0.028	1.84×10^{-5}	4.05×10^{-6}
0.021	1.83×10^{-5}	4.02×10^{-6}
0.014	1.82×10^{-5}	3.99×10^{-6}
0.007	1.81×10^{-5}	3.97×10^{-6}

Table 3a shows that decreasing the enlightenment rate to go for treatment (y) slightly decreases $R_0$and R_e respectively.

Table 3b: Impact of enlightenment rate to go for treatment (y) on S, E, I, I_T, V, R and B

y	S	E	I	I_T	V	R	B
0.070	500.86	4267.59	864.89	13175.90	4.11	507.34	1329.45
0.063	500.43	4266.92	870.31	13171.80	4.11	507.16	1330.77
0.056	500.00	4266.26	875.81	13166.60	4.10	506.98	1332.11
0.049	499.56	4265.58	881.37	13161.86	4.09	506.79	1333.46
0.042	499.12	4264.89	887.00	13157.06	4.09	506.61	1334.84
0.035	498.67	4264.20	892.77	13152.20	4.08	506.42	1336.23
0.028	498.22	4263.49	898.49	13147.28	4.07	506.23	1337.64
0.021	497.76	4262.78	904.34	13142.29	4.06	506.03	1339.06
0.014	497.30	4262.05	910.27	13137.24	4.05	505.84	1340.50
0.007	496.83	4261.32	916.28	13132.12	4.05	505.64	1341.97

Table 3b shows that decreasing the enlightenment rate to go for treatment (y), increases the size of the infectious classes. It can be concluded from Tables 3a and b that reducing the enlightenment rate to go for treatment can increase the possibility for the disease to spread faster.

Table 4a: Impact of enlightenment rate to go for vaccination (z) on R_0 and R_e

z	R_0	R_e
0.050	1.92×10^{-5}	4.21×10^{-6}
0.045	1.93×10^{-5}	4.23×10^{-6}
0.040	1.93×10^{-5}	4.25×10^{-6}
0.035	1.94×10^{-5}	4.27×10^{-6}
0.030	1.95×10^{-5}	4.29×10^{-6}
0.025	1.96×10^{-5}	4.31×10^{-6}

0.020	1.97×10^{-5}	4.33×10^{-6}
0.015	1.98×10^{-5}	4.35×10^{-6}
0.010	1.99×10^{-5}	4.37×10^{-6}
0.005	2.00×10^{-5}	4.39×10^{-6}

Table 4a shows that, a decrease in enlightenment rate to go for vaccination (z) increases R_0 and R_e, respectively and in consequence, increasing the tendency for the disease to spread over time

Table 4b: Impact of enlightenment rate to go for vaccination (z) on S, E, I, I_T, V, R and B

z	S	E	I	I_T	V	R	B
0.050	500.86	4267.59	864.89	13175.90	4.11	507.34	1329.45
0.045	501.51	4267.39	864.85	13175.52	4.10	507.32	1329.44
0.040	502.15	4267.20	864.81	13175.15	4.09	507.31	1329.43
0.035	502.80	4267.00	864.78	13174.77	4.07	507.30	1329.43
0.030	503.45	4266.80	864.74	13174.39	4.06	507.28	1329.42
0.025	504.12	4266.61	864.70	13174.01	4.04	507.27	1329.41
0.020	504.76	4266.41	864.66	13173.62	4.03	507.25	1329.41
0.015	505.42	4266.21	864.62	13173.24	4.02	507.24	1329.40
0.010	506.07	4266.01	864.58	13172.86	4.00	507.22	1329.39
0.005	506.73	4265.81	864.54	13172.47	3.99	507.21	1329.39

Table 4b shows that, decreasing the enlightenment rate to go for vaccination (z) gradually increases the susceptible class. It can be concluded from Tables 4a and 4b that the more the number of susceptible people, the higher the possibility for an epidemic case.

Table 5: Impact of zero and non-zero enlightenment rates

	$x = 0$, $y = 0.07$, $z = 0.05$	$x = 008$, $y = 0$, $z = 0.05$	$x = 008$, $y = 0.07$, $z = 0$,	$x = y = z = 0$	$x = 0.08$, $y = 0.07$, $z = 0.05$
R_0	2.08×10^{-5}	1.79×10^{-5}	2.01×10^{-5}	2.04×10^{-5}	1.92×10^{-5}
R_e	4.57×10^{-6}	3.94×10^{-6}	4.41×10^{-6}	4.49×10^{-6}	4.21×10^{-6}
S	4.72×10^{2}	4.96×10^{2}	5.07×10^{2}	4.73×10^{2}	5.01×10^{2}
E	4.27×10^{3}	4.26×10^{3}	4.27×10^{3}	4.27×10^{3}	4.27×10^{3}
I	8.66×10^{2}	9.22×10^{2}	8.65×10^{2}	9.24×10^{2}	8.65×10^{2}
I_T	1.32×10^{4}	1.31×10^{4}	1.32×10^{4}	1.31×10^{4}	1.32×10^{4}
V	3.88	4.04	3.97	3.67	4.12
R	5.08×10^{2}	5.05×10^{2}	5.07×10^{2}	5.06×10^{2}	5.07×10^{2}
B	1.33×10^{3}	1.34×10^{3}	1.33×10^{3}	1.34×10^{3}	1.33×10^{3}

Table 5 shows a spike in the size of the infectious class when the enlightenment rate of the exposed (x) and enlightenment rates to go for treatment (y) and vaccination (z) are zero, with the basic and effective reproduction numbers higher when the enlightenment rate of the exposed (x) is zero and lowest when the enlightenment rate to go for treatment (y) is zero. It can be concluded that, over time, the disease will result in an epidemic, whenever the rates of enlightenment approach zero.

Conclusion

A mathematical model was used to study the effect of enlightenment rate of the exposed and, enlightenment rates to go for treatment and vaccination. The study also considered the basic and effective reproduction numbers to ascertain which of the two gives early signal in the event of an epidemic occurring. The result in this work underscores the importance of sustained enlightenment campaigns in checking the spread of typhoid disease. It was shown that, the more the number of susceptible people in the population, the higher the possibility for the disease to spread faster. Furthermore, it was shown that, decreasing the enlightenment rate of the exposed (x) and enlightenment rates to go for treatment (y) and vaccination (z) can increase the possibility for the disease to spread faster. Additionally, it was revealed that, the basic reproduction number gives signal earlier than the effective reproduction number, if the disease will result in an epidemic. This is because some individuals in the population have immunity conferred on them, as is assumed in the effective reproduction number, hence delaying the signal for the occurrence of an epidemic.

This work is important to the scientific community since it has used the tools of mathematics to solve the crisis in health sector by indicating clearly the impact of vaccination and treatment to typhoid. It has brought in a clear relationship between biology, health science and mathematics which are ideal subjects in science. This work has revealed that, to consider target parameters in the outbreak of an infectious disease, the basic reproduction number gives a faster indication of an impending widespread than the effective reproduction number.

Disclaimer (Artificial intelligence)

Option 1:

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Option 2:

Author(s) hereby declare that generative AI technologies such as Large Language Models, etc. have been used during the writing or editing of manuscripts. This explanation will include the name, version, model, and source of the generative AI technology and as well as all input prompts provided to the generative AI technology

Details of the AI usage are given below:

1.

2.

3.

References

Adeboye, K. R., & Haruna, M. A. (2015). Mathematical model for the transmission and control of malaria typhoid co-infection using SIRS approach. *Niger. Res. J. Math 2*(2), 1–24

Butt, A. I. K., Shah, N. A., Ahmad, W., Botmart, T., & Ahmad, N. (2022). On implementation of a semi-analytic strategy to develop an analytical solution of a steady-state isothermal tube drawing model. *Sci. Rep., 12*(1), 7636

Butt, A. I. K., Rafiq, M., Ahmad, W., & Ahmad, N. (2023). Implementation of computationally efficient numerical approach to analyze a Covid-19 pandemic model. *Alex. Eng. J. 69*, 341–362

Driessche, P. V., & Watmough, J. (2002). Reproduction numbers and sub-thresholds endemic equilibrium for compartmental models of disease transmission. *Mathematical Bioscience,180*, 29 – 48

George, I. (2023). Diphtheria transmission dynamics: A sensitivity analysis. *FNAS Journal of Scientific Innovations, 5*(2), 131-141.

Healthline (2024). *Is typhoid fever contagious? What you need to know.* https://www.healthline.com/health/is-typhoid-contagious

Ivanoff, B., Levine, M. M., & Lambert, P. (1994). Vaccination against typhoid fever: Present status. *Bull. World Health Organ, 72*(6), 957

Musa, S. S., Zhao, S., Hussaini, N., Usaini, S., & He, D. (2021). Dynamics analysis of typhoid fever with public health education programs and final epidemic size relation. *Results Appl. Math. 10*, 100153

Mushayabasa, S., Bhunu, C. P., & Ngarakana-Gwasira, E. T. (2013). Mathematical analysis of a typhoid model with carriers, direct and indirect disease transmission. *Int. J. Math. Sci. Eng. Appl., 7*(1), 79-90

NHS (2024). Typhoid fever. https://www.nhs.uk/conditions/typhoid-fever/

Nsutebu, E. F., Martins, P., & Adiogo, D. (2003). Prevalence of typhoid fever in febrile patients with symptoms clinically compatible with typhoid fever in Cameroon. *Trop. Med. Int. Health, 8*(6), 575–578