Original Research Article

A new look at the Great Red Spot of Jupiter

Abstract

In a recent study using an Ocean General Circulation Model (OGCM) it was shown that the forcing by the surface vertical velocity generated a significant vorticity to that generated by the curl of the wind stress forcing. On Jupiter the surface vertical velocity is the only source of vorticity, and the Coriolis force is very similar to that in the sub-tropics on Earth. This leads to a plausible mechanism for the existence of the Great Red Spot (GRS) which is investigated in this paper. The properties which are modelled include the topography of the GRS, which is predicted to be a tabletop mountain of probable height about 1 km,situated on a surrounding gently sloping plain extending outwards over approximately 2000 km. On this plain, the upward vertical velocity brings fluid to the surface,where we propose it is trapped in a fluidic mixed layer probably of a few meters thickness beneath the intensely cold icy cap. This is analogous to the formation of a mixed layer in the oceans on Earth The time scale for heat exchange with the atmosphere is probably about 10 hrs, similar to the rotation period of Jupiter.

Keywords: Jupiter, Planetary evolution

1. Introduction

There has been an ongoing discussion on the origins and maintenance of the Great Red Spot on Jupiter, which is Jupiter's most conspicuous natural feature (https://www.nasa.gov>**jupiters-great-red-spot-2**). In this paper, we show how it is a topographic feature which arises from the equations of large scale fluid dynamics, applied to model the general circulation of Jupiter, in particular the Great Red Spot (Read 2024).The beauty of the vorticity modelis that the fluid dynamics can be easily applied on rotating bodies of all horizontal and time scales,

The principal difference between Earth and Jupiteris one of geometry. Earth has landmasses which punctuate its oceans and engender important wind fields, whereas Jupiter is exclusively fluidic. This difference favours the forcing by the surface vertical velocity on Jupiter relative to that by the wind stress curl on Earth.

2. The vorticity equation

The results presented in this paper, follow from the extended vorticity equation of Stommel (1948) for the two-dimensional circulation which is outlined below. The components of transport velocity (U,V) on a rotating spherical body in a Cartesian frame (Ox, Oy), where Ox is Eastward and Oy is Northward, are,

$$U = - \partial\psi/\delta y + \partial\phi/\delta x \quad \text{and} \quad V = \partial\psi/\delta x + \partial\phi/\partial y \qquad (1)$$

where ψ is the streamfunction, which satisfies the 2-D continuity equation, $U_x + V_y = 0$, and ϕ is the potential which satisfies the Poisson equation,

,

$$\partial^2_{xx} + \partial^2_{yy}\, \phi = w_o \qquad (2)$$

where w_o is the vertical velocity at the fluid surface.

The solution for ϕ is obtained from (2) by specifying w_o (x,y), and the solution for ψ is obtained by solving the vorticity equation obtained by eliminating the pressure gradients between the two transport momentum equations. In the original Stommel steady-state linear friction model this yields the vorticity equation,

$$f_o w_o + \beta\, (\partial\psi/\partial x + \partial\phi/\partial y) + R\, (\partial^2_{xx} + \partial^2_{yy}\, \psi) = 1/\rho\, (\partial\tau_{wy}/\partial x - \partial\tau_{wx}/\partial y) \qquad (3)$$

where (τ_{wx} , τ_{wy}) are the components of wind stress, and R(U, V) are the components of linear friction in which R is the coefficient of body friction, and the Coriolis parameter, f = 2 Ω sin θ where Ω is the angular rate of rotation of the body and θ is latitude, which is approximated by the linear relation,, $f = f_o + \beta y$ where $\beta = df/dy$, and R, f_o andβ are assumed to be constants. For $w_o = 0$, ϕ (x,y) = 0, and (3) reduces to the ocean general circulation vorticity equation of Stommel (1948).

3. **The streamline solution for ψ**

In more general terms, for a constant surface vertical velocity (w_o), (3) reduces to the vorticity equation,

$$\beta\, \partial\psi/\partial x + R\,(\partial^2_{xx} + \partial^2_{yy}\,\psi) = 1/\rho\,(\partial\tau_{wy}/\partial x - \partial\tau_{wx}/\partial y) - f\,w_o \quad (4)$$

Eq. (4) is the extended Stommel vorticity equation for ψ (x,y), which is independent of ϕ, but includes the effects of *both* the streamline forcing through the wind stress, and the potential forcing, through the surface vertical velocity. The solution of (4) for ψ (x, y) in an ocean basin has been obtained in Bye et al (2024).

The focus here however is on the solution of (2) for ϕ, which is independent of ψ

4. **The potential solution for ϕ**

In the absence of wind stress forcing, the general circulation is fundamentally radial in which the 2-dimensional transport equation (2) is,

$$\partial^2\phi/\partial r^2 + 1/r\,\partial\phi/\partial r - \phi/r^2 = w_o\,, \quad 0 \le r \le r_o \qquad (5)$$

where r is the radial co-ordinate, and r_o is the radius of the circulation, and no dependence on the azimuthal co-ordinate is assumed, and ϕ (r) [$L^3\,T^{-1}$] is the potential and w_o (r) [LT^{-1}] is the vertical velocity source for the circulation (Bye et al 2024).

4.1 The one-dimensional dynamics

The solution of (5) is,

$$\phi(r) = 1/3\, w_o\, r^2\,, \quad 0 \le r \le r_o \qquad (6)$$

Eq. (6) shows how the vertical velocity, w_o , causes the potential (ϕ(r)) which drives the radial velocity,

$$v(r) = 2/3\, w_o\, r\,/H \qquad (7)$$

where H is the depth of the circulation.The 1-D radial pressure equation is,

$$\partial v/\partial t + v\partial v/\partial r = -\, g\,\partial\eta/\partial r \qquad (8)$$

in which g is the acceleration of gravity, η is the surface elevation, and

$$v\,H = \partial\phi/\partial r \ (9)$$

Hence on substituting for v from (7) in (8), using (9), we obtain,

$$[\partial/\partial t\ (2/3\ w_o /H) + (2/3\ w_o/H)^2]\ r \ = - g\ \partial\eta/\partial r \qquad (10)$$

Eq. (10) shows how the vertical velocity (w_o) gives rise to the surface topography (η)

In the steady-state, on integrating (10), we have,

$$\eta\ (r) = \eta_o – 2/9\ (w_o/H)^2\ r^2/g \qquad (11)$$

which is the topographic equation for a circular table-top feature of height, η_o , The origin of this feature can be investigated using the time-dependent pressure equation (10).

5. Evolution of the topographic features

Within (10), we will assume that there is only one disposable variable. This is dependent on the depth of the substrate, namely, H (t). Hence (10) reduces to the equation,

$$[\ \partial/\partial t\ A/H + (A/H)^2]\ r \ = - g\ \partial\eta/\partial r \qquad (12)$$

where $A = 2/3\ w_o$is a constant, Eq.(12) is easily solved for an exponential depth relation,

$$H = H_o \exp\ (\gamma\ t\ ) \qquad (13)$$

in which$\gamma > 0$ for a growing substrate and $\gamma < 0$ for a subsiding substrate, and H_o is a constant depth. On substituting for H in (12) we obtain,

$$A/H\ [A/H – \gamma]\ r = - g\ \partial\eta/\partial r \qquad (14)$$

and hence on integrating wrt r,

$$\eta = \eta_o – ½\ [2/3\ w_o/H – \gamma\ ]\ [2/3\ w_o/\ H]\ r^2/g\ (15)$$

in which $2/3\ w_o/H > \gamma$occurs for a table-top, and $2/3\ w_o/H < \gamma$occurs for a crater.

The interpretation of this result is the following. For the establishment of a table-top, the vertical velocity (w_o) must be greater than 3/2 dH/dt, whereas for the formation of a crater, 3/2 dH/dt is greater than w_o. Since from (13)$dH/dt = \gamma H$, crater formation is much more likely to occur as the depth of the substrate increases.

6. The Great Red Spot of Jupiter

The Great Red Spot (GRS) enables us to view the early stage of topographic development on Jupiter, during which table-top features were favoured over cratonic features.

The results in Sections 4 and 5 will now be used to interpret observations of the GRS, especially with respect to corresponding results from Earth. The express purpose of this comparison follows from the finding reported in Bye (2021) that Earth and Jupiter both have the property that the ratio of ordinary matter to dark matter is very close to unity, enabling unstable modes to exist, and strongly suggesting that their dynamical behaviours should be similar and vigorous on both planets.

We begin with the landscape. The topographic relations (11) and (15) provide a description of the Great Red Spot, which consists of a central core in the shape of a table-top mountain situated on a gently sloping plain over to the boundary regions. The key determinant for the height of the table-top is the outward radial velocity (v_o) at its radius (r_o), which from (7) is,

$$v_o = 2/3\ w_o r_o / H \qquad (16)$$

At the edge of the GRS v_o is deflected by a right angle to the right hand side, due to the radial pressure gradient, to become an azimuthal velocity which can be measured from spatial satellite data'. From (11), $\eta_o = 2/9\ (w_o/H)^2\ r_o^2/g$, and hence on substituting for (w_o/H) from (16), we obtain the inertial relation,

$$\eta_o = \tfrac{1}{2}\ v_o^2\ /g \qquad (17)$$

Observations from Hubble indicate that $v_o \approx 100$ m s^{-1} (Wong et al 2020), and hence on substituting for v_o in (17) with the surface acceleration of Jupiter, g = 25 m s^{-2}, we predict that the height of the table-top,η_o= 1100 m, which sits on a central plain that extends out to a radius of 2000 km , on which (11) shows that the height deceases outwards according to the quadratic relation in radius that gives rise to the table top appearance.

The question may be asked: what is the significance of the altitude estimates discussed above. From the perspective of an oceanographer, the above analysis of Jovian dynamics corresponds with a two-layer ocean model, which has often been used to investigate various aspects of ocean dynamics. On Jupiter, the forcing of the circulation arises from the vertical velocity rather than the wind stress curl.

We speculate also that the effects of the decrease in temperature with altitude maycontribute to the colour pattern in the images, in a similar manner to the effect of snow on images of mountains on Earth.

An overlying quasi-zonal banded structure however characterises the appearance of Jupiter (Read 2024). This can also be interpreted using (2),which for a meridionalsolution is independent of Ox,and reduces to the simple expression,

$$d^2\phi/dy^2 = w_o (18)$$

On integrating (17) for w_o = const, with the boundary conditions, $\phi(y) = 0$ at $y = L$ and $y = -L$, we obtain,

$$\phi (x ,y) = ½ w_o (y – L)^2 (18)$$

Eq. (18) represents a linear field directed along Ox, which is symmetrical about Oy, and has an emergentcentral transport, $\phi(0) = ½ w_o L^2$, caused by the vertical velocity,w_o. This appears to be a prototype for a ridge topography, which on the f-plane is independent of orientation.

The meridional pressure equation is,

$$dv^2 /dy = - g d \eta /dy (19)$$

in which from (7), $v = 2/3 w_o y /H$, and hence on substituting for v and integrating,assuming that $\eta = 0$ at $y = 0$, we obtain,

$\eta = - ½ (2/3 w_o y/H)^2 /g$, $-L \le y \le L$ (20) In other words, a ridge occurs along Ox . On substituting fory in (20), its maximum height is,

$$\eta_o = 2/9 (w_o/H)^2 L^2 /g \qquad (21)$$

which is consistent with (11), and since $L << r_o$, the ridge height is much less than the height of the GRS.

Turning our attention to the surface exchange of heat,we recall that in the study of upwelling along the south coast of Australia (Bye and James 2024), the observations suggested that the major source of upwelling was through an upward vertical velocity (the Flinders Upwelling) which brought cool water locally to the surface. The upward vertical velocity (w_o) was independent of the wind stress, and of thermodynamic origin in whichthe surface heat exchange coefficient, $\lambda = \rho\ C_P\ w_o$where C_P is the specific heat at constant pressure and ρ is the fluid density. On Earth, in general, the upwelled water is cooler than the surface water. On Jupiter, the same mechanism operates, but the upwelled fluid is warmer than the intensely cold surrounding atmosphere, and an icy lid occurs.Hence we suggest that a surface fluidic layer may be produced on Jupiter, which is analogous to the formation of a surface mixed layer on Earth.

The period of rotation of Jupiter, T = 9.85 hrs, and the latitude of the GRS, $\theta = 22^o$ (Read 2024) yield, $f = 1.45\ 10^{-4}\ s^{-1}$, and hence the Jovian Coriolis parameter is very close to values in the subtropical latitudes on Earth, The orbital radius of Jupiter ($R_J = 71.4\ 10^3$ km) however is much greater than that of Earth, where $R_E = 6.4\ 10^3$ km. Hence the β – parameter, $\beta = 2\ \Omega \cos\theta/R$, is much smaller on Jupiter. This means that the circulation on Jupiter tends to be governed by an f-plane model, rather than a β-plane model. This is apparent in the satellite imagery of which the GRS, with a rotation period of 69 hrs, is the highlight (Wong 2020).

7The cause of the vertical velocity (w_o)

This theoretical model of the GRS is of course largely predicated on the existence and magnitude of the vertical velocity (w_o). This has been investigated for the continental shelf of South Australia in Bye and James (2024) and for the Southern Ocean and the North Atlantic Ocean in Bye et al (2024). In both situations the vertical velocity, w_o was found to be a prime cause of upwelling.

Here we suggest that it plays the same role on Jupiter.In summary, the downward heat flux ($Q = Q_a + Q_o$) may be partitioned into a component due to the atmosphere (Q_a) and a component due to the ocean (Q_o) which are respectively: $Q_a= \lambda\ (T_a – T)$ and $Q_o= \lambda\ (T – T_s)$ where $\lambda = \rho\ C_P\, w_o$. . On Jupiter, Q_o is the component due to the icy cap in which $T – T_s<$ 0 , as for upwelling in the ocean onEarth, and the vertical velocity, $w_o = \lambda / \rho\ C_P > 0$ where ρ and C_P are properties of the icycap.

8 Conclusions

The results of this study show the many similarities between Jupiter and Earth, notwithstanding the differences in size and temperature between the two planets. This conclusion follows from the existence of the vertical velocity (w_o) which appears to be an innate physical property as discussed in Section 7

The similarities in evolution on Jupiter and Earth are a cause for celebration. We may have an unexpected elder twin in the Solar System.

References

1. Bye J.A. T. 2021 Dark Matter in the Planetary System. Intl. Astron and Astrophys. Res J. 2021 3(4) 31-38
2. Bye J.A.T. and C. E. James 2024SEAFRAME time series for the Southern Shelf reveal a neglected major upwelling mechanism.Trans. R. Soc. South AustraliaDOI:10.1080/03721426.2024 2340 764
3. Bye, J. A. T., Mitchell, J. G. and C. E, James 2024 Quantifying the role of surface velocity in ocean circulation: Antarctic Circumpolar Current impacts. Geophys. Res. Letters. under review
4. Read, P.I. 2024 The dynamics of Jupiter's snd Saturn's Weather Layers: A Synthesis After *Cassini* and *Juno* Ann Rev Fluid Mech. 2024 56 271-293
5. Schlichting,H, 1962 Boundary Layer Theory McGraw-Hill Book Co. Inc. 647 p.
6. Stommel, H. E. 1948 The westward intensification of wind-driven ocean currents Trans. Amer. Geophys. Union 20(2) 202 - 206
7. Wong, I., et al 2020 High-Resolution U/V Optical/IR Imaging of Jupiter in 2016 – 2019 The Astrophysics J. Supplementary Series 247 (2): 58 doi: 10.3847 / 1538 - 4365/ ab775f